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The Annual Inflation Rate and Inflation Targeting: A Different Perspective

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Abstract

Central banks use annual inflation-targeting regimes to achieve and maintain price stability. However, the annual inflation rate, considered as a moving average filter of the annualized inflation rate, shows undesirable statistical characteristics, namely the time delay and spurious cycles. The source of the cyclical process are the jumps and the seasonality in the ln CPI and, subsequently, that of the annualized inflation rate. Therefore, the annual inflation cycles cannot be directly influenced and adjusted, and the commonly pursued inflation-targeting policies are ineffective. Another area for improvement is methodological. The banking authorities target the slope estimate of the linear deterministic CPI trend model, but they should target the slope parameter. But this approach also has weak points.

1. Introduction

Over the last three decades, central banks in industrialized, developing, and emerging market economies have adopted inflation-targeting policies to control price level growth, thereby maintaining price stability. An extensive body of academic literature analyzes this monetary strategy from various economic and financial perspectives.

In this paper, the issue of inflation targeting (IT) is approached unconventionally, marking the shift in focus from generic economic concepts to statistical and econometric aspects. The paper follows Arlt (2023), who analyzes the properties of the annual inflation rate indicator in detail. The annual inflation rate can be understood as a one-sided moving sum of the monthly inflation rate or as a moving average of the annualized inflation rate, its features being the subject of long-term research. Koopmans (1974) indicates through the decomposition of the frequency response function that a one-sided simple moving average has a non-zero phase function representing its lag of half the length of this moving average. It follows that the information in the annual inflation rate is misinterpreted as it is assigned to the last date of the moving window. However, it represents the entire window and should be thus assigned to its middle date. The same result is drawn, for example, by Oppenhaim and Schafer (1989), Ladiray and Quenneville (2001), or Dagum and Bianconcini (2016). Slutzky (1937) and

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Youle (1927) showed that both weighted and unweighted sums and averages of purely random numbers produce new time series with spurious cyclical behavior reminiscent of the macroeconomic business cycle. The annual inflation rate also implies the risk of spurious cyclicality, which does not reflect the actual inflationary behavior.

The results mentioned above may lead to the misinterpretation of inflation trends and central banks´ erroneous monetary policy decisions that affect the entire economy. The present study aims to address this issue consistently.

Framed by an introduction and conclusion, the article consists of three main sections (2–4). Section 2 sets out the IT process, briefly describing how it works. Section 3 goes into the properties of the annual inflation rate. First, the time delay is detected, then an actual data-based model is constructed, and stationarity and non-invertibility are finally assessed. In this context, the spurious cyclical behavior of the annual inflation rate, manifested in increased persistence, is justified. Building on the insights from the previous parts, Section 4 explains why the current annual inflation targeting regime cannot work effectively and proposes an alternative approach.

2. Inflation Targeting

The first country to introduce inflation targeting in response to high inflation in the 1970s was New Zealand in 1989. IT then spread to other countries; for example, Canada adopted the IT regime in 1991, the UK in 1992, Sweden, Finland, Australia in 1993, and the Czech Republic in 1997. The first inflation targeters were advanced economies, developing and emerging-market economies having adopted IT since 1997. The IT principle, as described, for example, by Bernanke et al (1998), Hammond (2001), Truman (2003), Svensson (2010), Niedźwiedzińska (2023) and others, is simple and straightforward. The central bank creates forecasts of annual inflation rate development, comparing it with an expressly set goal, i.e., with the target inflation rate, which is considered appropriate for the economy. The difference between the forecast and the target determines the necessary adjustment of monetary policy so that the actual inflation approaches the target in the future.

The principal concern is the level and form of the target inflation rate. The inflation targets differ from country to country. Inflation targets are usually set slightly above zero because, according to Hammond (2011), measured inflation tends to overstate actual inflation. Another reason is, for example, that a positive target decreases the risk of deflation. According to Hammond (2011), the consensus seems to be accepted that the year-on-year price level increase above 3%–4% imposes higher welfare costs, and below 2%, the plausible gains from reducing inflation are unlikely to outweigh the advantages of a positive inflation target. Lockyer (2022) or Adam and Hennig (2020) deal with determining the optimal amount of the inflation target. All central banks use the headline Consumer Prices Index (HCPI) to measure the price level, with monthly data available. Most banks set point targets with symmetrical tolerance bands; others identify only the point targets or target ranges. Depending on the country, the inflation target time horizon is usually intermediate, i.e., two or more years; in other cases, it ranges either from 12 (exceptionally) to 18 or from 18 to 24 months.

Monetary policy committees vary in size, staffing, and organization across countries, and they hold regular meetings, usually 8–12 times a year. Setting optimal interest rates to reach the inflation targets is "technically" supported by the departments creating economic models for inflation and output forecasting. Central banks use, e.g., ARIMA, VAR, ECM, and DSGE models, both theoretically- and data-based.

3. Annual Inflation Rate and its Properties

This part is based on the article by Arlt (2023), which deals in detail with the justification of the time shift and the issue of the spurious cycling of the annual inflation rate. This section describes the main conclusions of this article to the extent necessary for understanding the principle of analysis and criticism of inflation targeting, presented in the following section.

Based on the monthly time series of the Consumer Price Index (CPI), three inflation rates are computed, monthly, annualized, and annual.

In calculating inflation rates, there are options for assuming either the geometric or the exponential growth of CPI. In statistical and economic practice, the assumption of regular geometric growth $p_t = p_{t-1}(1+\pi_t^{gm})$ is used for the computation of the monthly inflation rate in the form

$$\pi_t^{gm} = \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{p_t}{p_{t-1}} - 1, \tag{1}$$

where p_t is the CPI in month t. Similarly, the annual geometric growth $p_t = p_{t-12}(1+\pi_t^{ga})$ is the basis for the computation of the annual inflation rate, i. e.

$$\pi_t^{ga} = \frac{p_t - p_{t-12}}{p_{t-12}} = \frac{p_t}{p_{t-12}} - 1. \tag{2}$$

Then, the restriction for the annual inflation rate is given by a non-linear relationship

$$\pi_t^{ga} = \left[\prod_{i=0}^{11}(1+\pi_{t-i}^{gm})\right] - 1. \tag{3}$$

In the econometric modeling, the assumption of the regular exponential growth $p_t = p_{t-1}e^{\pi_t^m}$ is used for the computation of the monthly inflation rate of the form

$$\pi_t^m = \ln p_t - \ln p_{t-1} \tag{4}$$

and from the annual exponential growth $p_t = p_{t-12}e^{\pi_t^a}$ the annual inflation rate is computed, i. e.

$$\pi_t^a = \ln p_t - \ln p_{t-12} \,. \tag{5}$$

Then, the restriction for the annual inflation rate is given by a linear relationship

$$\pi_t^a = \sum_{i=0}^{11} \pi_{t-i}^m \,. \tag{6}$$

From the definition of Euler´s Number and Binomial Theorem, it follows that exponential growth can be expressed as

$$p_t = p_{t-1}e^{\pi_t^m} = p_{t-1}\left(1 + \pi_t^m + \frac{\left(\pi_t^m\right)^2}{2!} + \frac{\left(\pi_t^m\right)^3}{3!} + \ldots\right). \tag{7}$$

Comparing this relationship with the formula for geometric growth, it is evident that for a monthly inflation rate of several percentage points, the difference between these two growths is negligible because the third and other elements in the formula (7) parentheses are extremely small. The same holds for the annual growths.

