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Dynamic Conditional Systemic Risk Measures

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Abstract

In this paper, we introduce dynamic dependence to the measurement of a number of systemic default risk based on a procedure for a consistent estimation of individual and joint default risk. Focusing on set of univariate and multivariate conditional indicators, our analysis documents a rise in banking systemic fragility in the euro area from the onset of the Subprime Crisis on, with a substantial increase around the First Greek Bailout in May 2010. Our measures also capture significant events in the euro area, such as Mario Draghi's "whatever-it-takes" speech in mid-2012 and the Cypriot Banking Crisis of 2012-2013. The dynamic dependence versions of our measures provide a richer depiction of conditional default risk in the European banking system and in many cases show very different dynamics to their static counterparts. Our results are robust to different approaches for calculating correlations. These results have important policy implications and add to our understanding of systemic risk of European banks.

1. Introduction

The Global Financial Crisis showed that contagion can spread quickly throughout the international financial system due to the interconnectedness of global banks. The Crisis gave impetus to stricter and more comprehensive regulation of banking activities which aimed at improving the health of the banking system and bank resolvability to prevent defaults of large banks to turn into systemic banking crises. A major pillar in the new regulatory and supervisory framework is the consistent and timely assessment of the individual and systemic risks of global banks. Gramlich and Oet (2011) argue that under these new circumstances, the role of a supervisor is twofold. First, to monitor the stability of the portfolio of banks under supervision conditional on their individual financial soundness, and second, to analyze and assess the dynamics of systemic risk, especially in crisis times, characterized by extreme events. This paper uses a comprehensive procedure for assessment of joint default risk to improve upon a number of systemic risk measures that capture conditional systemic default risk within the European banking system.

We extend the regulatory systemic risk measurement toolbox by improving existing measures currently in operational use by the European Central Bank (ECB). In particular, we introduce dynamic dependence to a set of minimum cross-entropy systemic risk measures by Segoviano and Goodhart (2009), Radev (2022b) and Radev (2022c): The conditional probability of a bank defaulting if another bank defaults, the conditional probability of a bank defaulting if two other banks default

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simultaneously and the multivariate conditional probability of at least n banks defaulting, given a particular bank defaults. The first two conditional measures are important for investigating and monitoring specific channels of transmission of default risk throughout the banking system, while the multivariate conditional measure captures the overall risk in the system, taking into account the complete intrinsic dynamic dependence structure among European banks.

The procedure we use consists of three steps. First, we use CDS spreads to extract the perceived individual default risk of 10 European large and complex banking groups (LCBGs) using a CDS bootstrapping procedure (Hull and White, 2000; Gorea and Radev, 2014; Radev, 2022a). Using derivatives that are more sensitive to default risk, such as CDS, allows us to address the problem of the low frequency of default of European banks. Furthermore, our bootstrapping procedure allows us to arrive at expected probability of default using a single CDS spread observation, which significantly reduces input data requirements.

Our next step involves modelling of the multivariate probability density of the European banking system in a way that is consistent with the individual probabilities of default (PoDs). The method that we consider especially suitable for our purposes is the minimum cross-entropy procedure developed by Kullback (1959) and extended by Segoviano (2006) and Segoviano and Goodhart (2009). Segoviano (2006) calls the method Consistent Information Multivariate Density Optimization (CIMDO). The cross-entropy method relies on the intuition of the Merton Model that an institution defaults on its debt once its assets can no longer cover its liabilities, but by focusing on traded default-sensitive CDS data, it avoids the sorting of assets to fit a Merton Model framework, required for instance by the Sovereign Contingent Claims Analysis of Gray, Bodie, and Merton (2007) and Gray (2011). An important modelling innovation of our paper to the measures suggested in Segoviano and Goodhart (2009), Radev (2022b) and Radev (2022c) is the introduction of dynamic correlation to the measurement of conditional systemic risk of banks using rolling windows of changes in 5-year CDS spreads. This approach allows us to derive measures that follow more accurately the level of systemic risk at any given time. After recovering the dynamic multivariate probability density of the banking system, we proceed to our third and final stage – deriving a series of systemic risk indicators that analyze the fragility of the financial system to default events.

