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Hedging Extreme Risk of Wheat in Semiparametric CVaR Portfolios with Commodities

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Abstract

This paper investigates how to minimize the downside risk of wheat by making three five-asset portfolios with different types of commodities – precious metals, industrial metals and energy. The portfolio optimization process uses the complex semiparametric CVaR metric as targeted. For comparison purposes, portfolios with the classical parametric CVaR are also constructed. Considering the different attitudes of investors towards risk, all portfolios are constructed assuming two different levels of risk aversion. The preliminary equicorrelation findings reveal that energy commodities have the lowest integration with the wheat market, which is suitable for diversification efforts. The constructed portfolios indicate that the precious metals portfolio has the lowest CVaR and mCVaR risk, taking into account both probability levels. Gold dominates this portfolio due to the lowest second, third and fourth moments. Industrial metals also have good hedging capabilities, while energy commodities perform the worst.

1. Introduction

The volatile nature of agricultural commodity prices is a well-known phenomenon among agricultural producers and scholars. It happens because many supply- and demand-side factors affect agricultural production (see, e.g. Teodor et al., 2018; Tonin et al., 2020; Jia et al., 2022; Zmami and Ben-Salha, 2023). For example, oil and fertilizers, which are among the most important production inputs, are very prone to price fluctuations that are directly transmitted to the prices of agricultural products (Dawson, 2015). Besides, adverse weather conditions, plant diseases, low investment in the agricultural sector, and politically imposed export restrictions also influence the lower supply of agricultural commodities in the global market. On the other hand, rising demand for agricultural products by the two most populous countries (China and India) greatly impacts the global agricultural market from the demand side. Increased speculations by institutional investors on agricultural markets also contribute to the volatility of their prices.

On top of that, it should be emphasized that various global crises, such as the COVID-19 pandemic and the war in Ukraine, significantly contribute to the

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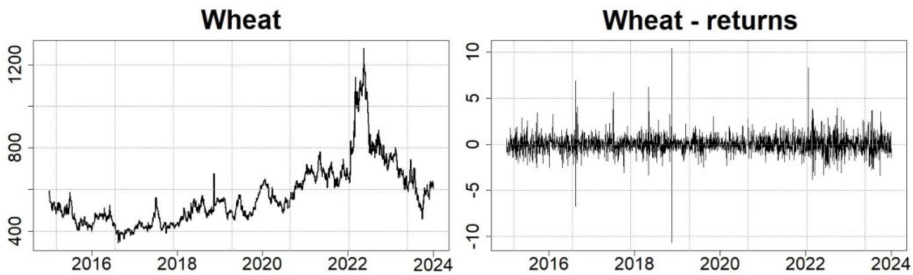
instability of the global agricultural markets. Figure 1 confirms this assertion, showing that spot wheat prices have experienced a lot of swings in the past decade, especially in early 2022, due to the war in Ukraine. Russia and Ukraine are among the world's largest wheat producers, and the outbreak of war caused a lot of lousy mood regarding the availability of wheat. Gloomy expectations about future developments in the global wheat market skyrocketed its price to over 1,200 USD cents per bushel in May 2022, while in August 2022, the price subsided significantly (see Figure 1).

All the factors mentioned above are responsible for the considerable rise of risk in the wheat market, which can be seen in the right plot of Figure 1. In order to mitigate the effects of these adverse developments, academics and practitioners try to find appropriate solutions. Generally speaking, hedging of high risk could be achieved by investing in diversified portfolios, and this paper explores an elaborate way to reduce extreme risk in the wheat market. In particular, we combine wheat with three globally well-known types of commodities – precious metals, industrial metals and energy commodities, in a five-asset portfolio. In this way, we try to determine which auxiliary assets are the best hedge for the extreme risk of wheat. To the best of our knowledge, none of the papers attempted to create a complex portfolio between wheat and the three groups of assets, which gives us the motivation for this research.

Figure 1 Empirical Dynamics of Spot Wheat Prices and Returns



Notes: The price of wheat is expressed in U.S. cents per bushel.

Three different groups of commodities are intentionally selected because various commodities might have different levels of integration, which can affect portfolio construction and performance. According to the Modern portfolio theory of Markowitz (1952), two factors have a crucial role in the performance of a portfolio – the individual risk level of assets and their mutual interdependence. Higher (lower) asset integration in the portfolio directly translates into worse (better) diversification results.

To obtain a preliminary insight into which combination of assets has a lower level of interdependence with wheat and, accordingly, a better hedging result, we first estimate the DECO-GARCH model of Engle and Kelly (2012). Equicorrelations are convenient for this process because they are time-varying, which means they can inspect the level of integration across time and under different market conditions. The DECO model retains the time-varying characteristics of the conditional correlation matrix but assumes that any pair of assets are equicorrelated. It produces a single dynamic correlation, i.e. equicorrelation, between all pairs of assets. The model elegantly and efficiently determines the interrelationship between a relatively large number of assets, which is the reason why many researchers used this model (see, e.g. Demiralay et al., 2019; McIver and Kang, 2020; Hung, 2021; Elsayed et al., 2021). Estimated equicorrelations serve as an indicator, helping us to understand and visualize the level of interlink between the assets in the portfolios across the sample. These findings can help us anticipate which auxiliary assets may perform best. After the portfolio construction, the initial assumption based on equicorrelations can be compared to the actual results to see if they coincide.

The most important aspect of the paper comes from the area of risk evaluation in a multi-asset portfolio. In other words, the paper tries to address the well-known problem of risk measurement bias, which appears when researchers use relatively simple risk measures, such as variance. In other words, variance equalizes positive and negative returns, while the only risk that investors are interested in comes from negative returns, known as downside risk. Advancement in this area was made by the introduction of the Value-at-Risk (VaR) metric by J.P. Morgan in 1994. Parametric VaR circumvents the problem of positive returns by observing only a specific quantile at the left tail of the standard normal distribution (Sajjad et al., 2008). It means that VaR actually calculates extreme risk, and the risk level depends on the degree of probability (Aloui and Ben Hamida, 2015; Bahloul et al., 2022). However, VaR struggles with its undesirable theoretical properties, such as the lack of subadditivity and non-convexity, which can create multiple local optima and unstable VaR rankings (Li et al., 2012). More importantly, a severe disadvantage of VaR is its inability to measure the losses beyond the threshold amount of VaR (Snoussi and El-Aroui, 2012), which could induce wrong investment decisions. Rockafellar and Uryasev (2002) try to resolve this issue by proposing parametric conditional VaR (CVaR), which controls the magnitude of losses beyond VaR.