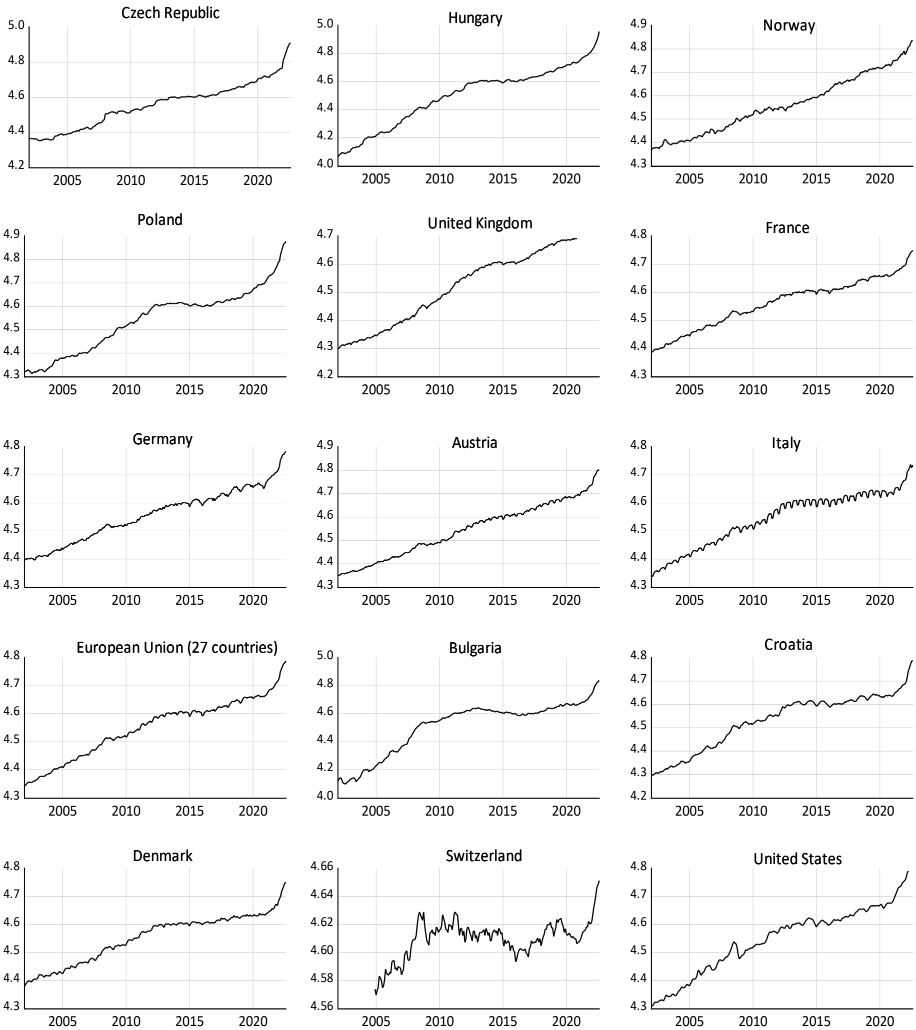
It is obvious why rates based on exponential growth are used in this article. It is the restriction (6), meaning that the annual inflation rate is a simple sum of twelve monthly inflation rates. This considerably simplifies econometric modeling (e.g., Tsay (2005)). The most practically applied econometric models are based on the linearity assumption. At the same time, the monthly and annual inflation rates based on the exponential growth are almost identical with those based on the geometric one, or the values are copied at a not great distance when extremely high. There is no loss or addition of information, distortion, or new cycle.

The monthly inflation rate based on exponential growth (as the monthly inflation rate based on geometric growth) removes both the stochastic and deterministic trends from the CPI, assuming that it is generated by a random walk process with a drift. When the level of inflation over one year is required, the annualized inflation rate in the form $\pi_t^{ma} = 12\pi_t^m$ is used. Like the monthly inflation rate, it includes strong seasonal and non-systematic components, which make the interpretation of the course of inflation somewhat tricky. The annual inflation rate can be expressed also as

$$\pi_t^a = \frac{1}{12}\sum_{i=0}^{11} \pi_{t-i}^{ma} \,. \tag{8}$$

The formulas (6) and (8) are a one-sided moving sum of twelve monthly inflation rates and a one-sided moving average of twelve annualized inflation rates set at the end of the calculation period.

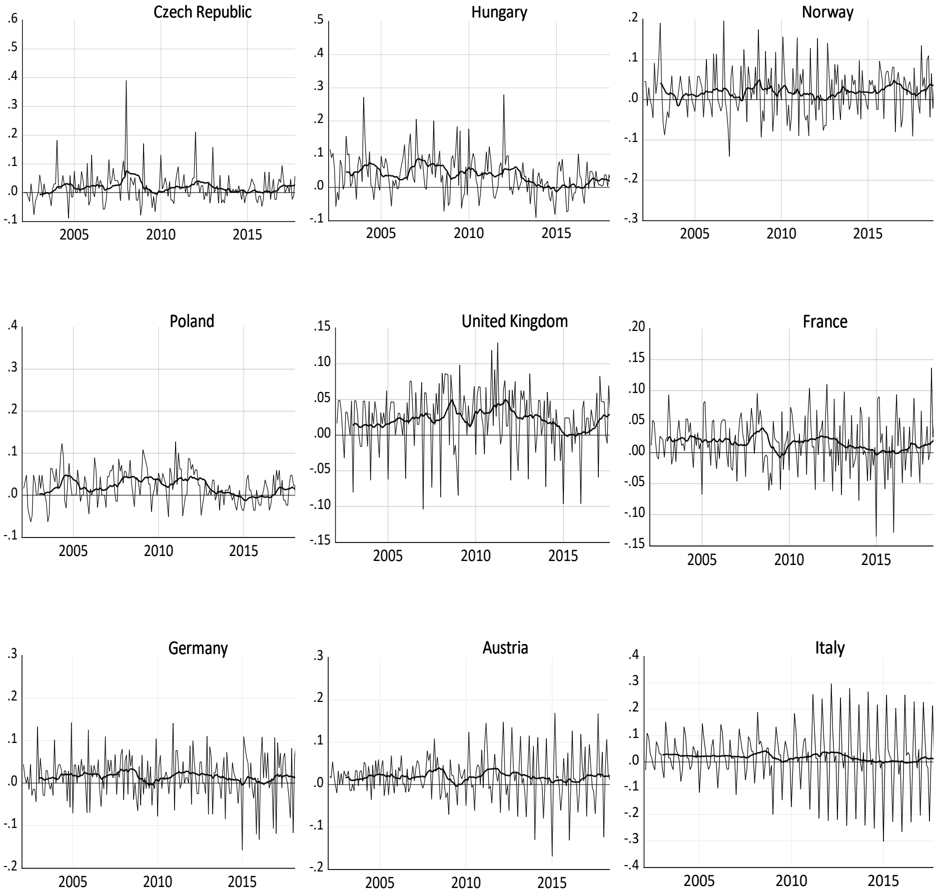
Figure 1 Consumer Price Index Logarithm



Source: Eurostat, 2015 = 100

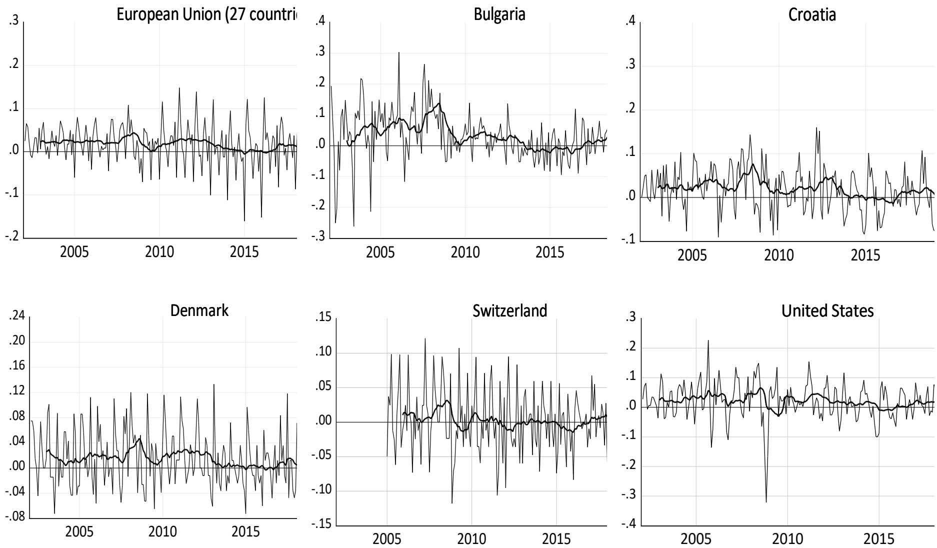
Figure 1 shows the log-transformed CPI in the five countries that have imposed IT regimes (the Czech Republic, Hungary, Norway, Poland; Jan. 2002–Aug. 2022, and the UK; Jan. 2002–Nov. 2020), four Eurozone countries (France, Germany, Austria, and Italy; Jan. 2002–Aug. 2022), the EU-27 (Jan. 2002–Aug. 2022), and the five countries that have not adopted an IT policy (Bulgaria, Croatia, Denmark; Jan. 2002–Aug. 2022), the US (Jan. 2002–April 2022) and Switzerland (Dec. 2004–Aug. 2022). All the time series are similar, characterized by upward trends and seasonal movements.