Our results show that banking systemic fragility has increased substantially since mid-2007. Several events seem to affect this dynamic: The Subprime Crisis, as well as the Greek fiscal issues and the subsequent attempts by European authorities to defuse the Sovereign Debt Crisis in the euro area. The dynamic dependence versions of our measures provide a richer depiction of conditional default risk in the European banking system and in many cases show very different dynamics to their static counterparts. This underlines the importance of acknowledging the changes in dependence that may occur in crisis times when measuring systemic default risk in the banking system.

We perform important extensions and robustness checks that extend the usefulness of our paper to policymakers and regulators. Data availability precludes us from updating the paper for the full sample of banks to the present day, but we manage to update the measures until mid-2017 for several large European banks that operate throughout the continent: Banco Santander, ING Groep and UniCredit. The new results show that European banks have been affected by the later stages of the Sovereign Debt Crisis, such as Mario Draghi's "whatever-it-takes" speech and the Cypriot Banking Crisis of 2012 and 2013. We also compare pairwise correlations derived from changes in CDS spreads to correlations derived using equity growth data and find no quantifiable differences in the results that could affect policy decision-making. This means that for our sample the dependence in the default region, captured by CDS-derived correlation, is similar to the general dependence, reflected by equity-derived correlations. The choice of the most appropriate data source that regulators should use will vary depending on the circumstances they dace. Such contingency analysis remains for future research.

Our paper is related to the broader literature on developing systemic risk measures for use by policymakers and regulators to monitor the risk in the financial system, as well as the literature on cross-entropy-based measures of banking risk. A number of systemic risk measures have gained popularity since the Global Financial Crises. These include the Conditional Value at Risk, or CoVaR, by Adrian and Brunnermeier (2016), the Marginal and Expected Systemic Shortfalls by Acharya and Richardson (2009) and Acharya et al. (2017) and the SRISK by Brownlees and Engle (2017). These measures use historical equity and balance sheet data to assess interactions of a single bank and an aggregated systemic index in the tail of a bivariate joint distribution and, while helpful to regulators, do not capture the complete dependence structure among banks in the financial system. That issue is addressed partially by the multivariate CDS-based measures of Segoviano and Goodhart (2009), who use the CIMDO approach developed in Segoviano (2006) to calculate unconditional joint probabilities of default, bivariate conditional probabilities and probabilities of at least one bank defaulting given a particular bank defaults. While extending the regulatory toolkit, these measures suffer from a major drawback – they use Gaussian distributions as prior distributions in the CIMDO model and assume that the banks are not correlated. Radev (2022b) shows that the independence in a Gaussian prior distribution transfers to the posterior CIMDO distribution and therefore the conditional measures based on the modelling in Segoviano and Goodhart (2009) are equivalent to their unconditional counterparts and do not provide any additional informational content beyond the unconditional case. This is not useful for policymakers who need to base their decisions on different scenarios of triggers and spillover effects during a crisis. This issue is first discussed in Gorea and Radev (2014), who provide a number of sensitivity checks and discuss drivers of correlated joint probabilities of default during the Sovereign Debt Crisis in the euro area. Radev (2022b) and Radev (2022c) extend the set of systemic risk measures using the cross-entropy approach by introducing the Conditional Joint Probability of Default and the unconditional Systemic Risk Measure (SFM). Radev (2022d) extends the Systemic Fragility Measure by incorporating the Leave-one-out (LOO) approach of Hue et al (2019). The new SFM-LOO measure presents a ranking of unconditional systemic risk and is the unconditional counterpart of the CoJPoD of Radev (2022c). While most previous measures use a static correlation matrix, Radev (2022d) also shows that the results change substantially when dynamic correlations are used to calculate the *unconditional* SFM and SFM-LOO. Our paper extends these previous efforts by introducing dynamic dependence to a number of *conditional* measures of default risk.

