

Time-Varying Risk Premium in the Czech Capital Market: Did the Market Experience a Structural Shock in 2008–2009?

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Abstract

Time-varying risk premiums and CAPM betas for several assets traded on the Prague Stock Exchange are estimated within a model which is derived as a restriction of a general stochastic discount factor model. The restriction takes the form of the Sharpe-Lintner capital asset pricing model. A time-varying risk premium for the whole market is then estimated within a restriction in the form of the Lucas-Breeden consumption-based capital asset pricing model. A multivariate GARCH-in-mean model is used to estimate the two restrictions. The estimation of the CAPM restriction seems to be favorable to the theoretical model, while the CCAPM seems to be less in accordance with the data. Models with dummies and tests of structural changes are used to show that the market experienced a significant shock in 2008–2009, but on the whole the tests do not give indisputable evidence that the shock had a lasting impact on the market.

1. Introduction

Modern financial economic theory focuses primarily on assessing the riskiness of various kinds of assets. This theoretical approach has been significantly supported by the world financial crisis. The paper presents an estimation of time-varying risk premiums of several issues traded on the Czech capital market and also of the market as a whole. Its contributions may be appreciated from two perspectives: first, dynamic CAPM and CCAPM models are estimated and the estimated coefficients are compared to the theoretical predictions of the model to assess the relevancy of the models for the data; second, with the use of dummies and structural break tests it is shown how the riskiness of capital assets and the capital market developed during the economic downturn of the Czech economy in 2008 and 2009 and whether or not the effects may be viewed as long-lasting.

In a sense, modern models of financial economics were spurred by the formulation of the consumption-based asset pricing model (CCAPM) proposed independently by Lucas (1978) and Breeden (1979). It can be shown that all preceding models of financial economics, such as the classic CAPM by Sharpe (1964), Lintner (1965), Mossin (1966), and Black (1972), the Black-Scholes model put forward by Black and Scholes (1973) and Merton (1973b), and the intertemporal capital asset pricing model by Merton (1973a), are closely related to the CCAPM. Analysis of the risk premium has been at the center of attention since the emergence of the so-called equity premium puzzle coined by Mehra and Prescott (1985) and further extended by Weil (1989). The problem of the equity premium puzzle points to the inability of standard representations of the CCAPM to fit the data, and analysis of the (time-varying) risk premium has been one of the dominant strands of research to

solve the problem. The others focus on the specification of the utility function and make use of either time inseparability, such as Epstein and Zin (1989), or habit formation, such as Constantinides (1990) and Cochrane and Campbell (1999), or make use of heterogeneous agents within the intertemporal framework introduced in this context by Constantinides and Duffie (1998). Generally, modern intertemporal economic models focus on the interaction between the financial and real economy, relating the prices of financial assets to real factors—see, for example, Cochrane (2005) for a nice review. Clearly, the economic reality of the past few years justifies such an approach.

I choose the restrictions of the stochastic discount factor model in the form of the CAPM and the CCAPM because the CAPM is a model widely used in practice (and many other practically oriented models—usually in the tradition of the arbitrage pricing theory—reflect the CAPM in some way) and the CCAPM plays a key role in structural macroeconomic modeling as a connection between real and financial markets.

I believe the CAPM/CCAPM should be estimated with time-varying risk premiums for three reasons: first, it is directly implied by the intertemporal approach; second, the results obtained can be used for further analysis of the behavior of markets, as done in this paper; and, third, it may provide one with useful information, especially when applying financial evaluation models in the short run (risk premiums and betas are the usual inputs into these models).

Čihák and Mitra (2009) analyze the impact of the crisis on both the financial and real sectors of emerging European economies. They present the increased risk in terms of higher sovereign spreads and show the dependence of higher sovereign spreads on both economic and financial risks.

Fedorova and Vaihekovski (2010) use a version of the world CAPM and relate the excess returns of some Eastern European stock markets to three sources of risk: global risk, as measured by the return of the US stock market; emerging markets risk, as measured by the aggregated emerging markets portfolio; and exchange rate risk, as measured by the trade-weighted US currency index and bilateral exchange rates against the US dollar. In the case of the Czech economy, they find dependence of stock market risk on specific emerging markets risk and exchange rate risk measured by the bilateral exchange rate. However, the results are not comparable with what is presented in this paper.

Guillaumin and Boukari (2010) use an international CAPM and estimate capital market risk for several Central European economies, which they decompose into global risk, currency risk, and local risk. The estimate for the Czech economy puts most of the weight of the total price of risk on the price of currency risk.

Borys (2011) tests the CAPM and factor-augmented CAPM in the Visegrad economies. The estimates show little support for the original CAPM. However, the estimated betas or risk premiums (not reported) are not considered as time-varying.

The question of structural changes in financial markets has long attracted the attention of researchers. The usual approach used by numerous studies, such as Jin et al. (2010), makes use of the univariate GARCH and the model is applied directly to stock indices, not individual issues. With the use of dummy variables, possible changes in the variance equation of the GARCH model are examined.

Chang-shuai and Qing-xian (2011) use a panel GARCH to assess possible structural changes in some European stock indices (the Czech market was not included). Their paper is especially relevant because it covers the recent turbulence in 2008–2009. They conclude that only one structural change took place, namely, in 2003—their tests did not find the developments during the 2008–2009 turbulence to be a structural change.

This paper is closely related to the studies by Bali (2008) and Bali and Engle (2010). Bali (2008) uses an intertemporal CAPM (Merton, 1973a) as a theoretical framework and estimates time-varying risk premiums using a multivariate GARCH-in-mean model for various portfolios taken as assets. The analysis was performed for the US stock market. He is able to derive reasonable values of the parameter of relative risk aversion, but the estimated magnitudes of the risk premiums seem to be rather low as compared to the equity premium puzzle literature (Mehra and Prescott, 1985). Bali and Engle (2010) use the same approach for individual stocks only, which, I believe, enables one to keep the analysis more attached to economic theory, as opposed to the application of GARCH models directly to stock indices or portfolios.

Chauvet and Potter (2001) pointed to the nonlinear nature of the relation between the conditional volatility of market returns and expected returns. This was analyzed by Han (2011), who distinguishes between two components of the total risk premium: one stemming from the traditional covariance between individual and market returns and one reflecting the volatility of market returns itself as another source of risk premium. He finds that the relation between the risk premium and market volatility is negative, which may distort the traditional notion of a positive relation between risk and return.

Hédi and Fredj (2010) use a multivariate GARCH-in-mean framework in the context of the US stock market and find support for structural breaks during the world financial crisis.

The rest of the paper is divided as follows. In the first part, the necessary theoretical background to the models used to estimate time-varying risk premiums is given. In the second part, the econometrical model and the data used in the estimations are presented. In the third part, the results of the analysis are presented and discussed. The key findings are summarized in the conclusion.