Figure 2 Annualized and Annual Inflation Rates



Source: Author´s own computations

Figure 2 Annualized and Annual Inflation Rates Continued



Source: Author´s own computations

Figure 2 captures the annualized and annual inflation rates in the above 14 countries and the EU (27 countries). Annual inflation rates smooth out the annualized ones by cyclical behavior.

3.1 Annual Inflation Rate Time Delay

The moving average property can be investigated through the frequency-domain representation of time series by decomposing the frequency response function.

Frequency response, gain, phase, and time shift functions of a moving average of the length $m_1 + m_2 + 1$ have the following forms

$$H(\omega) = \frac{1}{m_1 + m_2 + 1} \frac{\sin[\omega(m_1 + m_2 + 1)/2]}{\sin(\omega/2)} \mathrm{e}^{-i\omega(m_2 - m_1)/2}. \tag{9}$$

$$G(\omega) = |H(\omega)| = \left| \frac{\sin[\omega(m_1 + m_2 + 1)/2]}{(m_1 + m_2 + 1)\sin(\omega/2)} \right|, 0 \le f \le 1/2, \tag{10}$$

$$\varphi(\omega) = -\omega(m_2 - m_1)/2, \; \tau(\omega) = -(m_2 - m_1)/2, 0 \le f \le 1/2. \tag{11}$$

It follows from (11) that in the symmetric moving average, where $m_1 = m_2$, both the shifts are equal to zero. When using an asymmetric moving average, where

$m_1 = 0$, phase as well as time shifts for $0 < f < 1/12$ (i.e., for both trend and cyclical movements longer than one year) are

$$\varphi(\omega) = -\omega(m_2 / 2), \tau(\omega) = -(m_2 / 2) \cdot \qquad (12)$$

As the annual inflation rate (8) is an asymmetric moving average with m_2= 11, it lags behind the monthly and annualized inflation rates by $\tau(\omega) = -5.5$ months. For illustration, in the case of extremely high January monthly or annualized inflation rates, when the other values in the time series decrease symmetrically towards the past and into the future for at least five months and then do not change, the highest value of the annual inflation rate is in June and July, i. e. on average by five and half month later than it is logically expected to be. The delay given by the construction of the annual inflation rate leads to serious interpretation problems, especially in the case of unstable CPI development, manifested by high values or seasonal irregularities in the monthly and annualized inflation rates. A careful reading of the graphs in Fig. 2 makes this clear.

The properties of moving averages have long been known, the above results being based on Koopmans (1974), and Oppenhaim and Schafer (1989). However, Arlt (1998), Arlt and Bašta (2008, 2010) and Arlt (2021) were the first to demonstrate that even the annual inflation rate is delayed relative to the annualized and monthly rate as well as to the Consumer Price Index.

3.2 Annual Inflation Rate Model

Table A1 in Appendix presents the results of the ln CPI HEGY test (cf. Hylleberg *et al.*, 1990; Beaulieu and Miron, 1992; Franses and Hobijn, 1997) for the group of 14 countries and the EU-27, computed using the EViews 11 software. AIC and BIC information criteria (cf. Akaike, 1974 and Schwarz, 1978, respectively) are used (the latter in parentheses) for setting the number of lags in the test model. Regression models include constants, trends, and eleven seasonal dummies. It is clear from Table A1 that regular unit roots are not rejected in all the cases. The maximum number of non-rejected seasonal unit roots is three (Switzerland); two or one roots are not rejected in five cases and all seasonal unit roots are rejected in nine cases (5% sign. level). When the BIC is applied, the results are similar. Suppose the regression equations include only constants and trends; using the AIC, there are at most four non-rejected seasonal unit roots (the EU-27), three and two unit roots are not rejected in three, and one root in four cases, all seasonal unit roots being rejected in seven cases. When applying the BIC, a maximum of four non-rejected unit roots occurs in one case (the EU-27). It is, therefore, clear that regular differencing is sufficient to make the ln CPI stationary, this transformation representing the monthly rate of inflation. The seasonal difference in time series that contains only some seasonal unit roots leads to over-differencing.

The HEGY test results indicate the following model for ln CPI

$$\ln p_t = \alpha + \beta t + \sum_{s=2}^{12} \delta_s D_{st} + z_t , \qquad (13)$$

where t is a time variable, D_{st} is a seasonal dummy and z_t is an nonstationary, I(1) stochastic process

$$z_t = \sum_{j=1}^{t} e_j \,, \tag{14}$$

assumed to be driven by a stationary, invertible, zero-mean SARMA(p, q)(P, Q) model, termed a stochastic trend. The monthly inflation rate is then stationary with the model

$$\pi_t^m = (1-B)\ln p_t = (1-B)[\alpha + \beta t + \sum_{s=2}^{12}\delta_s D_{st} + \sum_{j=1}^{t} e_j] = \sum_{s=1}^{12}\gamma_s D_{st} + e_t \,, \tag{15}$$

where $\gamma_1 = \beta - \delta_{12}, \gamma_2 = \beta + \delta_2, \gamma_s = \beta + \delta_s - \delta_{s-1}$ for s = 3, 4, …, 12, and B is the backshift operator ($B^j y_t = y_{t-j}$).

Empirical analysis of monthly inflation rates for the countries mentioned above results in three stationary models of the e_t components in (15):

(A) SARMA(1, 0)(0, 0)

$$e_t = \phi_1 e_{t-1} + a_t \,, \tag{16}$$

(B) SARMA(0, 0)(1, 0)

$$e_t = \phi_{12} e_{t-12} + a_t \,, \tag{17}$$

(C) SARMA(1, 0)(1, 0)

$$e_t = \phi_1 e_{t-1} + \phi_{12} e_{t-12} + a_t \,, \tag{18}$$

where a_t are the white noise innovations. Parameter estimates as well as the P-values of the t-tests for models (15), (16), (17), and (18), are presented in Table A2 in Appendix (computations made using EViews 11). Durbin-Watson statistics varying between 1.7851 and 2.2636, the Ljung-Box tests reject the null of no autocorrelation in seven out of fifteen cases. The ARCH(1) and ARCH(5) tests reject the null of no conditional heteroscedasticity in ten cases. The Jarque-Bera tests reject the null of normality in fourteen out of fifteen cases. All diagnostic tests are performed at a 5% level of significance. The coefficients of determination R^2 lie in the interval between 0.3283 and 0.8688.

The annual inflation rate, which is represented by the seasonal difference of the CPI logarithm, takes the following model form

$$\pi_t^a = (1-B^{12})\ln p_t = (1-B)(1+B+B^2+\ldots+B^{11})\ln p_t =$$

$$\cdot\cdot = (1-B)(1+B+B^2+\ldots+B^{11})\left[\alpha + \beta t + \sum_{s=2}^{12}\delta_s D_{st} + \sum_{j=1}^{t} e_j\right] = 12\beta + \sum_{j=t-11}^{t} e_j \cdot \tag{19}$$

Seasonal differencing removes the deterministic trend, deterministic seasonality, and stochastic trend from the ln CPI, adding, however, truncated cumulative stationary innovations as a result of over-differencing (cf., e.g., Maravall, 1995) which creates the spurious cyclical behavior of the annual inflation rate.

The source of this effect can also be looked at from another angle. The seasonal difference consists of the regular difference and the truncated cumulation. The regular difference forms the monthly inflation rate from the ln (CPI), and the truncated cumulation then forms the annual inflation rate from the monthly inflation rate. So, part of the seasonal difference is restriction (6), which causes the spurious cycle when most of the seasonal unit roots are not in the ln (CPI). To illustrate, from formulas (15) and (19), it follows that a single jump in the CPI, which manifests as an unusually high monthly inflation rate, say in January, increases the annual inflation rates in the following months until December of the same year. However, it no longer affects annual inflation rates in the next months.