The paper that is most related to our modelling approach is Jin and De Simone (2014), where the authors apply a BAKK approach (Engle and Kroner, 1995) to model the correlation dynamics within the *unconditional* joint probability of default (JPoD; Segoviano and Goodhart, 2009) of the riskiest five banks in Luxemburg at any given time. While using a more complicated modelling of dependence than our setup, that paper has a number of drawbacks. First, it introduces a large number of new parameters to be estimated alongside the already computationally-heavy CIMDO approach. This forces the authors to use only 5 banks of their broader sample of banks at a time. Second, they rank their banks by assets to decide which one to calculate the index for at any given time. While useful in assessing general unconditional systemic vulnerability through JPoD, this selection approach provides inconsistent results when we want to trace particular channels of spillover effects within the financial system (e.g., which bank is most likely to be next to default) and is therefore not useful for monitoring of contagion effects.

Our paper is also related to the literature of information contagion by Acharya and Yorulmazer (2008), who postulate that a default of a given bank may provide valuable information about the default of other banks that are considered similar to it, or of the entire financial system. We extend the earlier contributions to the literature by Radev (2022b) and Radev (2022d) by introducing dynamic dependence into the modelling of conditional systemic risk.

We have several major contributions to the existing literature. First, we add to a broader research agenda with the main purpose to shed more light on the distress vulnerability of the European financial system during the recent crises (see Gorea and Radev, 2014; Acharya and Steffen, 2015; Radev, 2022b; Radev, 2022c and Radev, 2022d). Second, the current work enhances the multivariate probability measures of banking systemic risk developed by Segoviano and Goodhart (2009), Radev (2022b) and Radev (2022c) by introducing dynamic dependence into the modelling of conditional systemic risk. Last, we provide sensitivity checks for alternative modelling of dependence that are useful for policy decision-making, which is often based on incomplete data from varying sources.

The paper is organized as follows. Section 2 presents our methodology for deriving marginal and joint probabilities of default. In section 3, we introduce our probability measures and provide guidelines for their calculation. Section 4 describes briefly our dataset, while section 5 presents our empirical results. Section 6 concludes.

2. Methodology

Our approach to estimate multivariate conditional probability measures comprises three steps. First, we extract PoDs from CDS spreads with the CDS bootstrapping procedure in Hull and White (2000). Second, we use a minimum cross-entropy approach to construct a *multivariate* probability distribution consistent with the *individua*l probabilities of default (Kullback, 1959; Segoviano, 2006) and with the dynamics of the dependence among banks in the system (Radev, 2022d). Last, we calculate a number of systemic distress measures to trace the risk in the banking system in the euro area.

2.1 Deriving Marginal Probabilities of Default

In this paper, we employ a CDS bootstrapping procedure to estimate probabilities of default (PoD).[1] The procedure follows Hull and White (2000) and is based on a cumulative probability model that incorporates recovery rates, refinancing rates and cumulative compounding. The method uses CDS contracts of different maturities to calibrate hazard rates of particular time horizons to estimate cumulative probabilities of default.

The intuition behind the CDS bootstrapping is to use default-sensitive contracts traded in the insurance market such as credit default swaps that aim to protect the insurance buyer against the default of the underlying asset, to reverse-engineer the probabilities of default that clear the market for these instruments such that no party has an arbitrage advantage (the no-arbitrage condition on financial markets). There are many approaches and proxies that try to derive PoDs from CDS contracts, but the most popular, sound and consistent modeling procedure that uses the whole term structure (if available) of CDS spreads of individual entities is described by Hull and White (2000). We outline CDS bootstrapping in more detail in the online appendix.

This method could be used for both sovereign and corporate probability of default estimation. The resulting risk measures are risk-neutral probabilities of default and satisfy the no-arbitrage condition in financial markets (Hull, 2006). Risk-neutral probabilities are the probabilities that make market participants indifferent to buying or selling an asset under the respective market conditions. Risk-neutral probabilities differ from the actual (or physical) probabilities that take into account the risk-aversion of market participants. In practice, the latter could be derived from the former using empirical approximations of risk-aversion, such as the Sharpe's Ratio (Sharpe, 1966). We decided to report risk-neutral probabilities, since they are typically larger than their respective physical counterparts and thus deliver more conservative estimates of default risk. This paper deals with very rare events and we argue that policymakers should employ more conservative estimates to monitor the default risk of the banks under their monitoring mandate.