However, parametric VaR and CVaR have a stringent assumption that they are valid only if the distribution has the Gaussian function. Daily commodity time series usually do not follow a normal distribution, which means that CVaR could be a biased risk measure, as well as variance and VaR. This happens because CVaR uses only the first two moments for risk estimation, while the third and fourth moments remain neglected. In this regard, this paper leaps forward, trying to overcome the two-moment bias. In other words, we refer to Favre and Galeano (2002), who addressed the issue of the two-moment bias by introducing the semiparametric or modified VaR (mVaR). This theoretical risk metric is based on the Cornish-Fisher expansion (Cornish and Fisher, 1938), where the third and fourth moments have their role in calculating downside risk. Since the two-moment bias applies for both parametric VaR and CVaR, we use semiparametric CVaR or modified CVaR (mCVaR) in the portfolio optimization process as a better risk approximation than VaR. The paper merges complex and elaborate semiparametric CVaR algorithm with Markowitz's portfolio optimization process. Theoretically speaking, semiparametric CVaR penalizes the unfavourable characteristics of distribution, such as negative skewness and high kurtosis, and rewards positive features, such as positive skewness and low kurtosis (see e.g. Chai and Zhou, 2018). Setting mCVaR as a target in the portfolio optimization process can eliminate all listed shortcomings in the risk measurement, resulting in a more accurate and realistic risk assessment. In order to compare how much CVaR underestimates downside risk, we construct the portfolios with both minimum CVaR and minimum mCVaR. If portfolios have high negative skewness and high kurtosis, there is no doubt that CVaR will be significantly lower than the mCVaR counterpart; the only question is how much.

In order to be thorough in the analysis, the paper also addresses one aspect that could have great importance for market participants: the attitude of investors towards risk. In other words, market agents have different risk preferences, meaning some are risk-takers and some are risk-averters. However, this factor could also affect the structure of the semiparametric portfolio. Therefore, for every group of assets, we construct two portfolios that target downside risks at different probability level. The first reflects risk-averters' situation, and the other is closer to investors willing to accept higher risk. In the former situation, this would be the case when downside risk is calculated at 99% probability, which means that in the worst 1% of returns, the investor will have a certain amount of loss. On the other hand, downside risk is calculated at a 95.84% probability when investors are more relaxed about the size of the risk. The reason for setting such an unusual level of confidence is that mVaR can be inconsistent with a higher probability than 95.84%, according to Cavenaile and Lejeune (2012). Therefore, at 95.84%, it would mean that there is a chance of 4.16% that investors will suffer a certain level of loss, and this should portray the situation of an investor who is more open to risk. For all portfolios, the same procedure is applied, which allows us to see whether and how much the structure of the portfolios differs when the level of downside risk is targeted at different hedging preferences.

The contribution of the paper is four-fold. First, this is the first paper that tries to mitigate the extreme risk of wheat in a multi-asset portfolio with different commodities. Second, the paper is comprehensive in the research because it additionally uses a complex multivariate DECO-GARCH model to make a preliminary estimate of which group of commodities might best fit into a portfolio with wheat. Third, two elaborate risk measures of extreme risk are used in multi-asset portfolio optimization. Fourth, all portfolios are estimates at different probability levels, which reflects different risk tolerances.

Besides the introduction, the rest of the paper is structured as follows. The second section gives an overview of the extant literature. The third section explains the methodologies used, the DECO-GARCH model, and the construction of the CVaR and mCVaR portfolios. The fourth section introduces the dataset with descriptive statistics. The fifth section presents the results in three subsections. The sixth section discusses the results and considers possible implications. The last section concludes.

2. Literature Review

This section presents recent papers that used precious metals, industrial metals, and energy assets for hedging purposes. For instance, Das et al. (2022) investigate the impacts of contemporaneous and lagged implied oil volatility (OVX) jumps on precious metals (gold, palladium, platinum, and silver) with a focus on the hedging properties of precious metals. They report that gold returns are relatively less responsive against contemporaneous and lagged OVX jumps; thus, gold acts as a strong hedge against OVX jumps. However, other metals (copper, palladium, platinum, and silver) do not serve as a hedge against contemporaneous OVX jumps. They conclude that gold is the contemporaneous metal of choice when risk perception is high, or investors are averse to risk. The study of Naeem et al. (2022) researches the diversification properties of precious metals for African stock markets. They find that gold offers the strongest safe haven and hedging potential for African equity markets. The quantile-coherency analysis indicates a low safe haven ability for precious metals in the long-run. On the other hand, palladium provides both a safe haven and hedge opportunities in the short-term, while platinum holds only its hedging potential in the same spectrum. Peng (2020) examines precious metals' hedge and safe haven properties in China's financial markets, including stock, bond, commodity futures, and foreign exchange markets. He uses the DCC-GARCH models and shows that precious metals are strong hedges for the bond market and diversifiers for other financial markets. In addition, he asserts that precious metals can serve as a safe haven in market turmoil.

As for industrial metals, Adekoya and Oliyide (2020) try to hedge oil market risk with seven commonly traded industrial metals. They find that the nature of shocks, whether demand- or supply-based, determines the hedging ability of the industrial metals. They contend that the metals cannot hedge the oil supply shocks regardless of the estimation model, but virtually all metals can effectively hedge all the other three demand-based oil shocks. Also, they state that it is safe to include industrial metals in the oil portfolio since it is stable even when oil price exhibits significant instabilities. Umar et al. (2019) investigate the conditional correlation and the resulting optimal hedge ratios between the Credit Default Swap (CDS) spreads of the U.S. metal and mining industries and the prices of copper, platinum, silver and gold. They find that copper provides the best possible hedge for dealing with the U.S. metals and mining industries' credit risks. Chen et al. (2022) research the strength and network characteristics of spillovers among non-ferrous metals and sub-sectoral clean energy stocks from the time and frequency perspectives, also analyzing the portfolio diversification and hedging with non-ferrous metals for sub-sectoral clean energy stocks. They report that non-ferrous metals work best as hedging assets for developer stocks.

Abuzayed et al. (2022) examine the dynamic co-movements and portfolio management strategies between U.K. stock indices and both gold and crude oil futures markets during the Brexit referendum (2016) and Brexit day (2021). Regarding the Brexit referendum period, gold and crude oil provide a diversification opportunity for the U.K. stock portfolios, where investors should give more weight to gold than stocks in their gold-stock portfolios. On the other hand, they should allocate more funds into stocks than oil to minimize risk in their oil-stock portfolios. They reported that crude oil appears more effective than gold in reducing stock portfolio exposure to downside risk and improving hedging effectiveness. The paper of Mensi et al. (2021) researches the volatility transmission between crude oil and four precious metals and also investigates whether oil can be considered as a hedge or safe-haven asset against four precious metals. They conclude that Brent oil is a diversifier and a weak safe haven for precious metals, which means that a portfolio with Brent and precious-metals futures can yield better hedging effectiveness. Batten et al. (2021) study hedging stocks with oil using the Dynamic Conditional Correlation model, which allows them to calculate optimal hedge ratios and corresponding hedge portfolio returns. They find distinct economic benefits from hedging stocks with oil, although the hedging effectiveness is both time-varying and market-state-dependent.

3. Used Methodologies

3.1 DECO Model

Equicorrelations are used as an auxiliary tool to show which group of assets is more integrated. This information could indicate which portfolio is a better risk-minimizer since the level of interdependence between assets in a portfolio directly affects the diversification efforts. The DECO model is a simpler version of the classical DCC model of Engle (2002), and it is introduced to overcome the dimensionality problem of the DCC model. In other words, estimating dynamic correlations with many time series in the DCC model is cumbersome and computationally expensive (Demiralay et al., 2019). DECO elegantly circumvents this issue by assuming that all pairwise correlations in the DCC framework are equal or equicorrelated.