Equation (13) represents the univariate model of ln(CPI). Other model types can be imagined, univariate or multivariate, with different explanatory variables. However, whatever this model is, the definitions of monthly inflation rate (4), annual inflation rate (5), and restriction (6) will always apply. The model of the monthly and annual inflation rate must be based on the definitions of these rates, and at the same time, the restriction (6) must apply. Thus, the effect of the spurious cycle must be present in the annual inflation rate regardless of the type of model for the CPI and the monthly inflation rate. If the annual inflation rate model were constructed directly, restriction (6) validity would not be guaranteed, and the annual inflation rate would be artificially detached from the CPI and the monthly inflation rate, leading to meaningless conclusions and other problems, see below.

Annual Inflation Rate Stationarity

The annual rate of inflation (19) based on the SARMA(0, 0)(1, 0) model has the following MA(∞) representation

$$\pi_t^a = 12\beta + \sum_{i=0}^{\infty} \phi_{12}^i B^{12i} \sum_{j=0}^{11} a_{t-j} \cdot \tag{20}$$

It follows that the unconditional mean of the annual inflation rate is

$$E(\pi_t^a) = 12\beta \tag{21}$$

and the unconditional variance has the form

$$D(\pi_t^a) = \frac{12\sigma_a^2}{1-\phi_{12}^2} \cdot \tag{22}$$

Hassler and Demetrescu (2004) derived the autocorrelation function of the annual inflation as

$$\rho_{\pi^a}(h) = \phi_{12}^r - \phi_{12}^r \frac{s}{12}(1-\phi_{12}), h = 12r + s, s = 1,...,12, r = 0,1,... \cdot \quad (23)$$

Shock weights of the model (20) quickly converge to zero after the twelfth lag. The unconditional mean, unconditional variance, and autocorrelation functions are constant in time, implying that the annual inflation rate is stationary. Although it has a higher persistence than the monthly rate of inflation, which manifests itself as a spurious cyclical behavior, it always ultimately returns to its equilibrium state, to the unconditional mean.

For the annual inflation rate with a stochastic AR(1) element of the form (16), Arlt (2023) obtained similar results. The same holds for the annual inflation rate with the SARMA(1, 0)(1, 0) element (18).

The cyclical behaviors of annual inflation rates are illustrated in Fig. 2. It is obvious that annual inflation rates fluctuate around the estimates of the unconditional means $12\hat{\beta}$.

It follows from models (15) and (19) that restriction (6) guarantees the stationarity of the annual inflation rate. Suppose this restriction is not considered, and a model is sought directly for the annual inflation rate. In that case, spurious cycling causes a fundamental problem, and identifying the correct model is practically impossible, as illustrated by the simulation study in Arlt (2023), which will be presented here in an abbreviated version.

A fundamental question that must be answered when searching for a suitable time series model (univariate or multivariate) is whether the analyzed time series is stationary or not. From the above, it follows that the ADF test (Dickey and Fuller, 1979) and the PP test (Phillips and Perron, 1988) should reject the tested hypothesis of the presence of a unit root. Based on the simulation study, the powers of both tests were determined, i.e., the probability that the tests correctly reject the null hypothesis of unit root presence in the annual inflation rate.

Data with $T = \{200; 300\}$ observations were generated based on the models (19), (16) and (17), where a_t are standard Gaussian white noise innovations and parameters ϕ_1 and ϕ_{12} of the models (16) and (17) with values 0, 0.2, 0.4, 0.6, 0.8, 1.0 and $\beta = 0.002$. The powers of ADF and PP unit root tests in Tables A3 and A4 in Appendix were computed from 5000 replications. For the simulation study, the **urca** package for R (Pfaff, Zivot and Stigler, 2016) was used.

The powers of ADF and PP tests were found to be low and decreases with growing parameters ϕ_1 and ϕ_{12}. For model (17), they are lower than for model (16), and logically, they are also lower for shorter time series. Even if the annual inflation rate is stationary, both the ADF and PP unit root tests too often do not reject the zero hypothesis of the unit root presence. The reason for these erroneous conclusions is precisely a high spurious persistence manifested as spurious cycling, which is more robust with a higher value of the parameters ϕ_1 and ϕ_{12}. The risk of a false conclusion of the test for the analysed time series can be estimated by comparing the estimates of ϕ_1 and ϕ_{12} parameters in Table A2 with the corresponding columns in Tables A3 and A4 in Appendix. Similar issues are discussed, for instance, by Hassler and

Demetrescu (2004) and Frances (1991), the latter dealing with the HEGY unit root test.

Table A5 in Appendix presents the results of ADF and PP tests (EViews 11) on annual inflation rates for group of fourteen countries and EU 27. The only exception where the null hypothesis was rejected was the annual inflation rate in Norway; the PP_c test rejected the null hypothesis of the presence of the unit root at the 10% significance level and $PP_{c,t}$ at the 5% significance level. In other cases, the null hypothesis was not rejected. One might argue that such a strong result was caused by the sudden acceleration of CPI growth in 2021 and 2022. However, if the time series were shortened, the unit root tests would still mostly fail to reject the tested hypothesis (Arlt (2023)). When the time series is extended, the values will revert to the mean value, and the fluctuation in the series will remain.

What are the implications of identifying the annual inflation rate as a type I(1) non-stationary time series? The inverse transformation to truncation cumulation leads to the following relationship between the monthly and annual inflation rates

$$\pi_t^m = \sum_{i=0}^{\infty} (B^{12i} - B^{12i+1}) \pi_t^a \, , \tag{24}$$

from which it is clear that the monthly inflation rate is also non-stationary of I(1) type. The ln(CPI) received from the monthly inflation rate through its discrete cumulation (inverse operation to regular differencing) must be non-stationary of the I(2) type. However, this contradicts the results of the empirical analysis in Section 3.2, where ln(CPI) is shown to be non-stationary of I(1) type, and the monthly inflation rate is stationary. It is clear here that disregarding the restriction (6) and modelling the annual inflation rate directly does not allow retroactively reconstructing and predicting the monthly inflation rate and the consumer price index, which is why this procedure is entirely wrong.

The practical impossibility of correctly determining the character of the time series of the annual inflation rate is a fundamental problem for constructing one-dimensional and multidimensional prediction models because they make it possible to build misleading, non-converging forecasts with indefinitely expanding forecast intervals. The multidimensional models additionally do not allow for the distinction of true and spurious relationships between time series and exclusion of nonsense relationships.

But there is also a problem with the interpretation of the development of the annual inflation rate because its identification as non-stationary implies that a significant fluctuation does not have the character of a cycle that disappears after a certain period but instead of a trend that can, on the contrary, expand. Cases of such annual inflation rate developments can be found in history, but they appeared rarely and resulted from tragic war events or non-standard social development caused, for example, by revolutions.

Annual Inflation Rate Non-Invertibility

Non-converging AR(∞) representation of the annual inflation rate (19) with the SARMA(0, 0)(1, 0)

$$(1-\phi_{12}B^{12})\sum_{i=0}^{\infty}(B^{12i}-B^{12i+1})\pi_t^a = (1-\phi_{12})\beta + a_t \tag{25}$$

indicates the non-invertibility of the model (20). This is due to the absence of most seasonal unit roots in the ln CPI (e.g., Franses and Taylor, 2000; or Maravall, 1995). As is evident, using the model (25) to make forecasts is impossible because of an infinite number of lags with nonconverging weights. The annual inflation rate can only be predicted using the MA model representation (20) in a truncated form

$$\pi_t^a = 12\beta + \sum_{i=0}^{k}\phi_{12}^i B^{12i}\sum_{j=0}^{11}a_{t-j} \,. \tag{26}$$

When the parameters ϕ_{12} and β are known, the forecast with the horizon h and prediction threshold T has the form

$$\pi_T^a(h) = 12\beta + \sum_{i=0}^{k}\phi_{12}^i B^{12i}\sum_{j=0}^{11}a_T(h-j)\,, \tag{26}$$

where $a_T(l) = \pi_{T+l}^a - \pi_{T+l-1}^a(1) = a_{T+l}$ for $l \le 0$, and $a_T(l) = 0$ for $l \ge 1$ (e.g., Wei, 2006, 94). Apparently, the forecasts for the horizon $h > 1$ converge very quickly to the unconditional mean of the annual inflation rate, for $h > 12$ practically equaling it, i.e., $\pi_T^a(h) \approx 12\beta$. The annual inflation rate MA model with stationary components (16) and (18) leads to similar results, whereas the model with an AR(1) component converges even faster.