We use all available maturities from 1 to 5 year of CDS spreads to recover bank PoDs and adjust for quarterly premium payments and accrual interest, as suggested by Adelson, Bemmelen, and Whetten (2004). As risk-free rates we use all available maturities of AAA Euro Area bond yields from 1 to 5 years. The recovery rate is uniformly set at 40 %, both for banks and sovereigns, as this is the prevailing assumption in literature and practice.[2] The resulting series are cumulative probabilities of default. To arrive probabilities with the one-year horizon of interest to policy makers, they have to be annualized, using the following formula:

$$PoD_t^{\text{annual}} = 1 - (1 - PoD_t^{\text{cum}})^{\frac{1}{T}}, \#(1)$$

[1] CDS bootstrapping has no direct relation to statistical bootstrapping that uses past values, which may be autocorrelated. The procedure uses contemporaneous CDS spreads with different time horizons of default protection from 1 to 5 years from the current time t on.

[2] For a discussion on how different recovery rates affect the PoD estimates, please refer to Gorea and Radev (2014).

where T is the respective time horizon (T=5 for 5-year PoD) and PoD_t^{annual} is the annualized version of the cumulative PoD_t^{cum} .

2.2 Recovery of the Multivariate Probability Density

Since joint credit events are rarely traded in the insurance market, we need to impose a certain structure on the system's multivariate probability density to allow the transition from individual to joint PoDs. We base our methodology on the concept of cross-entropy, introduced by Kullback (1959). Our approach minimizes a cross-entropy objective function that updates a prior distribution to arrive at a posterior distribution governed by a set of constraints. The constraints are chosen to ensure that the posterior distribution is consistent with the individual PoDs.

Cross-entropy is related to and shares most of its vocabulary with Bayesian statistics. The goal of this method is to arrive at a reasonable approximation of an unknown joint asset distribution that captures as many of the intrinsic characteristics of the available data (such as dependence, fat tails, skewness, etc.) as possible by adjusting some prior "guess" multivariate distribution. In practice, what the procedure described below achieves is to arrive at fatter tails by shifting probability mass from the center of the joint probability distribution to its tails beyond a fixed Merton-like threshold. This must be achieved in a manner that delivers a consistent tail mass with the individual probabilities of default that are derived from individual CDS spreads. Since markets are incomplete, there are no traded baskets of CDS for every possible contingency and therefore, these approximations are considered reasonable. In banking literature, usually a normal distribution without cross-entity correlation is chosen as a prior (see Segoviano, 2006; and Segoviano and Goodhart, 2009). Gorea and Radev (2014) show empirically, and Radev (2022a) proves analytically that the choice of correlation structure for the prior distribution is important for the final correlation structure in the posterior distribution, and since bank assets are correlated, the authors argue in favour of using a static correlation matrix for the prior distribution as an improvement over the zero-correlation model in Segoviano (2006). As an extension to Gorea and Radev (2014), Jin and De Simone (2014) use the BEKK model of Engle and Kroner (1995) to arrive at time-varying covariance for a 5-dimensional portfolio of banks operating in Luxembourg. Gorea and Radev (2014) provide additional sensitivity checks and find that using a distribution with fatter tails as a prior, such as a t-distribution with a low number of degrees of freedom yields only marginally different joint probabilities of default. The reason is that the multivariate probability measures discussed in the literature summarize the mass in the tails of the joint distribution and not necessarily the shape of the distribution in the tails. Given the limited benefits and the significant increase in the computational time when using a more complicated distribution, especially at higher dimensions, Radev (2022a) argues in favor of using a joint normally distributed prior.

To proceed, let the financial system be represented by a portfolio of n sovereigns or banks: X_1, X_2, to X_n., with their log-assets being x_1, x_2, to x_n. The cross-entropy approach then minimizes the following Lagrangian:

$$
\begin{aligned}
L(p,q) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)\ln\left[\frac{p(x_1,x_2,\dots,x_n)}{q(x_1,x_2,\dots,x_n)}\right]dx_1\cdots dx_{n-1}dx_n \\
&+\lambda_1\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)\mathbf{I}_{[\bar{\mathbf{x}}_1,\infty)}dx_1\cdots dx_{n-1}dx_n - PoD_t^1\right] \\
&+\lambda_2\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)\mathbf{I}_{[\bar{\mathbf{x}}_2,\infty)}dx_1\cdots dx_{n-1}dx_n - PoD_t^2\right] \\
&+\cdots \\
&+\lambda_n\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)\mathbf{I}_{[\bar{\mathbf{x}}_n,\infty)}dx_1\cdots dx_{n-1}dx_n - PoD_t^n\right] \\
&+\mu\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)dx_1\cdots dx_{n-1}dx_n - 1\right]
\end{aligned}
\tag{2}
$$