Like the DCC model, DECO is also estimated using the two steps. The first step estimates univariate conditional volatility, while the second generates equicorrelations. We make portfolios with commodities, so we do not expect the presence of the asymmetric effect in the time series. Therefore, we use some form of the symmetric ARMA-GARCH(1,1) model in the first step. This model can tackle autocorrelation and heteroscedasticity problems in the time series of the selected commodities. The mean and variance equations of the symmetric GARCH model are presented in equations (1) and (2):

$$y_{i,t} = C + \phi y_{t-1} + \varepsilon_{i,t}; \qquad \varepsilon_{i,t} \sim z_{i,t}\sqrt{\sigma_{i,t}^2} \tag{1}$$

$$\sigma_{i,t}^2 = c + \alpha\varepsilon_{i,t-1}^2 + \beta\sigma_{i,t-1}^2 \tag{2}$$

where ϕ denotes the autocorrelation parameter of the first lag-order, which is sufficient to solve the autocorrelation problem. *C* and *c* are constants in the mean and variance equations. $y_{i,t}$ represents 5×1 vector of commodities, where i marks the i^{th} asset, while $\varepsilon_{i,t}$ is a 5×1 vector of error terms. $z_{i,t}$ is independently and identically distributed process in the mean equation, and it follows the Gaussian distribution. In equation (2), β describes the persistence of volatility, while α measures the ARCH effect. The second equation serves to handle the heteroscedasticity problem.

The starting point in explaining the DECO model is the classical DCC model of Engle (2002). The DCC model has positive definiteness of the variance-covariance matrix (H_t), which is presented in equation (3):

$$H_t = D_t^{1/2} R_t D_t^{1/2} \tag{3}$$

where $R_t = [\rho_{ij,t}]$ is the conditional correlation matrix, while the diagonal matrix of the conditional variances is given by $D_t = diag(\sigma_{1,t}^2 \ldots, \sigma_{n,t}^2)$. Engle and Kelly (2012) explained that ρ_t can be modelled by using the cDCC process of Aielli (2013) to obtain the conditional correlation matrix Q_t, and then taking the mean of its off-diagonal elements. They called this method the dynamic equicorrelation (DECO) model, and the scalar equicorrelation is defined as:

$$\rho_t^{DECO} = \frac{1}{n(n-1)}\left(J'_n R_t^{cDCC} J_n - n\right) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}, \tag{4}$$

where $q_{ij,t} = \rho_t^{DECO} + a_{DECO}\left(u_{i,t-1} u_{j,t-1} - \rho_t^{DECO}\right) + b_{DECO}\left(q_{ij,t} - \rho_t^{DECO}\right)$, which is the $(i,j)^{th}$ element of the matrix Q_t from the cDCC model of Aielli (2013). $u_t = [u_{1,t}, \ldots, u_{n,t}]'$ is the standardized residuals from the GARCH model, $u_{i,t} = \varepsilon_{i,t}/\sigma_{i,t}^2$. $n \times n$ is the unconditional covariance matrix of u_t, while a and b are non-negative scalars satisfying $a + b < 1$. Scalar equicorrelation is then used to estimate the conditional correlation matrix:

$$R_t = (1 - \rho_t) I_n + \rho_t J_n, \tag{5}$$

where J_n is $n \times n$ matrix of ones, and I_n is the n-dimensional identity matrix. The process in equation (5) calculates how a group of assets co-move in the portfolio with a single time-varying correlation coefficient, which is called equicorrelation.

3.2 Portfolio Optimization with CVaR and mCVaR Minimizing Goals

This paper constructs three five-asset portfolios to achieve minimum CVaR and minimum mCVaR goals. In other words, we combine the two complex risk algorithms (CVaR and mCVaR) with Markowitz's (1952) portfolio optimization procedure. It is an improvement in creating the minimum risk portfolio because the existing studies, according to our knowledge, have only used parametric CVaR (see, e.g. Vo et al., 2019; Braiek et al., 2020; Shen et al., 2021; Luan et al., 2022). The paper optimizes portfolios with both CVaR and mCVaR for comparison purposes, which will then show whether and how much the CVaR risk is lower.

The starting point in explaining minimum CVaR and mCVaR portfolios is the construction of a long-only minimum variance portfolio, which is achieved by solving equation (6):

$$min\ \sigma_p^2 = min\ \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, i \neq j}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j}, \tag{6}$$

where σ_p^2 is the portfolio variance, σ_i^2 is the variance of a particular asset i, w_i denotes the calculated weight of an asset i in the portfolio. $\rho_{i,j}$ is the pairwise Pearson correlation between the assets i and j. The sum of all asset-weights in the portfolio must be equal to one, which is a necessary condition in every multivariate portfolio optimization process, and all asset-weights are somewhere between zero and one.

$$\sum_{i=1}^{N} w_i = 1\,; \quad 0 \leq w_i \leq 1 \tag{7}$$

Every portfolio with minimum variance has the corresponding mean value, which is the weighted average portfolio return (r_p), and it can be calculated as in equation (8).

$$r_p = \sum_{i=1}^{n} w_i r_i \tag{8}$$

The first and second moments (r_p and σ_p) from equations (8) and (6) are necessary elements in calculating parametric VaR: $VaR_p = r_p + Z_\alpha \sigma_p$. Z_α is the left quantile of the normal standard distribution. This paper tries to construct portfolios where investors have two different risk preferences. Therefore, α is calculated at the probabilities of 99% and 95.84%, indicating risk-averting and risk-taking strategies, respectively. Finding the integral of VaR is the way to calculate CVaR.

$$CVaR_\alpha = -\frac{1}{\alpha}\int_0^\alpha VaR(x)dx \tag{9}$$

Expression (10) shows how the minimum CVaR portfolio can be optimized.

$$\min CVaR_p(w), \qquad \sum_{i=1}^{n} w_i r_i \tag{10}$$

However, the CVaR portfolio can be biased because it only considers the first two moments. The problem can be solved by considering all four moments of a distribution, and this is where mCVaR comes to the fore because it uses all four moments. Analogous to CVaR, mCVaR is the integral of mVaR, while mVaR is calculated as: $mVaR_\alpha = r_p + Z_{CF,\alpha}\sigma_p$, where $Z_{CF,\alpha}$ is the non-normal-distribution percentile adjusted for the higher-order moment information, according to the

Cornish–Fisher Expansion:

$$Z_{CF,\alpha} = Z_\alpha + \frac{1}{6}(Z_\alpha^2 - 1)S + \frac{1}{24}(Z_\alpha^3 - 3Z_\alpha)K - \frac{1}{36}(2Z_\alpha^3 - 5Z_\alpha)S^2 \tag{11}$$