2. Stochastic Discount Factor Model and its Restrictions

The stochastic discount factor model is based on a fundamental pricing equation (see Smith and Wickens, 2002, Smith et al., 2003, and Cuthbertson and Nitzsche, 2004):

$$P_t = E_t[M_{t+1}X_{t+1}] \quad (1)$$

where P refers to the price of an asset at time t , E is an expectation operator, with expectations taken with respect to the information set available at time t , and M is a stochastic discount factor transforming the future pay-off X at time $t+1$ into the price at time t . Thus, the price of an asset follows a stochastic process adapted to the available information set. The idea in equation (1) can be easily expressed using gross returns:

$$1 = E_t[M_{t+1}R_{t+1}] \quad (2)$$

where R_{t+1} is the gross return on the asset between periods t and $t+1$. Applying the formula for covariance and substituting for the gross risk-free return, equation (2) can be further restated:

$$E_t(R_{t+1} - R_t^f) = -R_t^f \text{cov}_t(M_{t+1}, R_{t+1} - R_t^f) \quad (3)$$

where R_t^f refers to the gross risk-free return. If investors were risk-neutral, the expected gross return on an asset would be equal to the gross risk-free return, as the covariance term would be zero. In other words, the gross excess return would be zero. Therefore, risk aversion is reflected in the covariance term and it is obvious that negative covariance between the stochastic discount factor and gross returns (or equivalently gross excess returns) pushes the expected gross returns on an asset above the gross risk-free return. The interpretation of the covariance term depends on the exact formulation of the model. The following two specifications or restrictions of the model will be used: CAPM and CCAPM.

To derive the classic CAPM relation, the stochastic discount factor is assumed to be a linear function of the return on wealth:

$$M_{t+1} = a_t + b_t R_{t+1}^W \quad (4)$$

where R_{t+1}^W is the gross return on wealth between periods t and $t+1$. Substituting for the stochastic discount factor in (3), the result may be stated as:

$$E_t(r_{t+1} - r_t^f) = \rho_t \text{cov}_t(r_t^W - r_t^f, r_{t+1} - r_t^f) \quad (5)$$

where small letters denote simple (not gross) returns and ρ is a parameter dependent on the risk-free rate and b from (4), which are both time-variant but known at time t . From equation (5) it is clear that the excess returns on an asset, the risk premium, are generated by the covariance between the (excess) returns on wealth and the (excess) returns on the asset. This covariance is referred to as systematic risk and it is time-varying. Equation (5) can be easily restated as a beta model. For a comprehensive analysis of stochastic discount factor models, see Cochrane (2001) or Duffie (2001):

$$E_t(r_{t+1} - r_t^f) = \beta_t E_t(r_{t+1}^W - r_t^f) \quad (6)$$

The beta is expressed as the ratio of the covariance between the (excess) returns on wealth and the (excess) returns on an asset and the variance of the (excess) returns on wealth. It is straightforward that beta is just an expression for a probability limit of a regression coefficient of a model where the (excess) returns on an asset are regressed on the (excess) returns on wealth. The return on wealth is normally proxied by the return on a market portfolio. The fact that all the parameters of the model are time-varying is crucial for both correct estimation of such a model and application of the model for analysis of financial markets. Indeed, in the original formulation of the CAPM by Sharpe (1964) the model was static and, thus, all the coefficients were time-invariant. Although the CAPM restriction was derived for real returns, it will be tested in nominal terms.

The second restriction of the model in the form of the CCAPM by Lucas (1978) and Breeden (1979) is derived from the following intertemporal optimization

problem. Assume a representative agent who maximizes the current value of expected utility, which is derived from consumption:

$$\max_C E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma} \quad (7)$$

where β is a subjective discount factor dependent on the marginal rate of time preference and the intratemporal utility function takes the form of a power utility function where parameter σ refers to both the intertemporal elasticity of substitution and relative risk aversion. The maximization is subject to the restriction:

$$W_{t+1} = (1 + i_t)W_t + P_t Y_t - P_t C_t \quad (8)$$

where W refers to nominal wealth, i denotes the nominal return on wealth, P is a price index, and Y is real income. The initial value of nominal wealth is given. This is a dynamic programming problem and the corresponding solution is captured by the transversality condition and the Euler equation, which takes the form:

$$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + i_{t+1}}{1 + \pi_{t+1}} \right] \quad (9)$$

where π refers to inflation. The product of the subjective discount factor and the ratio of marginal utilities is the actual form of the stochastic discount factor in (2), while the ratio of the gross nominal return to the gross inflation rate is just the gross real return in (2). Assuming that the stochastic discount factor and real returns are jointly normally distributed, the no-arbitrage condition (3) is:

$$E_t (i_{t+1} - i_t^f) + \frac{1}{2} \text{var}_t (i_{t+1}) = \sigma \text{cov}_t (\Delta c_{t+1}, i_{t+1} - i_t^f) + \text{cov}_t (\pi_{t+1}, i_{t+1} - i_t^f) \quad (10)$$

where Δc refers to the logarithmic change in consumption. The second term on the left refers to the Jensen inequality. The risk premium is represented by the right side of (10). This means that the risk premium is given by the covariance between consumption growth and (excess) nominal returns on wealth and by the covariance between inflation and (excess) nominal returns on wealth. A risk-averse investor requires higher nominal returns at times of higher inflation. Positive covariance between consumption returns and (excess) nominal returns means that investors get higher returns when the economy is growing, which in turn means that such assets do not enable them to smooth their consumption well over time, therefore they require a higher premium on such assets. Indeed, the CAPM restriction is just a special case of the CCAPM.

3. Multivariate GARCH-in-Mean Model and Data

The appropriate test of the models requires us to estimate the time-varying covariance terms in a multi-dimensional setting. The multivariate GARCH-in-mean framework accompanied by the BEKK model for the variance-covariance matrix by Engle and Kroner (1995) is used in this paper. The mean and variance equations take the form:

$$\mathbf{x}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Lambda} \text{vech}\{\mathbf{H}_t\} + \boldsymbol{\varepsilon}_{t+1} \quad (11)$$

$$\boldsymbol{\varepsilon}_{t+1} / I_t \sim Stt(\mathbf{0}, \mathbf{H}_{t+1}) \quad (12)$$

$$\mathbf{H}_{t+1} = \boldsymbol{\Omega}'\boldsymbol{\Omega} + \mathbf{B}'\mathbf{H}_t\mathbf{B} + \mathbf{A}'\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{A} \quad (13)$$

where $\mathbf{x}$ is a vector of endogenous variables, $\boldsymbol{\mu}$ is a vector of constants, $\boldsymbol{\Lambda}$ is a matrix of coefficients whose first row is restricted to comply with the particular model and whose other elements are set to zero, *vech* is a mathematical operator which transforms the lower triangular component of matrix $\mathbf{H}$ into a vector, $\boldsymbol{\varepsilon}$ is a vector of residuals which follow the multidimensional Student's *t* distribution with time-variant variance-covariance matrix $\mathbf{H}$, $\boldsymbol{\Omega}$ is a lower triangular matrix, $\mathbf{B}$ is a diagonal matrix, and $\mathbf{A}$ is a diagonal matrix. Kočenda and Poghosyan (2010) use the multivariate GARCH-in-mean framework to estimate the time-varying risk premium in the foreign exchange market.