The first integral in Equation (2) represents the cross-entropy probability difference (see Kullback (1959)) that minimizes the distance between a prior distribution guess $q(x_1,x_2,\dots,x_n) \in R_n$ and a posterior distribution $p(x_1,x_2,\dots,x_n) \in R_n$ that reflects empirical market data on individual probabilities of default. PoD_t^1, PoD_t^2 to PoD_t^n stand for the expected probabilities of default of the respective entities, derived from CDS prices. With $\mathbf{I}_{[\bar{\mathbf{x}}_1,\infty)}$, $\mathbf{I}_{[\bar{\mathbf{x}}_2,\infty)}$ to $\mathbf{I}_{[\bar{\mathbf{x}}_n,\infty)}$ we denote a set of indicator variables that take the value of one if the respective entities' default thresholds x_1, x_2, to x_n are crossed and zero otherwise. The default thresholds are the same as in the classic structural model (Merton, 1974). μ, λ_1, λ_2 to λ_n are the Lagrange multipliers of the constraints. The optimal posterior distribution is then:[3]

$$
p^*(x_1,x_2,\dots,x_n) = q(x_1,x_2,\dots,x_n)\exp\left\{-\left[1+\mu+\sum_{i=1}^{n}\lambda_i\mathbf{I}_{\mathbf{x}_i,\infty)}\right]\right\} \tag{3}
$$

Therefore, in order to derive the optimal posterior distribution, all we need is the prior distribution with an appropriate dependence structure (e.g., multivariate Gaussian density with an empirical correlation matrix), the optimal Lagrange multipliers and the individual default thresholds. The resulting posterior joint distribution has two main properties: first, it reflects the market consensus views about the default region of the unobserved asset distribution of the system, and second, it possesses fat tails, even if the starting assumption is a multivariate Gaussian distribution.

2.3 Dynamic Dependence

Since the minimum cross-entropy procedure that we employ has strong conceptual parallels with the classical structural model (Merton, 1974), an important issue is the appropriate choice of correlation structure that captures the dependence among banks. Gorea and Radev (2014) and Radev (2022a) show that if independence is incorporated in the prior distribution of the minimum cross-entropy setup as in

[3] See the Appendix and Radev (2022a) for a complete solution of the multivariate minimum cross-entropy problem.

Segoviano (2006) and Segoviano and Goodhart (2009), it transfers also to the posterior distribution. Hence, the values of the probability measures derived via the cross-entropy approach may not incorporate the true interdependence between entities: Even if we observe intertemporal dynamics in these measures, it is driven only by the individual probabilities of default of each member of the system and not by interdependence.

In Merton (1974) dependence is defined as pairwise correlation between corporate assets returns. Huang, Zhou, and Zhu (2009) proxy for asset return correlation with equity return correlation referring to the notion that equity is a call option on the underlying assets of a firm. Zhu and Tarashev (2008) approximate asset return correlation based on single-name CDS spreads. Since the minimum cross-entropy approach effectively uses individual CDS spreads to proxy individual asset return dynamics of banks, we argue that it is natural to use CDS spreads to calculate asset correlation as well as these will reflect the dependence in the tail of the distribution. Indeed, Gorea and Radev (2014) and Radev (2022b) improve upon the approach by Segoviano (2006) and Segoviano and Goodhart (2009) by substitution the identity matrix inherent to the CIMDO approach with an empirical correlation matrix derived using the 5-year CDS spreads of the respective entities.