S and K denote measures of skewness and kurtosis of a portfolio. Similar to expression (10), the minimum semiparametric CVaR portfolio can be optimized as in expression (12):

$$\min mCVaR_p(w), \qquad \sum_{i=1}^{n} w_i r_i \tag{12}$$

In order to quantitatively estimate how much the downside risks of wheat are reduced in the commodity portfolios, we calculate Hedge effectiveness indices (HEI). Therefore, regarding a specific risk measure (RM), i.e. CVaR or mCVaR, HEI_{RM} can be calculated in the following way:

$$HEI_{RM} = \frac{RM_{wheat} - RM_{portfolio}}{RM_{wheat}} \tag{13}$$

4. Dataset and Descriptive Statistics

The study uses daily spot prices of wheat and near-maturity futures from the Chicago mercantile exchange of the selected commodities – precious metals (gold, silver, platinum and palladium), industrial metals (aluminium, copper, zinc and tin) and energy commodities (Brent oil, natural gas, gasoline and heating oil). Wheat is combined with each group of commodities, creating, in this way, the three five-asset portfolios to find the best combination of assets that produce minimum CVaR and mCVaR risk. We choose futures rather than spot commodities because futures markets process new information faster, making futures prices more realistic. Besides, futures involve buying and selling contracts, not physical assets, which makes futures more suitable for hedging purposes. The data span covers the period between January 2015 and November 2023, and all assets are collected from the stooq.com website. All the time series are transformed into log-returns (r_i) according to the expression: $r_i = 100 \times log(P_{i,t}/P_{i,t-1})$, where P_i is the price of a particular commodity. Wheat is synchronized separately with each group of assets, and this implies slightly different lengths of the portfolios. Table 1 shows descriptive statistics of the selected commodities, containing the first four moments, the Ljung-Box tests of the level and squared returns and the DF-GLS test of stationarity. Wheat is presented in all three Panels because the time series of wheat is slightly different in the portfolios due to synchronization.

According to Table 1, energy commodities have the highest variances, while gold has the lowest risk. The value of the second moment plays a crucial role in calculating CVaR, which means that gold is likely to have a significant share in the CVaR portfolios. All assets have relatively low negative skewness values, which is essential to notice because higher confidence levels in calculating modified CVaR are conditional on the skewness value, according to Cavenaile and Lejeune (2012). Table 2 reports these values, and it can be seen that a higher 99% confidence level is in line with the skewness values of the assets. Consequently, this means that the mCVaR portfolios will also have adequate skewness, which is essential for the reliability of the results. It is interesting to note that gasoline has very high positive skewness, which indicates that some observations are found far on the right tail of the distribution, and this could have a significant role in calculating modified CVaR. Besides gasoline, wheat also has relatively high positive skewness. As for kurtosis, gasoline has a very high value, which indicates the presence of outliers. When the mCVaR portfolios are constructed, the share of gasoline will show which gasoline characteristic prevails – positive skewness or high kurtosis. All time series, except copper, zinc and Brent oil, have autocorrelation, while all time series report heteroscedasticity. DECO-GARCH can handle these issues when estimating equicorrelations. In the end, it should be said that all time series are stationary according to the DF-GLS test, which is an obligatory condition in the GARCH modelling.

Table 1 Descriptive Statistics of the Selected Assets

	Mean	*Variance*	*Skew.*	*Kurt.*	*JB*	*LB(Q)*	*LB(Q^2)*	*DF-GLS*
Panel A: Wheat and precious metals								
Wheat	0.002	0.871	0.553	8.973	3406.2	0.008	0.000	-10.332
Gold	0.011	0.401	-0.054	7.120	1567.3	0.068	0.000	-4.741
Silver	0.008	0.772	-0.469	9.063	3474.2	0.040	0.000	-4.889
Platinum	-0.008	0.712	-0.314	8.113	2449.6	0.002	0.000	-13.178
Palladium	0.002	0.984	-0.446	12.265	7995.9	0.000	0.000	-43.640
Panel B: Wheat and industrial metals								
Wheat	0.000	0.879	0.484	8.788	3180.3	0.015	0.000	-10.490
Aluminium	0.003	0.551	0.041	5.407	535.4	0.068	0.000	-9.887
Copper	0.008	0.573	-0.110	4.748	286.5	0.454	0.000	-5.695
Zinc	0.002	0.692	0.057	4.512	212.3	0.992	0.000	-10.025
Tin	0.003	0.666	-0.610	9.002	3463.4	0.123	0.000	-7.773
Panel C: Wheat and energy commodities								
Wheat	0.000	0.880	0.502	8.675	3130.5	0.016	0.000	-10.501
Brent oil	0.008	1.144	-0.936	17.670	20612.4	0.456	0.000	-3.196
Natural gas	-0.005	1.559	-0.048	5.826	753.5	0.045	0.000	-6.196
Gasoline	0.109	2.091	4.956	56.711	281160.6	0.000	0.000	-4.137
Heating oil	0.007	1.126	-0.751	14.550	12785.8	0.080	0.000	-7.225

Notes: J.B. stands for the value of Jarque-Bera coefficients of normality, L.B. (Q) and L.B. (Q2) tests refer to the p-values of Ljung-Box Q-statistics for level and squared returns of 10 lags. Assuming only constant, 1% and 5% critical values for the DF-GLS test with 5 lags are -2.566 and -1.941, respectively.

Table 2 Minimum Skewness for mVaR Consistency

Confidence level	96.0%	97.5%	99.0%	99,5%	99.9%
Minimum skewness	-3.3	-1.62	-0.98	-0.79	-0.59

Source: Cavenaile and Lejeune (2012)

According to Markowitz's theory, the risk of assets in the portfolio plays a crucial role in the portfolio's construction. We make portfolios with the minimum CVaR and mCVaR metrics, so it is helpful to know the size of downside risks of the selected commodities. Table 3 contains these values, which are also calculated at the two different probability levels. The CVaR and mCVaR measures can be used later to explain the calculated shares of assets in the portfolios. As can be seen, all CVaR values are lower than the mCVaR counterparts, which is expected since mCVaR takes into account all four moments, and all skewness and kurtosis values have no-normal properties (see Table 1). It inevitably reflects in higher mCVaR numbers, as Table 3 suggests. Only in the case of gasoline at 95.84%, mCVaR is lower than CVaR, which indicates that at this confidence level, positive skewness has the upper hand over high kurtosis in calculating mCVaR. Wheat has slightly different CVaR and mCVaR values between the portfolios, and this is because of the synchronization with different commodities.

Table 3 Calculated CVaR and mCVaR Risk Values of the Assets

Portfolio with precious metals			***Portfolio with industrial metals***			***Portfolio with energy commodities***		
Panel A: Downside risk at the 99% probability level								
	CVaR	mCVaR		CVaR	mCVaR		CVaR	mCVaR
Wheat	-2.320	-4.184	Wheat	-2.341	-4.249	Wheat	-2.346	-4.182
Gold	-1.057	-1.889	Aluminium	-1.466	-2.095	Brent oil	-3.041	-11.659
Silver	-2.050	-4.601	Copper	-1.520	-2.073	Natural gas	-4.158	-6.398
Platinum	-1.906	-3.875	Zinc	-1.841	-2.313	Gasoline	-5.461	-12.485
Palladium	-2.619	-7.406	Tin	-1.772	-3.974	Heating oil	-2.995	-9.796
Panel B: Downside risk at the 95.84% probability level								
	CVaR	mCVaR		CVaR	mCVaR		CVaR	mCVaR
Wheat	-1.861	-2.414	Wheat	-1.878	-2.463	Wheat	-1.882	-2.437
Gold	-0.846	-1.154	Aluminium	-1.176	-1.399	Brent oil	-2.438	-5.801
Silver	-1.643	-2.653	Copper	-1.218	-1.434	Natural gas	-3.337	-4.171
Platinum	-1.531	-2.299	Zinc	-1.477	-1.639	Gasoline	-4.360	-2.746
Palladium	-2.101	-3.943	Tin	-1.421	-2.313	Heating oil	-2.402	-5.060

5. Empirical Results

5.1 Equicorrelation Calculation

This section presents the results of the estimated DECO-GARCH models of the three portfolios. Estimated equicorrelations show how strongly the assets are integrated into the portfolios, where weaker integration potentially implies better hedging results of a portfolio, and *vice-versa*. Estimated equicorrelations are time-varying, which could indicate whether and how integration between the assets changes across the sample. Equicorrelations are not used in the portfolio construction process but only serve as preliminary findings. In order to be accurate in the analysis, we estimate the model with the best ARMA specification, and Table 4 shows these results.