Vector $\mathbf{x}$ has two components in the estimation of the CAPM (excess nominal returns on an asset and excess nominal returns on the market) and three in the case of the CCAPM (excess nominal returns on the market, real consumption growth, inflation). Matrix $\boldsymbol{\Lambda}$ is 2×2 in the case of the CAPM and λ_{11} is set to zero as there is no variance term in equation (5). The matrix is 3×3 in the case of the CCAPM and λ_{11} is set to $\frac{1}{2}$ according to (10). The other restrictions of the matrices $\boldsymbol{\Omega}$, $\mathbf{B}$, $\mathbf{A}$ are in the form of lower triangularity or diagonality, which limits the number of estimated coefficients, but the restrictions themselves are not in contradiction with the theory.

In the first step of the analysis, where the time-varying risk premiums within the restriction of the SDF model in the form of the CAPM are estimated, daily data retrieved from the Patria database are used. The behavior of the Prague Stock Exchange (PSE) is captured by the evolution of its index—the PX. The following titles traded in the SPAD system are considered: CETV, ČEZ (CEZ), Erste Bank (EB), Komerční banka (KB), Orco, Philip Morris (PM), Telefónica (Tel), and Unipetrol (Uni).

Stocks traded in the SPAD system are used because this is the most liquid segment of the PSE market. This is important for the underlying theoretical model used. The vast literature on frictions in the capital market, e.g. Aiyagari and Gertler (1998), shows that in such cases the pricing equations (6) or (10) may no longer be valid, as the premium is affected by other factors related to the nature of the frictions considered. I believe that the much lower liquidity of the other segments of the Czech capital market makes it difficult to run such an analysis for all the stocks which are traded on it (in contrast to developed capital markets, where lower liquidity may be present as an episode, not a distinct feature).

The sample starts on different trading days according to the availability of the data, but for all titles the sample ends on October 31, 2011. As far as this model is concerned, there are two series which enter each model: the excess nominal return on an asset and the excess nominal return on the market. Logarithmic approximation is used to compute the returns. To calculate the excess returns a proxy for the risk-free asset is needed. The three-month Pribor is used as a proxy (r^f).

In the second step a time-varying risk premium for the whole market (PX) within the restriction of the SDF model in the form of the CCAPM is estimated. For this purpose, monthly data either computed as averages of their daily counterparts or retrieved from the Czech Statistical Office are used. Quarterly seasonally-adjusted

Table 1 The table shows the means and standard deviations of the series.

The Jarque-Bera statistic and the t -statistic of the augmented Dickey-Fuller test are given. The first seven series are related to the CAPM and are annualized daily changes. The last three series are related to the CCAPM and are annualized monthly changes.

	Median	St. dev.	JB	ADF
PX— R^f	0.07661	3.77936	22151.68***	-58.89195***
CETV— R^f	-0.209	9.59636	16426.26***	-25.16959***
CEZ— R^f	0.21992	5.30432	18895.48***	-42.65299***
EB— R^f	-0.0236	7.19669	8844.903***	-30.85869***
KB— R^f	0.16067	5.72016	18992.81***	-49.73968***
Orco— R^f	-0.22329	9.35407	13923.73***	-34.19348***
PM— R^f	0.06122	4.66002	8004.014***	-50.87319***
Tel— R^f	0.00788	5.18084	5093.949***	-56.54476***
Uni— R^f	0.11525	6.27478	10742.22***	-53.64323***
PX— R^f	0.01385	0.75701	87.41073***	-9.66658***
C	0.02099	0.04142	21.01176***	-5.51306**
π	0.0211	0.07338	1137.357***	-3.57467*

Note: *, **, *** denotes rejection of the null at the 10%, 5%, 1% level of significance, respectively.

data on real consumption (C) are used and the series is transformed into monthly frequency. The transformation was carried out using a quadratic polynomial (to transform the quarterly series into monthly series, a quadratic polynomial is fit for each set of three consecutive points from the quarterly series and then used to fill in the missing points in the monthly series for that period so that the average of the interpolated high-frequency points matches the actual low-frequency values). The CPI index is used to compute inflation (π).

The CCAPM model is not estimated for individual assets because relevant data for the consumption series are not available. The model requires that asset demand and consumption be in accordance with each other, i.e., the consumption series representing the whole economy should be related to the capital asset capturing the whole economy—the market index as a proxy. When using the CCAPM for an individual asset and the consumption series for the whole economy one would have to assume that the demand for the asset is equally diversified as the demand for the index.

We work with annualized daily or monthly changes. *Table 1* gives the basic characteristics of the series. We can see that some of the assets considered in the analysis realized negative excess returns on average. The distribution of all the series does not follow normal distribution (the null of normal is rejected at the 1% level by the standard Jarque-Bera test) and all of them are stationary (the null of unit root is rejected at the 1% level by the standard ADF test, except for consumption and inflation, where it was rejected at the 5% and 10% level, respectively).

4. Results

First we give the results of the bivariate GARCH-in-mean models, which were used to estimate the time-varying risk premiums and betas for the selected assets. Then the estimate of the trivariate GARCH-in-mean model and the estimate

Table 2 Bivariate GARCH-in-mean estimates.

Multivariate Student's t distribution used for residuals. μ s are the intercepts of the mean equation. λ (element of Λ) is the in-mean effect: the effect of the covariance between the returns on the asset returns. Ω s are the elements of the lower triangular matrix of constants in the conditional moment equation. β s are elements of the diagonal matrix $\mathbf{B}$ relating present and forecasted variance-covariance matrices. α s are elements of the diagonal matrix $\mathbf{A}$ capturing the effects of shocks (residuals) on the forecasted variance-covariance matrix.