But the real situation is still more complicated. The parameters of MA representation (26) and the MA representations based on the components (16) and (18) are usually unknown and must be estimated. However, as shown in the simulation study in Arlt (2023), the estimates are biased and inaccurate. Moreover, as mentioned in Section 3.1, the annual inflation rate is delayed by approximately six months. Therefore, effective forecasts can be obtained for a maximum of six months, which are difficult to apply.

As mentioned above, respecting restriction (6) is necessary to identify the correct model of the annual inflation rate; on the other hand, respecting this restriction always and regardless of the type of model for the CPI and the monthly rate of inflation leads to a non-invertible model of the annual rate inflation. The non-invertibility is not the implication of the model type but the construction of the annual inflation rate indicator.

4. Inflation Targeting with the Annual Inflation Rate

Setting a specific inflation objective is a crucial in the IT process. As mentioned above, the inflation target is defined as a certain level of the annual inflation rate, discussed in Section 3 as a moving average (length 12) of the annualized inflation rate. The target is, therefore, a specifically smoothed (seasonally adjusted) annualized inflation rate. However, distinctive characteristics of annual inflation rate, such as the time shift and spurious cyclical fluctuations, prevent it from accurately reflecting the economic situation, especially in unstable periods. In addition, the annual inflation rate is generated by a non-invertible MA process, not allowing meaningful forecasts to be constructed. The predictions based on the truncated MA representation converge very quickly to the unconditional mean of annual inflation rates; at the target horizon of 18–24 months, they are virtually equal to this value.

The cyclical pattern of the annual inflation rate does not depend on the long-term ln CPI trend-cycle movement caused by the cumulated stationary stochastic elements because seasonal differencing removes it in the same way as regular differencing. Its source comes from the missing seasonal unit roots, shown in Table A1 in Appendix, containing the seasonal unit root test results for the countries analyzed. If no seasonal unit root is rejected in the ln CPI, cyclical movements do not load the annual inflation rate. The following hypothetical ln CPI model can illustrate this

$$\ln p_t = \alpha + \beta t + \sum_{s=1}^{12} \delta_s D_{st} + z_t , \tag{28}$$

where t is a time variable, D_{st} is a seasonal dummy variable and z_t is a seasonally integrated stochastic component

$$z_t = \sum_{j=0}^{t'} e_{t-12j} \quad \text{, where } t' = \text{int}\left[t/12\right] - 1 \text{ (int[.] rounds the number up),} \tag{29}$$

driven by a stationary and invertible zero-mean SARMA(p, q)(P, Q) process. Using the backshift operator, the monthly inflation rate model then takes the form

$$\pi_t^m = (1-B)\ln p_t = (1-B)[\alpha + \beta t + \sum_{s=2}^{12} \delta_s D_{st} + \sum_{j=0}^{t'} e_{t-12j}] = \sum_{s=1}^{12} \gamma_s D_{st} + \sum_{j=0}^{t'} \Delta e_{t-12j} , \tag{30}$$

where $\gamma_1 = \beta - \delta_{12}, \gamma_2 = \beta + \delta_2, \gamma_s = \beta + \delta_s - \delta_{s-1}$ for $s = 3, 4, \ldots, 12$, and $\Delta e_{t-12j} = e_{t-12j} - e_{t-12j-1}$. It follows that, apart from deterministic seasonality, the monthly inflation rate also contains stochastic seasonality guaranteed by the presence of all seasonal unit roots in the ln CPI.

The annual inflation rate can be expressed as

$$\pi_t^a = (1-B)(1+B+B^2+\ldots+B^{11})\left[\alpha + \beta t + \sum_{s=2}^{12} \delta_s D_{st} + \sum_{j=0}^{t'} e_{t-12j}\right] = 12\beta + e_t \tag{31}$$

Seasonal differencing removes the deterministic trend, deterministic seasonality, non-seasonal unit roots (stochastic trend), and all seasonal unit roots (integrated seasonality) from the ln CPI. The fundamental distinction between annual inflation rates (31) and (19) lies in the absence of the truncated cumulation of stationary innovations creating the cyclical component. The presence of all seasonal unit roots in the ln CPI manifests as a pronounced long-term cyclical seasonality. Other forms of ln CPI seasonality, which can be very diverse (incl. regular, constant, or no seasonality), lead to cyclically varying annual inflation rates. The presence of jumps in the ln CPI can also cause a specific cyclical behavior in the annual inflation rate. Some forms of the original time series′ seasonality and resulting seasonal differences are illustrated on simulated data in Franses, Paap, and Fok (2005). Only cycles in stationary components e_t can move from the ln CPI to the annual inflation rate. As shown in Table A2 in Appendix, however, the model parameters of these components are usually low, with cyclical components not occurring.

The above analysis yields a strong argument against the standard IT approach, justifying that there is no point in directly influencing cycles in the annual inflation rate; it can only be done by changing ln CPI seasonality or adjusting jumps in the ln CPI. Moreover, the source of spurious cycling cannot cause a long-term departure from the equilibrium state; the annual inflation rate always ultimately returns to the unconditional mean, even without outside intervention.

Price-level targeting could be used instead, but what instruments should be applied to achieve the desired ln CPI seasonality needs to be clarified. To the author's knowledge, none of the numerous studies that have examined price-level targeting deal with the link between CPI seasonality and cycles in the annual inflation rate.

The IT approach needs to be revised. The intention of the banking authorities to target a specific value of the annual inflation rate is wrong from several different points of view. It should be noted that, they target twelve times the estimate of the slope of the linear deterministic CPI trend model obtained as a moving average of twelve monthly inflation rates. First, the moving window is too narrow for the parameter estimate not to be affected by the nature of seasonality and short-term fluctuations in the ln CPI. Second, the linear deterministic trend model is incomplete, missing both the deterministic seasonal and stochastic parts. Third, as mentioned in Section 3.1, the interpretation of the annual inflation rate is incorrect. As is given by the average monthly rate in the 12-month wide window, it represents the whole window and should not be assigned to the last date in it.

Instead of the estimator, the parameter should be targeted, i.e., $\pi^{targ} = 12\beta$, where β is the slope of the ln CPI model (13). It represents the unconditional mean of the annual inflation rate (the same as the mean of the annualized inflation rate), i.e., the mean annual growth of the ln CPI. The β parameter itself characterizes the mean monthly ln CPI growth. The model (13) has an alternative form of

$$\ln p_t = \beta + \ln p_{t-1} + \sum_{s=1}^{12} \omega_s D_{st} + e_t, \tag{32}$$

where $\omega_1 = -\delta_{12}, \omega_2 = \delta_2, \omega_s = \delta_s - \delta_{s-1}$ for $s = 3, 4, \ldots, 12$, and $\sum_{s=1}^{12} \omega_s = 0$. So, it is clear that the ln CPI is driven by a random walk with the drift and short-term cyclicality given by the autoregressive term, which is consistent with the literature (see, e.g., Cobrae and Ouliaris, 1989; Hassler and Demetrescu, 2004; Kahn, 2009).