Table 4 AIC Values of DECO Models with Different ARMA Specifications

	ARMA(1,0)	*ARMA(1,1)*	*ARMA(2,1)*	*ARMA(2,2)*
PMP	9.303	9.307	9.308	9.309
IMP	8.827	8.831	8.831	8.830
ECP	14.494	14.464	14.455	14.468

Notes: The greyed value indicates the lowest AIC. PMP, IMP and ECP acronyms denote precious metals portfolio, industrial metals portfolio and energy commodity portfolio, respectively.

On the other hand, each ARMA-GARCH-DECO model is estimated using the normal and Student t multivariate distribution. Table 5 shows calculated AIC values, showing that all models with the Student t distribution have better performance. It signals the presence of peaks in equicorrelations.

Table 5 Calculated AIC Values of the Two DECO Models with Normal and Student t Distributions

	Precious metals portfolio		*Industrial metals portfolio*		*Energy commodities portfolio*	
	Normal	*Student t*	*Normal*	*Student t*	*Normal*	*Student t*
AIC value	9.069	8.630	8.480	8.305	14.315	13.717

Table 6 Parameter Estimates of the DECO-GARCH Models

		Wheat	*Gold*	*Silver*	*Platinum*	*Palladium*
GARCH parameters	α	0.100***	0.021**	0.029**	0.046***	0.069***
	β	0.831***	0.972***	0.966***	0.948***	0.908***
Diagnostic tests	LB(Q)	0.672	0.791	0.268	0.416	0.739
	LB(Q^2)	0.987	0.126	0.304	0.611	0.226
DECO parameters	a_{DECO}	0.035**				
	b_{DECO}	0.983***				
	M-shape	5.986***				
		Wheat	*Aluminium*	*Copper*	*Zinc*	*Tin*
GARCH parameters	α	0.091***	0.086***	0.064*	0.049***	0.183***
	β	0.854***	0.876***	0.823***	0.928***	0.755***
Diagnostic tests	LB(Q)	0.825	0.675	0.956	0.908	0.826
	LB(Q^2)	0.788	0.574	0.710	0.708	0.931
DECO parameters	a_{DECO}	0.006				
	b_{DECO}	0.987***				
	M-shape	9.523***				
		Wheat	*Brent oil*	*Natural gas*	*Gasoline*	*Heating oil*
GARCH parameters	α	0.095***	0.127***	0.089***	0.163***	0.140***
	β	0.840***	0.847***	0.907***	0.852***	0.843***
Diagnostic tests	LB(Q)	0.557	0.656	0.521	0.345	0.263
	LB(Q^2)	0.404	0.625	0.596	0.221	0.877
DECO parameters	a_{DECO}	0.003				
	b_{DECO}	0.996***				
	M-shape	5.248***				

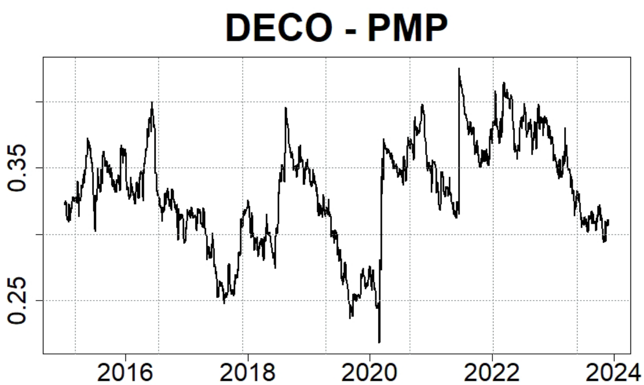
Notes: LB (Q) and LB (Q2) numbers indicate p-values at 10 lags. ***, **, * represent statistical significance at the 1%, 5% and 10% level, respectively.

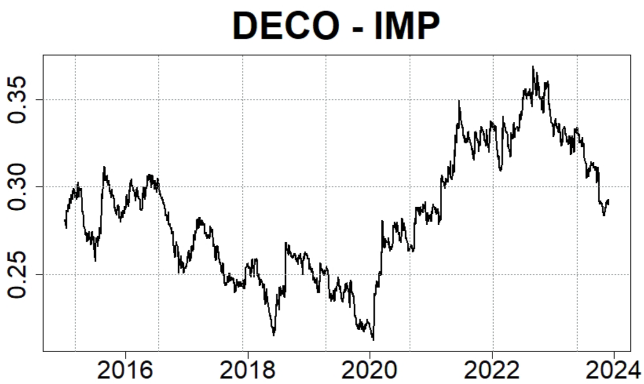
Table 6 contains the estimated parameters of the DECO-GARCH models. It can be seen that all α and β parameters are statistically significant. Statistically significant α parameters indicate the ARCH effect, which means that previous shocks impact the conditional variance. On the other hand, all β parameters are relatively high, which indicates the high volatility persistence in the commodity futures markets. As for the DECO parameters, both aDECO and bDECO parameters are statistically significant only in the case of the precious metal portfolio, meaning that market shocks affect equicorrelations.

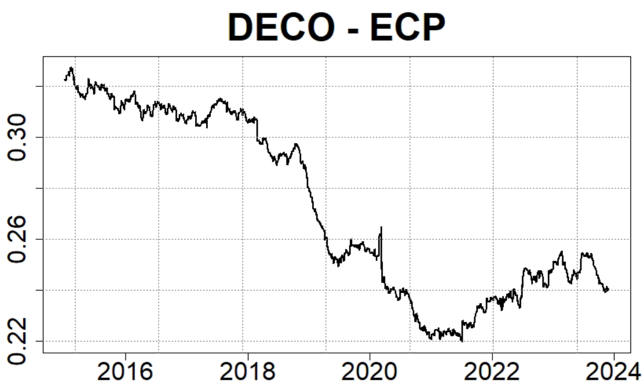
In contrast, all equicorrelations are dependent on the past equicorrelations. In the cases of the other two portfolios, a_{DECO} parameters are not significant, but this does not pose a severe problem because b_{DECO} parameters are crucial in estimating accurate equicorrelations. All M-shape parameters are highly statistically significant, which is in accordance with the AIC values in Table 4. As for the diagnostic tests, Table 6 shows that all autocorrelation and heteroscedasticity problems are resolved in the DECO-GARCH models.

Figure 2 plots estimated equicorrelations of the three portfolios, where it can be seen that all equicorrelations are time-varying but relatively low. In other words, the highest average equicorrelation ($\bar{\rho}_t^{DECO}$) is found in the precious metals portfolio, with a value of 0.332, while the lowest value, 0.272, is found in the portfolio with energy commodities. According to the preliminary DECO results in Figure 2, the energy commodity portfolio might produce the best hedging results, while precious metals are the worst. However, this does not necessarily have to be true because the level of asset integration in a portfolio is only one element of the portfolio performance. Even more important factor is the risk level of assets, and the energy commodities are among the riskiest (see Table 3), which is not a good trait for portfolio construction.