However, the fixed correlation matrices in Gorea and Radev (2014) and Radev (2022b), despite closer to reality, still do not reflect the true dynamics of dependence among banks and sovereigns. Early research in the contagion literature observes that in times of crises, correlation among stock markets increases (see, for instance, Forbes and Rigobon, 2002), a phenomenon, which Longin and Solnik (2001) associate with contagion in financial markets. Furthermore, using constant average correlation over the whole sample incorporates information not only from past periods, but also from future periods to the current period t. Therefore, this approach does not reflect reality, since the magnitude and duration of future crises and shocks are generally unpredictable.

To address these issues, we employ dynamic correlation matrices to calculate our multivariate conditional measures by computing pairwise correlations using the 5-year CDS spreads 3 months (60 days) prior to period t. We argue that the introduction of dependence structure dynamics to our approach in addition to the dynamics of individual probabilities allows us to arrive at measures that follow more closely the level of systemic risk at any given time. In Section 5.3.2, we compare the correlations derived from changes in CDS spreads to correlations from equity growth rates for several banks in our sample.

3. Dynamic Conditional Probability Measures

This section describes the conditional bivariate and multivariate measures, introduced in Radev (2022c). Section 5 will compare visually the dynamic correlation measures to their static versions in Radev (2022c).

3.1 Dynamic Probability of A Defaulting Given B Defaults

We start with the simplest extension beyond the unconditional joint probability framework: the probability of default of bank A (say UniCredit) given bank B (say Intesa Sanpaolo) defaults (P(A|B)), introduced in Radev (2022c).

Deriving P(A|B) is a direct application of the Bayes rule:

$$(A \mid B) = \frac{P(A,B)}{P(B)} \tag{4}$$

where $P(A,B)$ is the joint probability of default of banks A and B, while $P(B)$ is the marginal probability of default of bank B.

This measure is useful in analyzing particular channels of contagion from one bank to another or vice versa. Since $P(A \mid B)$ is rarely equal to $P(B \mid A)$,[4] we can discern which of both banks in the couple is more vulnerable to a default of its counterpart. For policymaker purposes, it can be incorporated in tables or heat maps with average conditional PoD containing all possible bivariate couples, akin correlation tables. In contrast to correlation tables, however, the corresponding values across the main diagonal of the PoD table will not be equal.

3.2 Dynamic Probability of A Defaulting Given B and C Default

The next indicator introduced in Radev (2022c) measures the conditional probability of default of a bank, given two other banks default simultaneously. In the Bayes' framework, mentioned above, this probability of default is defined as

$$P(A \mid B,C) = \frac{P(A,B,C)}{P(B,C)} \tag{5}$$

with $P(A,B,C)$ and $P(B,C)$ being, respectively, the joint probabilities of banks A, B and C, and of banks B and C defaulting. For instance, this will measure the probability of default of UniCredit, given Intesa Sanpaolo and BNP Paribas *jointly* default.

The procedure for the calculation of the measure is similar to the method in the previous subsection, but this time it involves 3– and 2–dimensional joint probabilities of default. The measure is particularly useful when measuring the risk of a bank run on several banks to spread further throughout the system.

3.3 Dynamic Conditional Probability of at Least N Banks Defaulting

Our final (and most complex) probability measure is the probability of at least *n* banks defaulting, given a particular bank defaults (PAN). This measure is a generalization of the probability of at least one (PAO) bank defaulting, introduced in Segoviano and Goodhart (2009) and aims at gauging the expected severity of a crisis stemming from a particular bank, and hence, the rate of *contagion penetration* in the financial system. In contrast to the Systemic Fragility Measure introduced in Radev (2022c), which reflects the overall *unconditional* fragility of the system, the PNBD is a *conditional* measure that gauges the level of systemic fragility in case of a default of one of its participants.

To define the measure, let us consider again a system of three banks, A, B and C.[5] The probability of at least one additional bank defaulting given a particular bank

[4] Actually, both measures are equal if and only if the individual unconditional probabilities are equal.
[5] The extension to higher dimensions, although more involving, is straightforward, as long as we keep account of the default contingencies to be added or subtracted.