	CETV	CEZ	EB	KB	Orco	PM	Tel	Uni
$\mu(1)$	0.08646 (0.14467)	0.24802 (0.19626)	0.15622 (0.09882)	0.19017 (0.17288)	0.05381 (0.13743)	0.11812 (0.09153)	0.12300 (0.09834)	0.14189* (0.08643)
$\mu(2)$	0.11911* (0.06283)	0.16037*** (0.05150)	0.12229* (0.06372)	0.20983*** (0.05319)	0.14942** (0.07497)	0.32732*** (0.06366)	0.22114*** (0.04941)	0.18790*** (0.05157)
$\lambda(2)$	0.00181 (0.00131)	0.00113* (0.00061)	0.00393** (0.00169)	0.00356* (0.00205)	0.00286 (0.00308)	0.01219*** (0.00459)	0.00619** (0.00256)	0.00205* (0.00114)
$\Omega(1)$	0.91139*** (0.06100)	0.84326*** (0.04148)	0.59815*** (0.04449)	1.12965*** (0.03489)	0.75153*** (0.04360)	1.20246*** (0.05687)	0.67939*** (0.01908)	1.17717*** (0.03919)
$\Omega(2)$	0.42079*** (0.06056)	0.35258*** (0.02504)	0.38828*** (0.04054)	0.40443*** (0.02585)	0.14214*** (0.04709)	0.20012*** (0.02226)	0.29435*** (0.02658)	0.26840*** (0.02530)
$\Omega(3)$	0.46049*** (0.05749)	0.31858*** (0.02070)	0.32883*** (0.02651)	0.40689*** (0.02665)	0.44365*** (0.04302)	0.51729*** (0.04090)	0.33533*** (0.01976)	0.48697*** (0.03418)
$\beta(1)$	0.95697*** (0.00280)	0.94055*** (0.00333)	0.95420*** (0.00273)	0.92419*** (0.00250)	0.95497*** (0.00263)	0.94364*** (0.00474)	0.93696*** (0.00197)	0.90060*** (0.00373)
$\beta(2)$	0.89916*** (0.00826)	0.94002*** (0.00331)	0.93620*** (0.00429)	0.93437*** (0.00355)	0.93242*** (0.00543)	0.92739*** (0.00572)	0.93694*** (0.00328)	0.92926*** (0.00397)
$\alpha(1)$	0.27076*** (0.00952)	0.30495*** (0.00959)	0.28557*** (0.00875)	0.34275*** (0.00640)	0.29188*** (0.01043)	0.21010*** (0.00939)	0.34369*** (0.00624)	0.40202*** (0.00827)
$\alpha(2)$	0.42282*** (0.01741)	0.32382*** (0.00910)	0.32559*** (0.01101)	0.32526*** (0.00824)	0.34994*** (0.01413)	0.34373*** (0.01311)	0.33813*** (0.00857)	0.34657*** (0.00928)
AC	12.90574*	8.98178	7.02811	8.99136	20.26130*	10.82727*	7.18177	8.12808
ARCH	32.21004*	15.72189	21.00874*	14.22010	32.20771*	14.90992	13.00180	13.10448
Log-L	-9238.553	-16404.56	-9555.757	-18093.86	-9727.633	-15096.7	-17192.2	-18475.32
AIC	11.64072	10.30418	10.42021	10.73140	11.64789	10.89556	10.22419	10.97052

Notes: AC: multivariate autocorrelation test—the Q-statistic for the first lag is reported. ARCH: White heteroskedasticity test—reported for the first lag.

*, **, *** denotes rejection of the null at the 10%, 5%, 1% level of significance, respectively.

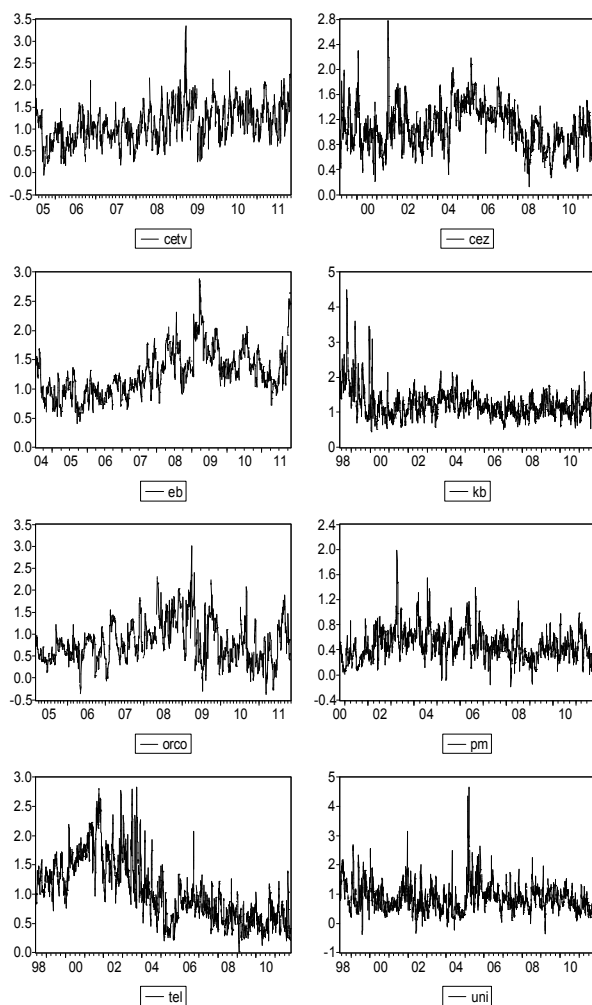
of the time-varying risk premium for the whole market are given. Attention is then turned to the stability of the estimated parameters and the question of possible structural changes.

4.1 CAPM, Betas and Risk Premiums

Table 2 presents the estimates of the bivariate GARCH-in-mean models. The estimates of the variance equation in all of the models are quite robust. As far as the mean equation is concerned, acceptable results were achieved in the cases of CEZ, EB, KB, PM, Tel, and Uni. Some of the models' residuals display some remaining autocorrelation and heteroskedasticity.

The estimate of $\mu(1)$ should be insignificant according to the theoretical model used (all the risk should be priced in accordance with the market risk). We can see

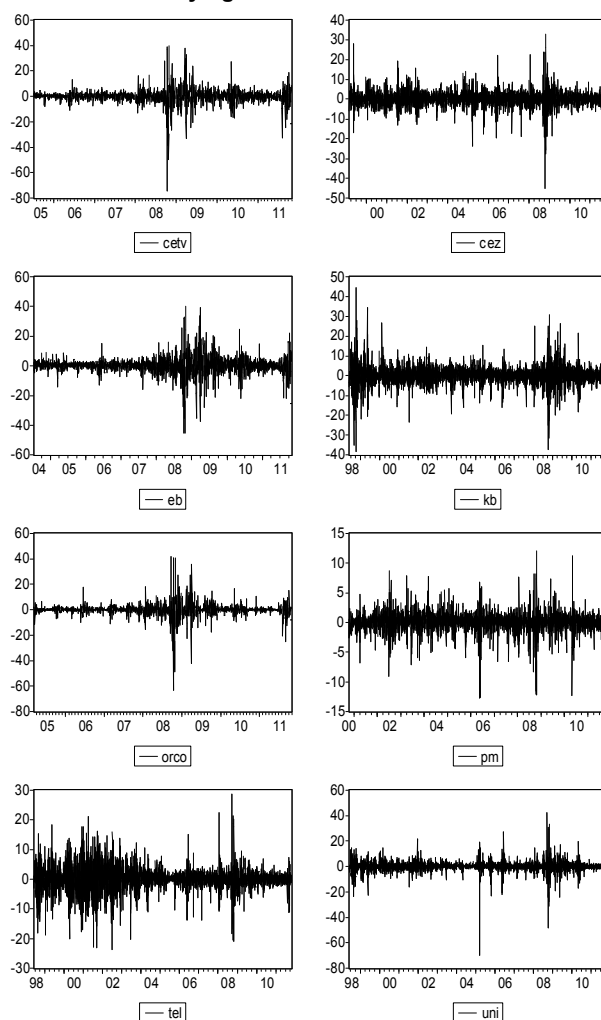
Figure 1 Estimates of Time-Varying CAPM Betas



that the estimates are insignificant in all cases but Uni. The estimate of $\lambda(2)$ measures the average influence of the covariance between the asset and the market. Remember that $\lambda(1)$ was set to zero. According to the theory, the estimates of $\lambda(2)$ should be positive.