Figure 2 Estimated Equicorrelations of the Three Portfolios







Notes: Abbreviations PMP, IMP and ECP indicate to precious metals portfolio, industrial metals portfolio and energy commodity portfolio, respectively.

5.2 Portfolio Construction at the 99% Confidence Level

This section presents the results of the constructed portfolios, where the two goals are targeted – minimum CVaR and minimum mCVaR at a 99% probability level. A high probability level in calculating downside risk indicates investors who want to avoid risk, and Table 7 contains the optimal shares of assets in the portfolios. For all shares of assets, we try to offer a rational explanation. To this end, Table 3 is of great help because it contains calculated downside risk levels of all assets. In addition, Table 8 presents calculated pairwise Pearson correlations between all portfolio assets, showing the importance of the covariance matrix in portfolio construction.

Starting with the precious metals portfolio, it can be seen that gold dominates in both CVaR and mCVaR portfolios with 83% and 73%, respectively. Gold takes the highest share in the portfolios because it has the lowest CVaR (-1.057) and mCVaR (-1.889). Wheat has a relatively high share of 15% and 27% in the CVaR and mCVaR portfolios, respectively, although wheat has the second-highest CVaR (-

2.320) and the third-highest mCVaR (-4.184). However, wheat has the lowest pairwise correlation with gold (0.070), the most dominant asset in the portfolio, which explains why wheat has a relatively high share. As for the other precious metals, only palladium has 2% in the CVaR portfolio, while silver and platinum have zero share. Gold has the lowest correlation with palladium (0.334), compared to gold-silver (0.777) and gold-platinum (0.533), which is the reason why palladium has a 2% share, although palladium has the highest CVaR (-2.619). These results are very similar to the paper of Živkov et al. (2022), who also found only gold and palladium in the five-asset portfolio with energy commodities.

Table 7 Calculated Shares of Assets in the Portfolios at a 99% Probability Level

Portfolio with precious metals			*Portfolio with industrial metals*			*Portfolio with energy commodities*		
	CVaR	***mCVaR***		***CVaR***	***mCVaR***		***CVaR***	***mCVaR***
Wheat	15%	27%	*Wheat*	18%	11%	*Wheat*	52%	52%
Gold	83%	73%	*Aluminium*	33%	33%	*Brent oil*	13%	16%
Silver	0%	0%	*Copper*	24%	39%	*Natural gas*	15%	24%
Platinum	0%	0%	*Zinc*	5%	17%	*Gasoline*	4%	6%
Palladium	2%	0%	*Tin*	20%	0%	*Heating oil*	16%	4%
Σ	100%	100%	*Σ*	100%	100%	*Σ*	100%	100%

In the portfolio with industrial metals, all assets have a share in the CVaR portfolio, while in the mCVaR portfolio, tin is excluded. Aluminium has the highest share in the CVaR portfolio (33%) because aluminium has the lowest CVaR (-1.466) due to the lowest variance (0.551). Tin has a 20% share in the CVaR portfolio; although tin has the third highest CVaR (-1.772), tin has the lowest correlation with the other industrial metals (see Table 8), which puts tin at the third place in the CVaR portfolio. Copper has 24% in the CVaR portfolio, probably because it has the second lowest CVaR (-1.520). Wheat has a relatively high 18%, albeit wheat is the riskiest asset in the portfolio, with a CVaR of -2.341. However, wheat has a very low pairwise correlation with all industrial metals (see Table 7), which explains the relatively high share of wheat. This result highlights the important role of the covariance matrix in the case of wheat.

On the other hand, the situation changes significantly in the mCVaR portfolio. Aluminium retains its share of 33%, but copper increases its share from 24% to 39%. This is because copper has a slightly lower mCVaR (-2.080) than aluminium (-2.135). Zinc takes the third position with 17% because it has the third lowest mCVaR (-2.313). Zinc has positive skewness (0.057) and relatively low kurtosis (4.512), which offsets the high variance (0.692) of zinc. All these factors are responsible for relatively low mCVaR, which gives the portfolio a relatively high share of zinc. Wheat decreases to 11% from 18% because it has the highest mCVaR (-4.249), and the only reason wheat has a share in the mCVaR portfolio is its very low correlation with the industrial metals. Tin performs the worst in the mCVaR portfolio because its share decreases from 20% to 0%. This is because tin has the second-highest mCVaR (-3.974) and a relatively high pairwise correlation with other industrial metals (see

Table 8). Tin has the highest negative skewness and the highest kurtosis, which explains the very high mCVaR of tin.

Table 8 Calculated Pairwise Pearson Correlations Between the Assets in the Portfolios

		Wheat	*Gold*	*Silver*	*Platinum*	*Palladium*
Portfolio with precious metals	*Wheat*	1	—	—	—	—
	Gold	0.070	1	—	—	—
	Silver	0.073	0.777	1	—	—
	Platinum	0.090	0.533	0.596	1	—
	Palladium	0.092	0.334	0.419	0.537	1
		Wheat	***Aluminium***	***Copper***	***Zinc***	***Tin***
Portfolio with industrial metals	*Wheat*	1	—	—	—	—
	Aluminium	0.116	1	—	—	—
	Copper	0.114	0.495	1	—	—
	Zinc	0.097	0.471	0.561	1	—
	Tin	0.076	0.323	0.394	0.384	1
		Wheat	***Brent oil***	***Natural gas***	***Gasoline***	***Heating oil***
Portfolio with energy commodities	*Wheat*	1	—	—	—	—
	Brent oil	0.130	1	—	—	—
	Natural gas	0.035	0.098	1	—	—
	Gasoline	0.051	0.271	0.063	1	—
	Heating oil	0.123	0.794	0.121	0.263	1

As for the energy commodity portfolio, wheat has the highest share in both CVaR and mCVaR portfolios, with a share of 52%. Wheat dominates the portfolios because all energy commodities have higher CVaR and mCVaR risks (see Table 3). Heating oil is the second one with 16%, while natural gas follows with 15% in the CVaR portfolio because heating oil has a lower CVaR (-2.995) than natural gas (-4.158). Surpassingly, Brent oil has only 13%, although it has the third lowest CVaR (-3.122). However, Brent has a relatively high correlation with wheat (0.130) and heating oil (0.794), which explains a relatively low percent of Brent in the portfolio. Gasoline has only 4% because gasoline is the riskiest energy asset (-5.461).

On the other hand, the situation changes significantly for some assets in the mCVaR portfolio. For example, natural gas increased its share from 15% to 24%, while Brent also improved its position from 13% to 16%. However, heating oil sinks from 16% to 4%. Natural gas increases its share because it has the second lowest mCVaR (-6.398). On the contrary, Brent has the highest mCVaR due to the highest negative skewness (-0.936) and very high kurtosis (17.670), but despite that, Brent has a higher share in the mCVaR than the CVaR portfolio. The covariance matrix also comes to the fore in this case, which means that Brent has relatively low pairwise correlations with wheat (0.130) and natural gas (0.098), the two most representative assets in the portfolio.