(say C) defaults is then

$$PAN(\text{ at least } 1 \mid C) = P(A \mid C) + P(B \mid C) - P(A, B \mid C) \tag{6}$$

where $P(A \mid C)$, $P(B \mid C)$ and $P(A, B \mid C)$ are the respective conditional probabilities for all possible default contingencies. Using this intuition, it is easy to proceed one step further and derive the probability of at least two banks (in this case A and B) defaulting given bank C defaults:

$$PAN(\text{ at least } 2 \mid C) = P(A, B \mid C) \tag{7}$$

In the limit (i.e. for N-1 additional entities defaulting), the PAN converges to the Conditional Joint Probability of Default (CoJPoD), introduced in Radev (2022b):

$$\text{CoJPoD}\left(\text{ System }_{-C} \mid C\right) = P\left(\text{ System }_{-C} \mid C\right) \tag{8}$$

where $\text{CoJPoD}\left(\text{ System }_{-C} \mid C\right)$ is the probability of the remaining banks in the system to default, given a bank C defaults.

3.4 Practical Considerations for Policymakers

This paper improves upon the family of measures introduced in Radev (2022c) by applying dynamic dependence structure to the prior distribution in the CIMDO procedure. For the purposes of practical implementation of the discussed measures, it is important to note that since their calculation involves a different number of banks, they are distinct measures and not variations of a single measure. Therefore, these measures should be interpreted with care. Although we use 10-dimensional distribution of banks in this paper, to arrive at the simpler conditional measures in Equation (4) and (5), we reduce the dimensions to the needed joint probabilities and individual probabilities by integrating over the values of the banks that we do not need. For instance, $P(A \mid B)$ involves a portfolio of two banks, A and B, and assumes independence with the rest of the banks in the system, and hence is based on a bivariate distribution, achieved by integrating over the remaining 8 banks. $P(A \mid B, C)$ involves three banks and therefore is based on a trivariate distribution, achieved by integrating over the remaining 7 banks. $PAN(\text{ at least } 1 \mid C)$ is based on all 10 banks and therefore stems from a 10-dimensional distribution.

Since the number of dimensions of the multivariate distribution matters in probability theory, the values of these three measures are not directly comparable. To see this, consider the numerators $P(A, B)$ and $\mathrm{P}(A, B, C)$ in equations (4) and (5), respectively. Intuitively, increasing the number of defaulting banks means that it is less likely that *all* banks will default, and therefore $\mathrm{P}(A, B, C)$ is smaller than $P(A, B)$. However, since we use a bivariate distribution in the latter case, we assume independence of banks A and B with bank C. Therefore, in the bivariate setting of equation (4), adding bank C in the joint PoD will be represented as $P(A, B) \cdot P(C)$, which is different to the $\mathrm{P}(A, B, C)$ in equation (5) which is derived from a trivariate distribution with a non-zero correlation structure. However, we can compare the values of PAN for a different number of defaulting banks, because they will all stem from the same 10-dimensional distribution, where we sum up the regions where at least n banks default given a particular bank, say BNP Paribas, defaults.

To sum up, we can compare the values of the pairwise probabilities $P(A \mid B)$ across different pairs, because they stem from bivariate (albeit different) distributions. We can also compare the different combinations of $P(A \mid B, C)$ probabilities, since they come from trivariate (albeit different) distributions. But we cannot compare the levels across $P(A \mid B)$ and $P(A \mid B, C)$ probabilities. These measures serve different purposes for policymakers and, e.g., may measure the vulnerability of a bank to the default of a particular bank or the joint default of a couple of banks. Since there may be unlimited contingencies policymakers may be interested in, there is an unlimited probability measures that may be created.

4. Data

We recover marginal probabilities of default using CDS premia for contracts with maturities from 1 to 5 years for the period 01.01.2007 and 31.12.2011. The bootstrapping procedure requires as additional inputs refinancing interest rates, which we choose to be the AAA euro area government bond yields for maturities from 1 to 5 years. The CDS spreads and the government bond yields are at daily frequency, which is also the frequency of the resulting probabilities of default. Our analysis covers 10 EA large and complex banking groups used by the European Central Bank to calculate various probability-based systemic risk measures, such as the Systemic Fragility Measure (Radev and Alves, 2012).[6] Each banking group has a minimum total assets value of EUR 200 Billion. The sample is presented in Table 1.