Figure 1 presents the estimated time-varying CAPM beta coefficients for the selected assets. The calculations of the betas were carried out according to the Sharpe-Lintner CAPM model (6) and were based on the estimates of the bivariate GARCH-in-mean models. The variability of the betas is rather significant and their average values are: 1.09 (CETV); 1.05 (CEZ); 1.27 (EB); 1.17 (KB); 0.82 (Orco); 0.45 (PM); 0.96 (Tel); 0.81 (Uni). The estimates do not differ too much from one in most cases. The lower estimate in the case of PM may be explained by the specificity of its production.

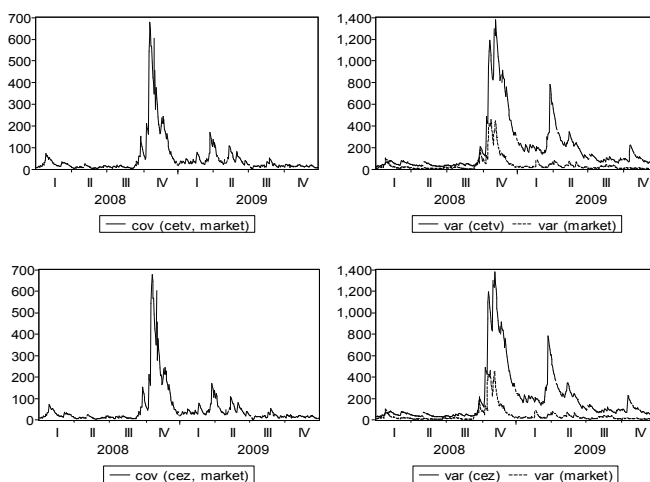
Figure 2 Estimates of Time-Varying CAPM CAPM Risk Premiums



Following up on the betas, the time-varying risk premiums based on the Sharpe-Lintner interpretation of the CAPM were calculated. The median risk premiums for the respective titles were: -11.5% (CETV); 15.4% (CEZ); -0.01% (EB); 15.4% (KB); 3.7% (Orco); 6.7% (PM); 0.9% (Tel); 7.7% (Uni). *Figure 2* presents the results. It is obvious that the volatility of the risk premiums significantly increased in the period of turbulence. This is partially due to the increased volatility of market excess returns and also due to changes and higher volatility of the betas in some cases. The risk premiums take both positive and negative values. The reason why risk premiums may be negative in this estimation is simple: with positive betas a negative excess return on the market index must translate into a negative risk premium. Comparing the estimated medians of the risk premiums with the medians of the excess returns from *Table 1*, which are: -20.9% (CETV); 22.0% (CEZ); -2.4% (EB); 16.1% (KB); -22.3% (Orco); 6.1% (PM); 0.8% (Tel); 11.5% (Uni), we can see that the model is clearly insufficient in the cases of CETV and Orco. It should be noted, however, that in these two cases the sample was the shortest used in the analysis.

Finally we present the estimated covariances between the excess returns and the excess market returns and the respective variances, which are the primary output of the estimations. The results are given in *Figure 3*. In all cases it is obvious that the variances of the assets' returns and the covariances between the assets' returns and the excess market returns increased during the period of turbulence; the key shock took place at the end of the third quarter of 2008, which corresponds to the external shock spurred by the fall of Lehman Brothers. The increase in the variances of the individual assets was significantly higher than the increase in the variability of the market returns in all cases. The increases in the covariances were not in line with the increases in the variances of the market returns, which translated into temporary changes in the betas, as documented in *Figure 1*. The increased volatility of the market returns channeled into increases in the risk premiums and, especially, increases in their volatilities, as seen in *Figure 2*.

Figure 3 Time-Varying Covariances and Variances of Bivariate GARCH-in-Mean Models.
Period of 2008–2009 is displayed only for the sake of clarity.



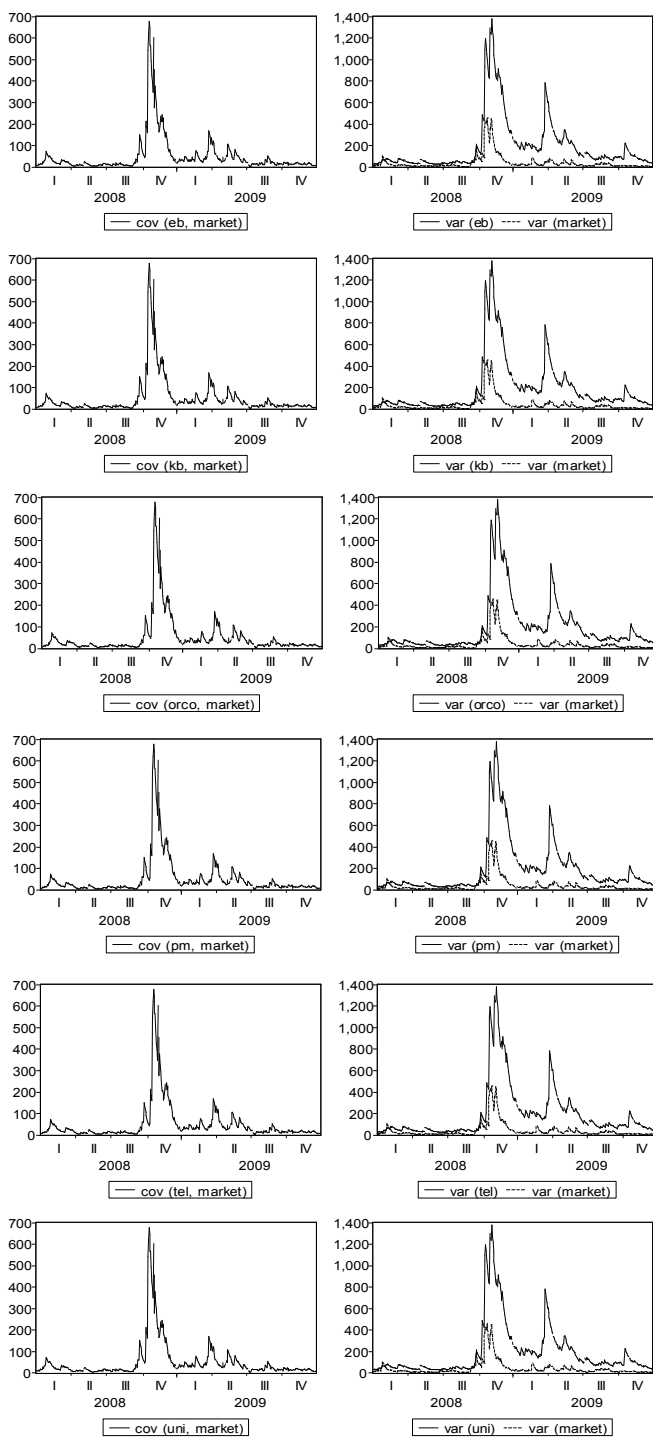


Table 3 Trivariate GARCH-in-mean estimates.