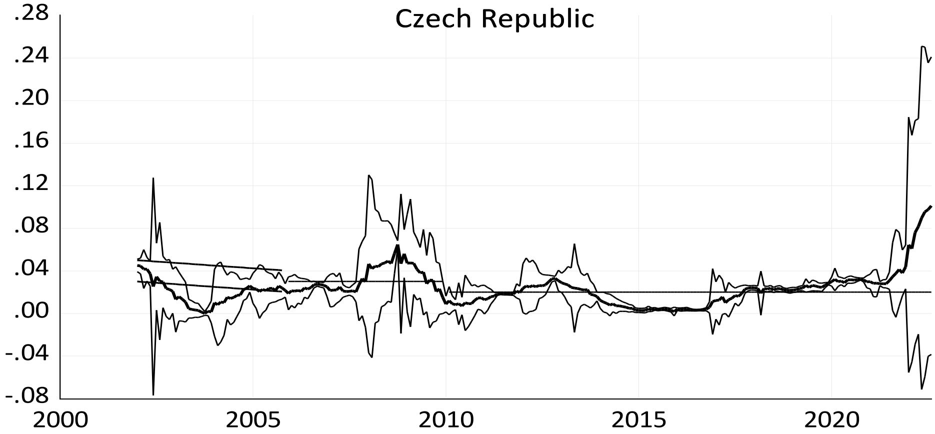
Verification of the inflation target achievement is then performed empirically by comparing the target, i.e., 12β, with its point and interval estimate, which can be interpreted as a point and interval estimate of the unconditional mean of the annual inflation rate or as a point and interval forecast of the annual inflation rate; see the passage on the annual inflation rate non-invertibility in Section 3.2.

From (8), it follows that the annual inflation rate can be written as $\pi_t^a = 12((1/12)\sum_{i=0}^{11}\pi_{t-i}^m)$, i.e., twelve times the average monthly inflation rate computed from the moving window of the width $\lambda = 12$ months. As mentioned above, for particular time series (i.e., realizations of the stochastic process), this average monthly inflation rate is, in fact, the moving point estimate of the slope of the linear deterministic trend model of the ln CPI. Generally, the ML estimate of the parameter β from the model (13), computed from a moving window of the width λ, i.e., $\hat{\beta}_{t,\lambda}$, represents the same, but at a more complex and precise level. The time series of the annual inflation rate can then be written as $12\hat{\beta}_{t,\lambda}$, its shape depending not only on the time t, but also on the width λ. With the widening window, the annual inflation rate indicator is less affected by the nature of seasonality and irregularities in the monthly inflation rate, the cycles being longer, less pronounced, and flatter. However, more considerable delay still has adverse effects, especially in the case of significant spurious cycling of the annual inflation rate, i.e., in certain types of seasonality and extreme values in the monthly inflation rate.

The question remains: How wide should the moving window be for β parameter estimation? It is clear from the above that the twelve months are not enough. The window width must be chosen to reasonably eliminate the risk of spurious cyclicality with a reasonable delay; it is always a compromise, and there is no objective criterion.

The following illustration analyzes inflation targeting in the Czech Republic based on the CPI from Jan. 2000 to Sept. 2022 (source: the Czech Statistical Office). Figure 3 plots the annual inflation rate in both point and interval form (95% tolerance limit) computed as the point and interval estimates of the parameter β in the model (13) and (16) for the window of the width $\lambda = 24$ months. The horizontal lines represent the inflation targets.

Figure 3 Annual Inflation Rate - 24 Months Window, Interval and Point Inflation Target (3–5% Linearly Declining Band from January 2002, 3% from January 2006, 2% from January 2010)

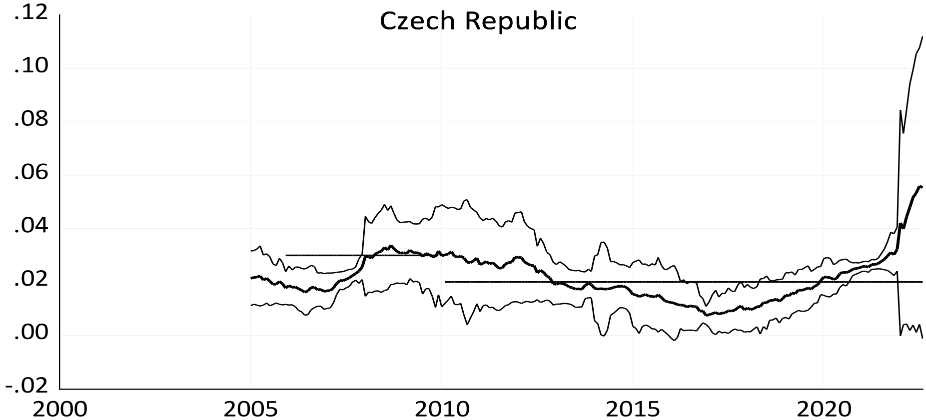


Source: Author´s own computations, https://www.cnb.cz/en/monetary-policy/inflation-target/history-of-cnbs-inflation-targets/

Section 3.1 suggests that this annual inflation rate is delayed by twelve months, the date-specific values in Fig. 3 indicating the annual (point and interval) rates of inflation valid a year ago, the latest point and interval inflation rate representing the point and interval forecasts set a year ago for any time horizon. These values should be compared with the inflation target to assess its performance. As shown in Fig. 3, the inflation target was met until about 2014; the annual inflation rate was below the target between 2014 and 2017, rising above it in 2018 and accelerating its point growth from 2021 onwards. Since then, inflation intervals have been widening, increasing future developments´ uncertainties.

However, as analyzed before, the annual inflation rate calculated from a 24-month window still contains some spurious cyclicality. Thus, it makes sense to extend the window to 60 months. The point and interval annual inflation rates and a 3% and 2% inflation targets are captured in Figure 4 below.

Figure 4 Annual Inflation Rate - 60 Months Window, Point Inflation Target (3% from January 2006, 2% from January 2010)



Source: Author´s own computations
https://www.cnb.cz/en/monetary-policy/inflation-target/history-of-cnbs-inflation-targets/

The delay of this annual inflation rate is 30 months, its level being significantly lower than the previous one. Suppose the annual inflation is moved back by two and a half years. In that case, a similar cyclical behavior like in the Czech Republic's CPI can be seen (cf. Fig. 1), which can be understood as a subjective confirmation of the correctness of the window length. The annual inflation rate has been above the inflation target since mid-2018.

The weak points of this approach can be seen in Fig. 3 and 4. First, the spurious cycle of the annual inflation rate depends on the subjectively chosen length of the moving window. Second, the delay makes it difficult to verify that the inflation target has been met. The only way to solve this problem is to construct a forecast of the annual inflation rate. However, as argued in Arlt (2023), the annual inflation rate is a time series generated by a non-invertible process that cannot be used for forecasting. The forecast can only be obtained through the CPI or monthly inflation rate forecast. However, the longer the forecast horizon, the less accurate the forecast, especially for jumps and irregular seasonality. Third, as the annual inflation rate increases, the confidence interval widens, i. e., even when it is evident that the target is being exceeded, this method indicates otherwise. This follows from the fact that the annual inflation rate with a window of various lengths can only be obtained as an estimate of β parameter of the CPI model (20). Logically, a sudden increase in CPI values increases the standard error and, thus, the confidence interval. It follows from the work of Arlt (2023) that the annual inflation rate model cannot be used because the standard error of the estimate would always be considerably overestimated.

Conclusions

To achieve price stability, central banks implement annual inflation-targeting policies. The annual inflation rate, however, was found to have some statistical weaknesses. The asymmetry of the one-sided simple moving average of the annualized inflation rate has proven to account for a 5.5-month lag behind the annualized rate of inflation and the CPI. The empirical analysis of the CPI revealed a similar behavior of all the time series examined – for two groups of five countries each that do and do not practice inflation targeting, respectively, for the four selected eurozone countries and the EU. The time series share linear deterministic and stochastic trends, deterministic seasonality, and stationary non-seasonal and seasonal movements. A regular unit root and the absence of most seasonal unit roots in the ln CPI appear as a spurious cyclical behavior in a stationary annual inflation rate.

The stationarity is guaranteed by the restriction that the annual inflation rate is an aggregate of twelve monthly inflation rates. Suppose this restriction is not respected, and the annual inflation rate is modeled directly regardless of the monthly inflation rate and consumer price index. In that case, spurious cycling causes a fundamental problem; the unit root tests do not reject the tested hypothesis of the presence of a unit root, and the series is considered nonstationary. Consequently, the annual inflation rate models do not allow retroactively reconstructing the monthly inflation rate and the consumer price index. In addition, they make it possible to build misleading, non-converging forecasts with indefinitely expanding forecast intervals, and the multidimensional models do not allow for the distinction of true and spurious relationships between time series and exclusion of nonsense relationships.