5.3 Portfolio Construction at the 95.84% Confidence Level

This section tries to answer how the structure of the portfolios changes if downside risk measures are calculated at a lower probability level. CVaR observes a particular area of a distribution on the left tail, and the surface of this area depends on the level of probability. In other words, a higher probability implies a narrower left tail area *and vice-versa*. In the multivariate CVaR portfolio construction, the CVaR value of every asset in a portfolio is considered to construct the portfolio with minimum CVaR. The primary input in calculating CVaR is variance, and variance does not vary with the change of probability. Therefore, if the CVaR of assets is observed at a higher (lower) probability, the portfolio with a higher (lower) probability CVaR would be created. However, the structure of the portfolios remains the same because variance, as the main ingredient in calculating CVaR, does not change.

On the other hand, in calculating modified CVaR, skewness and kurtosis are also taken into account, and these values are sensitive to the observed probability level. Generally speaking, kurtosis at a higher probability could be significantly higher than kurtosis at a lower probability due to the narrower left tail area, which could affect the portfolio's structure. Unlike variance, skewness and kurtosis can change significantly at different levels of probability, which could have severe repercussions on the portfolio's structure. This assertion is going to be tested empirically in this section.

Table 9 contains shares of the assets in the CVaR and mCVaR portfolios when a probability is 95.84%. Comparing Table 9 with Table 7, it can be seen that the CVaR portfolio structures are unchanged. This confirms that different probability levels do not affect the variance used in calculating CVaR. This explains the unchanged structure of the CVaR portfolios at different probability levels.

Table 9 Calculated Shares of Assets in the Portfolios at a 95.84% Probability Level

Precious metals portfolio			*Industrial metals portfolio*			*Energy commodities portfolio*		
	CVaR	***mCVaR***		***CVaR***	***mCVaR***		***CVaR***	***mCVaR***
Wheat	15%	26%	*Wheat*	18%	15%	*Wheat*	52%	55%
Gold	83%	74%	*Aluminium*	33%	35%	*Brent oil*	13%	15%
Silver	0%	0%	*Copper*	24%	36%	*Natural gas*	15%	20%
Platinum	0%	0%	*Zinc*	5%	11%	*Gasoline*	4%	7%
Palladium	2%	0%	*Tin*	20%	3%	*Heating oil*	16%	3%
Σ	100%	100%	*Σ*	100%	100%	*Σ*	100%	100%

On the other hand, comparing the mCVaR portfolios, it can be seen that the structure changes to a greater or lesser extent. For instance, gold increases its share in the precious metal portfolio from 73% to 74%, while wheat decreases from 27% to 26%. The reason for this result lies in the fact that the mCVaR of gold decreases more between the two portfolios (from -1.057 to -0.846) than the mCVaR of wheat (from -4.184 to -2.414).

In the industrial metals portfolio, the changes are more pronounced. Wheat, tin and aluminium increased from 11% to 15%, from 0% to 3%, and from 33% to 35%, respectively. On the other hand, copper decreases from 39% to 36%, and zinc decreases from 17% to 11%. This happens because skewness and kurtosis have significantly different values in the 95.84% portfolio, which is directly reflected in the calculated mCVaR values and the share of assets in the portfolio. In particular, the mCVaR of wheat reduces from -4.249 to -2.463, in the case of tin from -3.974 to -2.313, and aluminium from -2.095 to -1.399. The risk decline of wheat, tin and aluminium is greater than that of copper (from -2.073 to -1.434) and zinc (from -2.313 to -1.639). This explains why the shares of wheat, tin and aluminium increase while the other two metals decrease at the higher probability portfolio.

In the energy commodity portfolio, the share of wheat increases from 52% to 55%, and gasoline from 6% to 7%. On the other hand, the share of Brent, natural gas and heating oil decreases. Brent falls from 16% to 15%, natural gas has the biggest decline, from 24% to 20%, while heating oil decreases from 4% to 3%. The reason is the same as in the cases of precious and industrial metals, i.e., the mCVaR decline of Brent and gasoline is more significant than the decline of Brent, natural gas, and heating oil.

The results clearly show that probability level does not affect the CVaR portfolio's structure, but this significantly affects the mCVaR portfolio. This happens because probability does not affect variance but affects skewness and kurtosis, and this strongly confirms the assertion stated at the beginning of this section.

6. Discussion and Implications of the Results

This section comments on the performances of the created portfolios and reveals which portfolio has the best hedging abilities considering both risk-averse and risk-tolerant investors. Table 10 contains the CVaR and mCVaR values of the created portfolios as well as the calculated hedge effectiveness indices. HEIs are also presented because the downside risk of wheat differs in the three portfolios due to synchronization. Therefore, HEIs are more accurate indicators of extreme risk reduction than the values of CVaR and mCVaR. Figure 3 illustrates the mCVaR efficient frontier lines of the created portfolios and presents the spatial position of all assets in the portfolios and the value of the minimum mCVaR risk of the portfolios.

Comparing Tables 10 and 3, it can be seen that all CVaR and mCVaR values of the created portfolios are lower than the downside risks of wheat, which is a clear sign that all portfolio optimizations are successful. Also, it is evident that all CVaR risks are lower than mCVaR risks in the portfolios, which means that the third and fourth moments play an essential role in calculating downside risk. All downside risk measures are higher at 99% than 95.84% probability, which is expected.

As for the hedging performances of the portfolios, Table 10 shows that the portfolio with precious metals reports the lowest downside risk, the industrial metals portfolio is the second best, and the energy commodity portfolio is the worst. According to HEIs, PMP and IMP have significant advantages in both CVaR and mCVaR portfolios compared to ECP at both probability levels. Precious metals are better hedging instruments than industrial metals when the target is CVaR (0.571 vs 0.515). On the other hand, the advantage of PMP over IMP is more minor in the mCVaR portfolio (0.626 vs 0.625) at 99% probability and (0.581 vs 0.550) at 95.84% probability. At both probability levels, ECP significantly lags behind PMP and IMP. These findings indicate that despite the lower equicorrelation of energy commodities, ECP performs the worst, which means that the level of risk is a decisive factor when it comes to the best hedging performances. In other words, investors must not rely only on equicorrelation results when deciding which auxiliary instruments to use for hedging.

We can find some parallels with other papers, although the existing studies did not hedge wheat, mainly oil, with precious and industrial meals. For instance, Ahmed et al. (2022) researched the relationship between oil and precious metal prices. They asserted that gold is a good hedging instrument due to the lowest tail risk among the four precious metals, which coincides with our findings. Adekoya and Oliyide (2020) find excellent hedging properties of precious metals, combining them with oil. Ali et al. (2023) investigate the interdependence between renewable energy tokens, precious metals, and industrial metals. Their portfolio analysis showed that including energy tokens in a metals-based portfolio presents diversification opportunities, which aligns with our findings.

Table 10 CVaR, mCVaR and HEI Values of the Created Portfolios

	99% probability level			*95.84% probability level*		
	PMP	***IMP***	***ECP***	***PMP***	***IMP***	***ECP***
CVaR	-0.993	-1.135	-1.791	-0.797	-0.911	-1.437
mCVaR	-1.565	-1.590	-2.590	-1.011	-1.107	-1.741
HEI_{CVaR}	0.571	0.515	0.236	0.571	0.515	0.236
HEI_{mCVaR}	0.626	0.625	0.381	0.581	0.550	0.285

Notes: Greyed numbers indicate the best portfolio.