Multivariate Student's t distribution used for residuals. μ s are the intercepts of the mean equation. λ s (elements of Λ) are the in-mean effects: the effects of the covariance between the returns and consumption and the covariance of the returns and inflation on the market returns. Ω s are elements of the lower triangular matrix of constants in the conditional moment equation. β s are elements of the diagonal matrix $\mathbf{B}$ relating present and forecasted variance-covariance matrices. α s are elements of the diagonal matrix $\mathbf{A}$ capturing the effects of shocks (residuals) on the forecasted variance-covariance matrix.

PX			
$\mu(1)$	0.16689* (0.10831)	$\Omega(5)$	0.00925 (0.00726)
$\mu(2)$	0.00957* (0.00529)	$\Omega(6)$	0.02947*** (0.00911)
$\mu(3)$	-0.02502 (0.04698)	$\beta(1)$	0.60535*** (0.13872)
$\lambda(2)$	2.35467* (1.22155)	$\beta(2)$	-0.00136 (0.25824)
$\lambda(3)$	15.23573* (8.23922)	$\beta(3)$	0.87453*** (0.07784)
$\Omega(1)$	0.47218*** (0.10449)	$\alpha(1)$	0.56144*** (0.06149)
$\Omega(2)$	0.00258 (0.00472)	$\alpha(2)$	0.79664*** (0.10445)
$\Omega(3)$	0.00379 (0.00335)	$\alpha(3)$	0.25026** (0.10190)
$\Omega(4)$	0.03039*** (0.00221)		
AC	9.72178	ARCH	86.15704
Log-L	354.259	AIC	-3.62644

Notes: AC: multivariate autocorrelation test—the Q-statistic for the first lag is reported. ARCH: White heteroskedasticity test—reported for the first lag.

*, **, *** denotes rejection of the null at the 10%, 5%, 1% level of significance, respectively.

4.2 CCAPM, Market Risk Premium

Next, the estimate of the trivariate GARCH-in-mean model is presented. The model was used to estimate the restriction of the underlying SDF model in the form of the CCAPM. The estimates of the model are given in *Table 3*.

The first two intercepts of the mean equation (coefficients μ) bear at least a 10% significance level. The estimated contributions of consumption growth and inflation to the risk premium (coefficients λ ; remember that $\lambda(1)$ was set at 0.5) have the expected signs and are statistically significant. The variance equation does not have such a high level of significance as in the cases of the bivariate models, but given the length of the series and the number of estimated coefficients this comes as no surprise.

Next, *Figure 4* presents the time-varying covariances between the nominal excess market returns and real consumption growth, the nominal excess market returns and inflation, and real consumption growth and inflation.

Between the years 2008 and 2009 there was an increase in both the covariance between the excess returns and consumption growth and the covariance between

Figure 4 Time-Varying Covariances and Variances of Trivariate GARCH-in-Mean Model

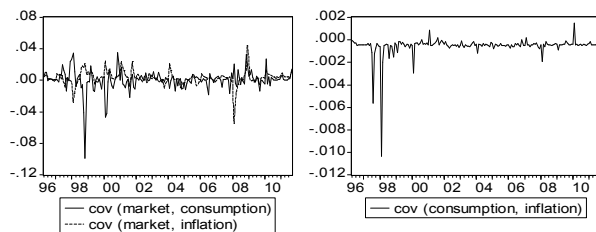
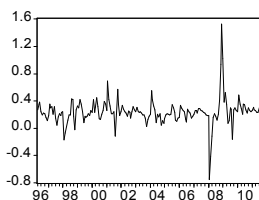


Figure 5 Time-Varying Risk Premium for Whole Market



the excess returns and inflation. This should translate into an increased risk premium in that period. The estimate of the time-varying risk premium for the market is given in *Figure 5*. The median of the estimate is 9.6%.

Fernandez et al. (2011) report an average market risk premium for the Czech economy at the level of 6.1%, with a maximum of 8.0%. Their analysis is based on a survey among academic economists, professional economists, and managers.

An abrupt decline and increase in the value of the premium, which was influenced by the overall volatility of the risk premium, is clearly visible in *Figure 5*.

4.3 Stability of the Estimated Models: Was There a Structural Shock?

To assess the stability of the results, the models were also estimated with dummies taking effect from August 2008, which was chosen on the grounds of an analysis of the index behavior (decline and increased variability) and the individual results from the bivariate models for the individual assets. *Tables 4* and *5* present the results.

Table 4 reports that the first dummy, $d(1)$, which directly affects the individual returns, was not found to be significant in any cases. The second dummy, $d(2)$, directly affecting the market returns, was weakly significant in six out of eight cases. The estimates of the parameters of the variance-covariance equation show a high level of stability; they stayed virtually unchanged. The estimates of $\lambda(2)$, the sensitivity of the individual returns to the covariance between the individual and market returns, changed rather negligibly. The most significant changes can be observed in the cases of the intercept coefficients. The estimates of parameter $\mu(1)$ stayed insignificant (except for Uni), in line with the underlying theoretical model.

The same analysis for the trivariate model reveals that two of the three dummies were significant. Apart from changes of the estimates in the mean equation, some noticeable changes of the estimates of α s are reported.

Table 4 Bivariate GARCH-in-mean estimates.

Multivariate Student's t distribution used for residuals. d_s are dummies in the mean equation, taking effect from August 2008. μ_s are the intercepts of the mean equation. λ (element of Λ) is the in-mean effect: the effect of the covariance between the returns on the asset returns. Ω_s are elements of the lower triangular matrix of constants in the conditional moment equation. β_s are elements of the diagonal matrix $\mathbf{B}$ relating present and forecasted variance-covariance matrices. α_s are elements of the diagonal matrix $\mathbf{A}$ capturing the effects of shocks (residuals) on the forecasted variance-covariance matrix.