Another problem is connected with the interpretation of the development of the annual inflation rate because its identification as non-stationary implies that a significant fluctuation in development does not have the character of a cycle that disappears after a certain period but a trend instead that can, on the contrary, expand. This can lead to confusing economic and political discussions with negative social consequences. The very presence of a spurious cycle (which can arise from just one high value of the monthly inflation rate!) is misleading. Still, its substitution for a trend can strengthen disproportionate and unjustified economic responses.

On the other hand, respecting the restriction that the annual inflation rate is an aggregate of twelve monthly inflation rates leads to a non-invertible MA(∞) representation of the annual rate inflation, meaning its non-converging AR(∞) representation, which does not allow to construct forecasts of annual inflation rates. Even the truncated MA model does not lead to reasonable forecasts.

Given a regular unit root presence, the long-run cyclical movement in the ln CPI does not correspond to that in the annual inflation rate since it is removed by seasonal differencing from the ln CPI. A short-term cycle may be present in the annual inflation rate due to strong autocorrelation transmitted from the ln CPI. If the ln CPI included all or most of the seasonal unit roots manifested in long-term cyclical seasonality, then the derived annual inflation rate would be stationary with neither additional persistence nor cyclical movements. However, most seasonal unit roots are missing in the real ln CPI, implying that the source of cyclical fluctuations in the annual inflation rate is a different form of ln CPI seasonality than the one mentioned above. Hence, cycles in the annual inflation rates cannot be influenced and modified directly, but only via the changes in ln CPI seasonality and outliers, which is a powerful argument against inflation targeting in its commonly used form. Another argument is methodological. The banking authorities target twelve times the estimate of the slope of the linear deterministic CPI trend model obtained as a moving average of twelve monthly inflation rate values. Correctly, they should not target the parameter estimate but its actual value. Verification of whether the inflation target is met is then performed by comparing it with this parameter's point and interval estimate. However, this approach has weak points. The width of the estimation window is always a compromised and subjective solution. The delay problem remains, making it difficult to verify that the inflation target has been met. As the annual inflation rate increases, the confidence interval widens, which makes it practically impossible to identify periods where the inflation target needs to be achieved.

APPENDIX

Table A1 HEGY Test Results for Unit Roots in CPI

		0	$\pi/6$ (1/12)	$\pi/3$ (2/12)	$\pi/2$ (3/12)	$2\pi/3$ (4/12)	$5\pi/6$ (5/12)	π (6/12)	
Country	*Lags*	t_1	$F_{11,12}$	$F_{7,8}$	$F_{3,4}$	$F_{5,6}$	$F_{9,10}$	t_2	#SUR
CR	0 (0)	0.9814	0.0002	0.0000	0.0002	0.0002	0.0000	0.0056	0 (0)
Hungary	1 (0)	0.9531	0.0178	0.0000	0.0000	0.0000	0.0000	0.0413	0 (0)
Norway	1 (1)	0.9730	0.0000	0.0002	0.0002	0.0000	0.0000	0.0056	0 (0)
Poland	0 (0)	0.8996	0.0000	0.0000	0.0000	0.0000	0.0000	0.0413	0 (1)
UK	0 (0)	0.9307	0.0002	0.0002	0.0002	0.0000	0.0000	0.0319	0 (0)
France	4 (1)	0.5677	0.0000	0.0000	0.0002	0.0054	0.0084	0.0483	0 (0)
Germany	2 (1)	0.4014	0.1236	0.0076	0.0002	0.0002	0.0000	0.0183	1 (0)
Austria	1 (1)	0.3709	0.0016	0.0829	0.0146	0.0443	0.0000	0.0056	1 (1)
Italy	1 (1)	0.4075	0.0000	0.3544	0.0000	0.3220	0.0000	0.0209	2 (2)
EU 27	1 (1)	0.6604	0.0000	0.0010	0.0000	0.0084	0.0000	0.0073	0 (0)
Bulgaria	10 (0)	0.5816	0.0076	0.0005	0.0000	0.0002	0.1217	0.0073	1 (0)
Croatia	4 (1)	0.3821	0.0000	0.0000	0.0005	0.0203	0.0021	0.0056	0 (0)
Denmark	4 (1)	0.7450	0.0000	0.0611	0.0106	0.8723	0.0047	0.0343	2 (1)
Switzerland	4 (1)	0.6043	0.0002	0.1973	0.0000	0.0802	0.0270	0.2467	3 (2)
US	4 (1)	0.6083	0.0000	0.0000	0.0000	0.0000	0.0000	0.0056	0 (0)

Notes: 1. Unit roots at the frequencies $f = 0$ and $f = 6/12$ are tested by one-sided *t*-tests, their low values supporting alternative hypotheses. Seasonal unit roots at the frequencies $f = 3/12$, 5/12, 1/12, 4/12, 2/12 are tested using joint F-tests. 2. *Lags* is the number of lags *p* in the test model, applying the AIC (BIC in parentheses). 3. The test results are in the form of P-values. 4. #SUR means the number of non-rejected seasonal unit roots (5% sign. level; by the BIC in parentheses). 4. The testing equations include constants, trends and eleven seasonal dummies. 5. All the time series are in the log transformation.
Source: Author´s own computations