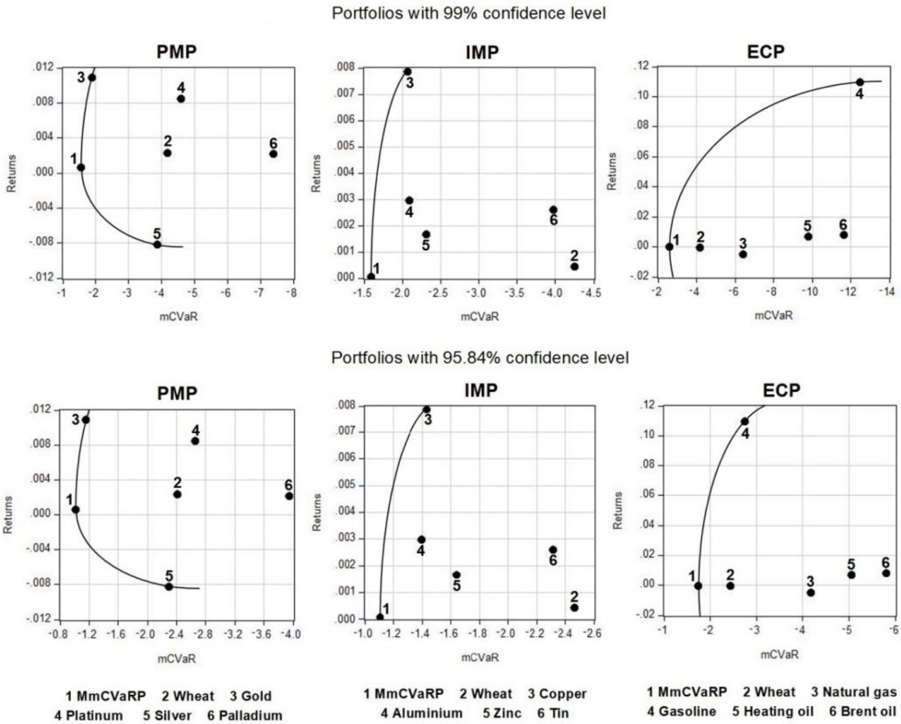
In order to give CVaR and mCVaR models more credibility, we check their adequacy from the aspect of forecasting, and Table 11 contains the results. We refer to Su et al. (2023) and split the whole sample into the in-sample and out-of-sample parts, where the in-sample covers the period between January 2015 and December 2021, while the out-of-sample includes the last two years. The standard coverage test of Kupiec (1995) is used to assess how well the in-sample data forecast the out-of-sample downside risk. According to the findings, PMP has very good forecasting abilities at the 99% confidence level when the target is CVaR. In comparison, the IMP forecast is excellent at 95% probability when the target is mCVaR. On the other hand, ECP has very bad forecasting results. These findings additionally confirm that metals are much better auxiliary instruments for wheat hedging than energy commodities.

Table 11 Results of Portfolio Forecasting

		99% probability level			*95.84% probability level*		
		PMP	***IMP***	***ECP***	***PMP***	***IMP***	***ECP***
CVaR	N	4	25	18	14	39	37
	Z-score	-0.371	9.367	6.001	-1.372	4.450	3.826
	Probability	0.710	0.000	0.000	0.170	0.000	0.000
mCVaR	N	2	6	19	7	20	38
	Z-score	-1.288	0.587	6.458	-2.971	0.074	4.053
	Probability	0.198	0.557	0.000	0.003	0.941	0.000

Notes: N is the number of failures. Greyed values indicate the highest probability and the best model in terms of forecasting.

Figure 3 Efficient Frontier Lines of the Created Portfolios



Notes: PMP, IMP and ECP denote precious metals portfolio, industrial metals portfolio and energy commodity portfolio, respectively.

In the end, we construct efficient frontier lines of the portfolios in Figure 3. In all plots, it can be seen that number one points are positioned at the far left, which confirms visually that all portfolio optimizations are efficient. There is a significant distance between points one and two in PMPs and IMPs, which means that precious and industrial metals are good hedgers of wheat. On the other hand, in the case of ECPs, the distance between points one and two is relatively small, which indicates the poor performance of ECPs.

The results bear several practical implications. The most important implication for investors in wheat is knowing which auxiliary assets are the best for reducing extreme risk in the wheat market. This is particularly important for investors who have exposure to wheat-related assets or industries, such as agricultural companies or food producers. According to the results, both precious and industrial metals can serve very well for wheat hedging, while energy commodities prove to be lousy diversification instruments.

For companies involved in producing, processing or distributing wheat-based products, hedging wheat prices can help manage input costs and stabilize profit margins. This is particularly important for companies that rely heavily on wheat as a raw material in their operations.

Since the paper also explores portfolios suitable for risk-tolerant investors or speculators, the results can be used by agents with specialized knowledge of the wheat market. They can take positions in wheat futures contracts or options and potentially profit from anticipated price movements in the wheat market. The construction of a diversified portfolio can enhance their earnings.

7. Conclusions

This paper investigates the downside risk of wheat hedging by combining wheat with three different types of commodities – precious metals, industrial metals, and energy- in the five-asset portfolios. The research uses portfolio optimization, targeting the semiparametric CVaR metric. Before portfolio construction, we estimate the three DECO-GARCH models to gain an insight into which assets are more integrated.

Estimated equicorrelations indicate that all assets in the portfolios are weakly integrated, which means that all commodities can be good hedging instruments based only on interdependencies. In order to be more thorough in the analysis, the paper tries to answer whether portfolio structure differs if portfolios are constructed at different probability levels. In other words, we distinguish between investors who are risk-averters and those who are more tolerant of risk.

In the CVaR risk-averse portfolios, the precious metals portfolio has the lowest CVaR, while gold dominates the portfolio because gold is by far the least risky asset. The situation is the same in the portfolio with a more tolerant attitude towards risk, i.e. lower probability. In other words, the precious metals CVaR portfolio stands as the best one, while the portfolio's structure does not change at all. This is because the different probability levels in calculating CVaR do not affect variance as the key element in computing CVaR.

On the other hand, in the mCVaR portfolio, the difference is evident between the risk-averse and risk-tolerant portfolios in terms of portfolio structure and risk level. This happens because skewness and kurtosis are sensitive to the probability at which they are calculated. In this regard, the precious metals portfolio is slightly better than the industrial metals portfolio at 99% probability, which means that both precious and industrial metals are good hedgers of wheat when investors are risk-averters. Comparing all CVaR and mCVaR portfolios at both probability levels, it is evident that CVaR is lower than mCVaR because CVaR considers only the first two moments.

This paper could be helpful for agents who work with wheat because it shows how to efficiently decrease wheat's extreme financial risk. An important contribution of the paper is introducing a new risk measure in the portfolio optimization process – semiparametric CVaR, which overcomes the deficiencies reported in the less perfect risk measures. The results indicate that the CVaR risk is lower than the mCVaR risk, which might mislead investors during extreme turbulence. In these specific market conditions, we recommend using the semiparametric CVaR measure.

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