	CETV	CEZ	EB	KB	Orco	PM	Tel	Uni
$d(1)$	-0.22264 (0.29524)	-0.42116 (0.36605)	0.01713 (0.20943)	-0.10689 (0.18478)	-0.11221 (0.27438)	0.20596 (0.18203)	-0.74789 (0.13359)	-0.07302 (0.18596)
$d(2)$	-0.19372* (0.10461)	-0.19873* (0.11232)	-0.23268* (0.12749)	-0.14137* (0.07177)	-0.19860 (0.13071)	-0.22169* (0.11951)	-0.15466* (0.09169)	-0.18149 (0.11596)
$\mu(1)$	0.15887 (0.18847)	0.37866 (0.28839)	0.16234 (0.12088)	0.21199 (0.19764)	0.10463 (0.16863)	0.06160 (0.10733)	0.14819 (0.10862)	0.16016* (0.08565)
$\mu(2)$	0.19372** (0.09653)	0.22136*** (0.05765)	0.22041*** (0.08159)	0.23239*** (0.05963)	0.21867** (0.09409)	0.36078*** (0.06859)	0.24864*** (0.05426)	0.22129*** (0.05667)
$\lambda(2)$	0.00265 (0.00227)	0.00159* (0.00062)	0.00514*** (0.00178)	0.00324* (0.00211)	0.00367 (0.00292)	0.01012** (0.00466)	0.00616** (0.00257)	0.00159* (0.00081)
$\Omega(1)$	0.91152*** (0.06140)	0.84083*** (0.04139)	0.59899*** (0.04448)	1.12898*** (0.03495)	0.74922*** (0.04352)	1.21426*** (0.05791)	0.68564*** (0.01991)	1.17172*** (0.03907)
$\Omega(2)$	0.42667*** (0.06220)	0.35023*** (0.02500)	0.39072*** (0.04091)	0.40384*** (0.02589)	0.14108*** (0.04746)	0.20087*** (0.02218)	0.29646*** (0.02684)	0.26673*** (0.02528)
$\Omega(3)$	0.46029*** (0.05789)	0.31670*** (0.02058)	0.32828*** (0.02643)	0.40568*** (0.02665)	0.44412*** (0.04366)	0.51548*** (0.04112)	0.33622*** (0.01988)	0.48671*** (0.03404)
$\beta(1)$	0.95716*** (0.00279)	0.94131*** (0.00330)	0.95396*** (0.00275)	0.92433*** (0.00250)	0.95507*** (0.00264)	0.94298*** (0.00484)	0.93672*** (0.00206)	0.90107*** (0.00371)
$\beta(2)$	0.89907*** (0.00829)	0.94029*** (0.00329)	0.93580*** (0.00431)	0.93439*** (0.00356)	0.93228*** (0.00548)	0.92719*** (0.00578)	0.93676*** (0.00333)	0.92921*** (0.00398)
$\alpha(1)$	0.26996*** (0.00950)	0.30405*** (0.00961)	0.28559*** (0.00892)	0.34242*** (0.00640)	0.29160*** (0.01047)	0.20974*** (0.00941)	0.34395*** (0.00643)	0.40131*** (0.00824)
$\alpha(2)$	0.42266*** (0.01732)	0.32353*** (0.00909)	0.32682*** (0.01098)	0.32543*** (0.00827)	0.35029*** (0.01418)	0.34453*** (0.01317)	0.33847*** (0.00864)	0.34688*** (0.00928)
AC	12.52245*	8.80027	6.98991	8.84902	21.10668*	10.61120*	7.16627	8.12455
ARCH	31.18477*	15.0114	19.52438*	14.21714	32.53552*	14.81005	12.90885	13.10061
Log-L	-9225.102	-16400.9	-9542.285	-18093.09	-9726.497	-15082.99	-17186.79	-18469.05
AIC	11.64830	10.30314	10.41907	10.73212	11.64892	10.89498	10.22520	10.97124

Notes: AC: multivariate autocorrelation test—the Q-statistic for the first lag is reported. ARCH: White heteroskedasticity test—reported for the first lag.

*, **, *** denotes rejection of the null at the 10%, 5%, 1% level of significance, respectively.

Table 5 Trivariate GARCH-in-mean estimates.

Multivariate Student's t distribution used for residuals. d s are dummies in the mean equation, taking effect from August 2008. μ s are the intercepts of the mean equation. λ s (elements of Λ) are the in-mean effects: the effects of the covariance between the returns and consumption and the covariance of the returns and inflation on the market returns. Ω s are elements of the lower triangular matrix of constants in the conditional moment equation. β s are elements of the diagonal matrix B relating present and forecasted variance-covariance matrices. α s are elements of the diagonal matrix A capturing the effects of shocks (residuals) on the forecasted variance-covariance matrix.

PX			
$d(1)$	-0.04339 (0.17930)	$\Omega(3)$	0.00273 (0.00339)
$d(2)$	-0.03411** (0.01105)	$\Omega(4)$	0.03109*** (0.00212)
$d(3)$	-0.02725* (0.01595)	$\Omega(5)$	0.01382* (0.00825)
$\mu(1)$	0.16572* (0.09604)	$\Omega(6)$	0.02720* (0.01463)
$\mu(2)$	0.02142** (0.00675)	$\beta(1)$	0.61164*** (0.13779)
$\mu(3)$	-0.02248 (0.02489)	$\beta(2)$	-0.00209 (0.01622)
$\lambda(2)$	3.71789* (2.14759)	$\beta(3)$	0.88441*** (0.10359)
$\lambda(3)$	16.44990* (9.25456)	$\alpha(1)$	0.57258*** (0.07269)
$\Omega(1)$	0.46004*** (0.10287)	$\alpha(2)$	0.68369*** (0.12169)
$\Omega(2)$	0.00195 (0.00458)	$\alpha(3)$	0.21020* (0.12457)
AC	9.76062	ARCH	85.01127
Log-L	367.129	AIC	-3.73258

Notes: AC: multivariate autocorrelation test—the Q-statistic for the first lag is reported. ARCH: White heteroskedasticity test—reported for the first lag.

*, **, *** denotes rejection of the null at the 10%, 5%, 1% level of significance, respectively.

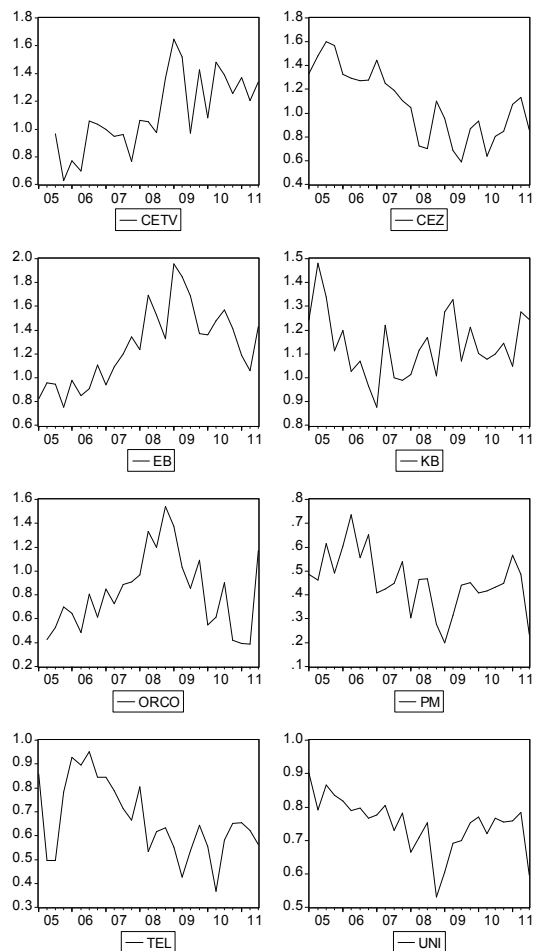
The analysis points to a rather low probability of significant structural change with a lasting impact on the market. This conclusion is based on the estimation of the CAPM models, which are clearly more statistically significant.