Table A2 Monthly Inflation Rate Model

Country	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8	γ_9	γ_{10}	γ_{11}	γ_{12}	ϕ_1	ϕ_{12}
CR	0.0124	0.0017	0.0017	0.0031	0.0030	0.0025	0.0026	-0.0002	-0.0029	0.0021	-0.0003	0.0011	0.4265	-
	(0.0000)	(0.0725)	(0.0835)	(0.0017)	(0.0026)	(0.0119)	(0.0119)	(0.8043)	(0.0089)	(0.0381)	(0.7344)	(0.2808)	(0.0000)	-
Hungary	0.0094	0.0057	0.0057	0.0061	0.0061	0.0025	0.0031	0.0008	0.0033	0.0050	0.0017	0.0011	0.5587	0.1620
	(0.0000)	(0.0003)	(0.0003)	(0.0001)	(0.0001)	(0.1061)	(0.0448)	(0.5947)	(0.0411)	(0.0022)	(0.2874)	(0.5008)	(0.0000)	(0.0276)
Norway	-0.0022	0.0066	0.0028	0.0035	0.0002	0.0013	0.0018	-0.0031	0.0080	0.0011	0.0016	0.0010	-	-
	(0.0255)	(0.0000)	(0.0039)	(0.0003)	(0.8046)	(0.1754)	(0.0607)	(0.0016)	(0.0000)	(0.2840)	(0.1095)	(0.3275)	-	-
Poland	0.0040	0.0023	0.0048	0.0054	0.0039	0.00016	-0.0007	-0.0010	0.0027	0.0039	0.0020	0.0016	0.5016	0.1967
	(0.0003)	(0.0337)	(0.0000)	(0.0000)	(0.0004)	(0.1346)	(0.5266)	(0.3408)	(0.0157)	(0.0004)	(0.0707)	(0.1502)	(0.0000)	(0.0130)
UK	-0.0055	0.0047	0.0029	0.0040	0.0026	0.0007	-0.0009	0.0038	0.0023	0.0012	0.0011	0.0034	0.1671	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1553)	(0.0927)	(0.0000)	(0.0000)	0.0194	(0.0370)	(0.0000)	(0.0146)	-
France	-0.0032	0.0039	0.0069	0.0022	0.0023	0.0014	-0.0022	0.0040	-0.0010	0.0010	0.0004	0.0025	0.1946	0.2367
	(0.0000)	(0.0000)	(0.0000)	(0.0018)	(0.0010)	(0.0463)	(0.0022)	(0.0000)	(0.1506)	(0.1499)	(0.5920)	(0.0007)	(0.0022)	(0.0009)
Germany	-0.0042	0.0055	0.0051	0.0020	0.0024	0.0016	0.0039	0.0003	-0.0007	0.0004	-0.0034	0.0068	-	0.2866
	(0.0002)	(0.0000)	(0.0000)	(0.0652)	(0.0253)	(0.1407)	(0.0004)	(0.7636)	(0.5488)	(0.9198)	(0.0020)	(0.0000)	-	(0.0001)
Austria	-0.0046	0.0036	0.0088	0.0027	0.0011	0.0014	-0.0028	0.0011	0.0060	0.0026	0.0013	0.0041	0.2012	0.4618
	(0.0003)	(0.0039)	(0.0000)	(0.0273)	(0.3611)	(0.2438)	(0.0259)	(0.3915)	(0.0000)	(0.0430)	(0.2957)	(0.0012)	(0.0007)	(0.0000)
Italy	-0.0114	-0.0003	0.0174	0.0064	0.0024	0.0027	-0.0103	0.0000	0.0116	0.0046	0.0006	0.0022	-	0.6908
	(0.0000)	(0.9003)	(0.0000)	(0.0039)	(0.2700)	(0.2160)	(0.0000)	(0.9656)	(0.0000)	(0.0428)	(0.8041)	(0.3296)	-	(0.0000)
EU 27	-0.0047	0.0038	0.0090	0.0046	0.0025	0.0017	-0.0023	0.0015	0.0035	0.0027	-0.0003	0.0030	0.3898	0.3032
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0048)	(0.0492)	(0.0092)	(0.0968)	(0.0001)	(0.0026)	(0.6993)	(0.0007)	(0.0000)	(0.0001)
Bulgaria	0.0057	0.0052	0.0020	0.0045	-0.0002	-0.0034	0.0074	0.0029	0.0008	0.0036	0.0036	0.0069	0.4685	0.2241
	(0.0019)	(0.0044)	(0.2774)	(0.0124)	(0.9265)	(0.0614)	(0.0001)	(0.1127)	(0.6428)	(0.0495)	(0.0485)	(0.0002)	(0.0000)	(0.0003)
Croatia	0.0014	0.0023	0.0064	0.0065	0.0048	0.0033	0.0010	0.0006	-0.0006	0.0016	-0.0002	-0.0014	0.4161	0.2048
	(0.2272)	(0.0479)	(0.0000)	(0.0000)	(0.0001)	(0.0056)	(0.3778)	(0.5950)	(0.6006)	(0.1697)	(0.8832)	(0.2234)	(0.0000)	(0.0035)
Denmark	0.0006	0.0077	0.0042	0.0039	0.0022	0.0005	0.0012	-0.0004	0.0034	0.0021	-0.0004	-0.0017	0.1415	0.5250
	(0.6647)	(0.0000)	(0.0036)	(0.0069)	(0.1252)	(0.7083)	(0.3957)	(0.7569)	(0.0185)	(0.1443)	(0.7672)	(0.2177)	(0.0165)	(0.0000)
Switzerland	-0.0033	0.0019	0.0031	0.0043	0.0006	0.0003	-0.0017	-0.0008	0.0012	0.0026	-0.0029	0.0001	-	0.3739
	(0.0000)	(0.0568)	(0.0021)	(0.0000)	(0.5266)	(0.7352)	(0.0855)	(0.4179)	(0.2347)	(0.0091)	(0.0040)	(0.8902)	-	(0.0000)
US	0.0037	0.0049	0.0065	0.0043	0.0036	0.0028	0.0005	0.0011	0.0022	-0.0009	-0.0031	-0.0029	0.4971	-0.1857
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0005)	(0.5675)	(0.1588)	(0.0066)	(0.2752)	(0.0001)	(0.0004)	(0.0000)	(0.0017)

Notes: The table contains parameter estimates and P-values (in parentheses).
Source: Author´s own computations

Table A3 Unit Root Test Power

T	*TEST*	ϕ_1							
		0.0	***0.2***	***0.4***	***0.6***	***0.8***	***0.9***	***0.95***	***1.0***
200	ADF_c	0.8288	0.8358	0.8540	0.8784	0.8678	0.7436	0.5214	0.1470
	$ADF_{c,t}$	0.5890	0.6160	0.6654	0.7050	0.7428	0.6288	0.4402	0.2136
	PP_c	0.8342	0.6668	0.4888	0.2746	0.0638	0.0158	0.0124	0.0384
	$PP_{c,t}$	0.4686	0.2392	0.1088	0.0366	0.0068	0.0026	0.0034	0.0118
300	ADF_c	0.9916	0.9900	0.9922	0.9938	0.9890	0.9190	0.6872	0.1312
	$ADF_{c,t}$	0.9318	0.9342	0.9416	0.9570	0.9470	0.8286	0.5696	0.1972
	PP_c	0.9984	0.9862	0.9558	0.8908	0.4932	0.0944	0.0234	0.0284
	$PP_{c,t}$	0.9478	0.8352	0.6836	0.4522	0.1120	0.0094	0.0066	0.0108

Notes: 1. The table reports the power of unit root tests. ADF_c and PP_c denote tests with a constant, $ADF_{c,t}$ and $PP_{c,t}$ denoting those with a constant and linear time trend. 2. The number of lags in the ADF test is determined by the AIC with the maximum number of lags $\lfloor 4(T/100)^{2/9} \rfloor$, the number of Newey-West lags in the PP test being $\lfloor 4(T/100)^{1/4} \rfloor$. 3. The annual inflation rate is generated by the models (14) and (11), with standard Gaussian white noise innovations a_t $_t$. The number of Monte Carlo replications is 5000.
Source: Author's own computations

Table A4 Unit Root Test Power

T	*TEST*	ϕ_{12}							
		0.0	***0.2***	***0.4***	***0.6***	***0.8***	***0.9***	***0.95***	***1.0***
200	ADF_c	0.8288	0.6164	0.3600	0.1800	0.0804	0.0712	0.0558	0.0532
	$ADF_{c,t}$	0.5890	0.3830	0.2016	0.1086	0.0648	0.0588	0.0552	0.0554
	PP_c	0.8342	0.5864	0.3264	0.1678	0.0844	0.0684	0.0598	0.0580
	$PP_{c,t}$	0.4686	0.2810	0.1454	0.0856	0.0554	0.0550	0.0616	0.0570
300	ADF_c	0.9916	0.9250	0.6852	0.3448	0.1222	0.0764	0.0630	0.0488
	$ADF_{c,t}$	0.9318	0.7370	0.4298	0.1972	0.0812	0.0698	0.0622	0.0572
	PP_c	0.9984	0.9446	0.7032	0.3402	0.1188	0.0788	0.0670	0.0590
	$PP_{c,t}$	0.9478	0.7274	0.3972	0.1794	0.0756	0.0666	0.0628	0.0574

Notes: 1. The table reports the power of unit root tests. ADF_c and PP_c denote tests with a constant, $ADF_{c,t}$ and $PP_{c,t}$ denoting those with a constant and linear time trend. 2. The number of lags in the ADF test is determined by the AIC with the maximum number of lags $\lfloor 4(T/100)^{2/9} \rfloor$, the number of Newey-West lags in the PP test being $\lfloor 4(T/100)^{1/4} \rfloor$. 3. The annual inflation rate is generated by the models (14) and (12) with standard Gaussian white noise innovations a_t The number of Monte Carlo replications is 5000.
Source: Author's own computations

Table A5 ADF and PP Test Results for Unit Roots in Annual Inflation Rates

	ADF		*PP*	
Country	***c***	***c, t***	***c***	***c, t***
CR	0.9501	0.9915	0.9805	0.9960
Hungary	0.9581	0.9998	0.9796	0.9999
Norway	0.5316	0.5576	0.0715	0.0442
Poland	0.9818	0.9995	0.9993	0.9999
UK	0.5773	0.7454	0.2171	0.4174
France	0.6941	0.9921	0.6083	0.9460
Germany	0.3499	0.7751	0.9551	0.9944
Austria	0.7533	0.9479	0.9715	0.9956
Italy	0.6904	0.9889	0.9184	0.9997
EU 27	0.7632	0.9989	0.9893	0.9999
Bulgaria	0.6176	0.9679	0.5712	0.9259
Croatia	0.6347	0.9755	0.9350	0.9993
Denmark	0.9179	0.9991	0.9920	0.9999
Switzerland	0.3111	0.7649	0.1863	0.5549
US	0.6349	0.9606	0.1650	0.4702

Notes: The test results are in the form of P-values. Letter *c* denotes a constant and *t* denotes a linear time trend. The number of lags in the ADF test is determined by the AIC, the number of Newey-West lags in the PP test being determined automatically.
Source: Author's own computations

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