4.4 Stability of the Estimated Parameters: Was There a Structural Shock?

Next, we focus more on the properties of the estimated time-varying betas and risk premiums within the CAPM. *Figures 6* and *7* present the quarterly averages of the series on which the tests were carried out. The sample starts in 2005, so that all issues are included and the examined period 2008–2009 is pinned down.

Figure 6 shows that some of the estimates of the time-varying betas may display trend behavior (nonstationarity), but this can hardly be attributable to the 2008–2009 turbulence. On the other hand, *Figure 7* reveals a common pattern in the estimated time-varying risk premiums in the second half of 2008: a rapid decrease and increase of the estimated risk premiums. This is in line with the estimated

Figure 6 Quarterly Averages of Estimated Time-Varying Betas



time-varying risk premium for the whole market captured in *Figure 5*. The application of ADF tests to the time-varying betas shows that they are mostly non-stationary (with the exceptions of CETV, KB, and Uni). On the other hand, the time-varying risk premiums are found to be stationary. Because of the possibility of a structural break, stationarity is evaluated using the test by Zivot and Andrews (1992), which allows for one undefined break in the data. The test was applied to both of the series depicted in *Figures 6* and *7* and the results are reported in *Table 6*.

The results for the risk premiums show that a break was located in 2009Q2 in all cases but PM, where it was located in 2009Q1. The series of risk premiums are mostly stationary with a break, except for CETV and PM (here, the *t*-statistic almost reaches the value for 10% significance). When applied so that a break only in the trend of the risk premium series was allowed, the results (not reported) showed that the series were not stationary (the null was not rejected in any cases). It follows

Figure 7 Quarterly Averages of Estimated Time-Varying Risk Premiums

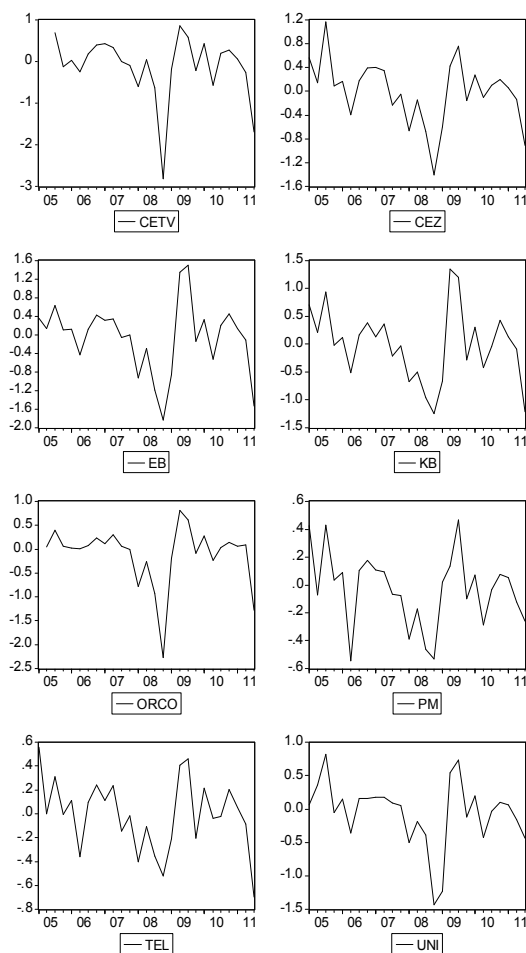


Table 6 Zivot-Andrews unit root tests.

A break was allowed in both the trend and the intercept. Lags of differences chosen by AIC. The period when the shock was detected by the test is reported. The *T*-statistic for the test is reported. The null is that the series exhibits a unit root without a break. The alternative is trend stationarity with a break.

	CETV	CEZ	EB	KB	ORCO	PM	TEL	UNI
beta	2008Q4	2009Q3	2009Q1	2007Q2	2008Q2	2007Q1	2007Q2	2009Q3
	-6.00009***	-3.89912	-5.45142**	-5.74603***	-4.08402	-4.57799	-5.03713*	-6.17798***
risk premium	2009Q2	2009Q2	2009Q2	2009Q2	2009Q2	2009Q1	2009Q2	2009Q2
	-4.54902	-5.41257**	-4.90459*	-5.80572***	-4.86899*	-4.77877	-5.29876**	-5.14070**

Notes: *, **, *** denotes rejection of the null at the 10%, 5%, 1% level of significance, respectively.

that the reported breaks in the risk premium series in *Table 5* were due to shifts in the intercepts. In the case of the betas, a break that corresponds to the period covered by the dummies in the previous exercise was located in only two cases.

5. Conclusions

In this paper we presented estimates of a time-varying capital asset pricing model and a consumption-based capital asset pricing model for the most liquid segment of the Czech capital market. The estimates of the CAPM for most of the sample came out as reasonable with respect to both the estimated coefficients of the model and the estimated time-varying betas and risk premiums. The estimates were mostly statistically significant with acceptable behavior of the residuals. The estimates indicate that sufficiently long series are necessary to carry out such an analysis. The estimate of the CCAPM for the market as a whole shows less reliability, but again it must be noted that the use of multivariate GARCH-in-mean models requires longer time series, as a large number of coefficients are estimated.

The estimates showed increased volatility of the covariances between the returns of the particular titles and market returns and also between market returns and the real economy in the period of financial market turbulence. This directly translated into increased volatility of the risk premiums in that period. The estimates of the CAPM restriction of the SDF model show that for some titles the sensitivity to systematic risk—beta—temporarily changed in that period. However, no permanent or systematic changes in the sensitivity to market returns were detected. The increased volatility of systematic risk also seemed to be transitory.

The application of dummy variables in the GARCH-in-mean models showed that from the point of view of individual assets there is little support for possible structural changes in the 2008–2009 period: the coefficients of the variance-covariance equation showed high stability and the dummy variables were only weakly significant. Some support came from the trivariate GARCH-in-mean as applied to the index, but the statistical validity of this model is much lower as compared to the bivariate versions, which were applied to the individual stocks.

A unit root test with the possibility of a break showed a break in the risk premium series of the individual assets in most cases due to a change in intercept. The same test applied to the betas showed much lower support for the possibility of a structural break.

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