Table 1 List of Euro Area Large and Complex Banking Institutions Used in our Analysis

Euro Area Large and Complex Banking Groups		
	Country code	***Name***
1	DE	Commerzbank AG
2	DE	Deutsche Bank AG
3	ES	Banco Bilbao Vizcaya Argentaria
4	ES	Banco Santander SA
5	FR	BNP Paribas
6	FR	Credit Agricole SA
7	FR	Societe Generale
8	IT	Intesa Sanpaolo SpA
9	IT	UniCredit SpA
10	NL	ING Groep NV

Table 2 presents the descriptive statistics of the 5-year CDS spreads of the 10 banks in our sample. The average 5-year CDS spread in the cross-section ranges from 82.8 basis points for BNP Paribas to 133.7 basis points for BBVA. We also

[6] The banks used in our analysis are listed in Table 1.

notice a substantial increase of CDS premia even for the safest bank in the beginning of the sample, ING Groep, from 4 to 251.8 basis points. However, this does not compare to the dynamics of the price for protection against the default of Intesa Sanpaolo, which starts at 5.4 basis points at the beginning of the period and reaches a maximum 635.5 basis points. We also notice that, on average, French, German and Dutch banks exhibit the lowest volatility in the price for protection against default.

Table 2 Descriptive Statistics of the 5-Year CDS Spread Series of 10 LCBGs

	BBVA	*B. Santander*	*BNP Paribas*	*Commerzbank*	*Credit Agricole*
Minimum	7.10	7.00	5.50	7.90	5.50
Mean	133.65	130.55	82.82	101.93	103.56
Maximum	473.17	473.54	361.62	364.65	363.31
Std. dev.	98.14	94.41	63.02	67.10	67.29
Nr. of obs.	1305	1305	1305	1305	1305

	Deutsche Bank	*ING Groep*	*Intesa Sanpaolo*	*Societe Generale*	*UniCredit*
Minimum	8.70	4.00	5.40	5.70	7.48
Mean	94.39	94.42	111.53	107.96	103.49
Maximum	317.28	251.79	625.50	434.14	398.89
Std. dev.	49.69	54.76	108.98	82.24	70.80
Nr. of obs.	1305	1305	1305	1305	1305

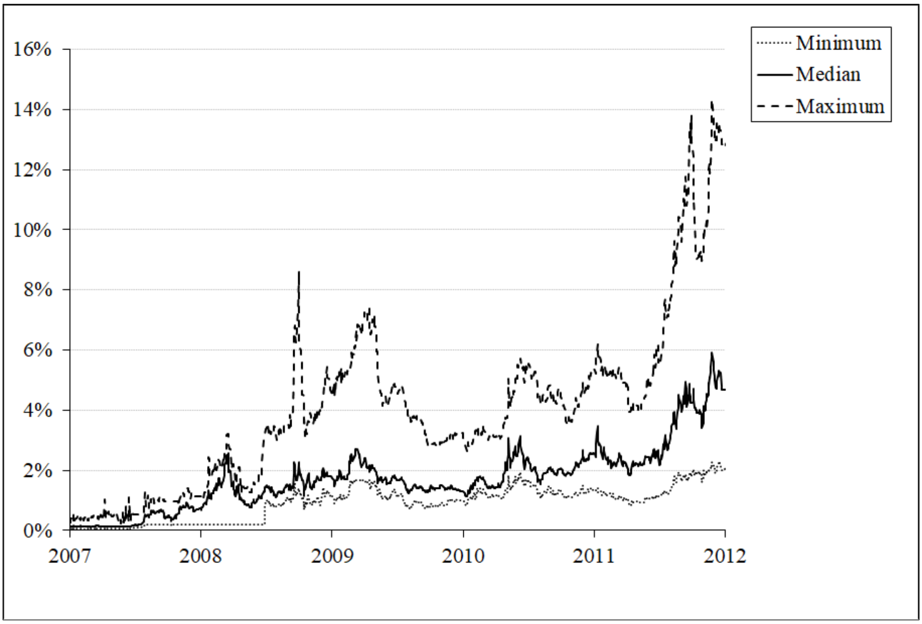
Notes: BBVA, Santander, BNP PARIBAS, Commerzbank, Credit Agricole, Deutsche Bank, ING Groep, Intesa Sanpaolo, Societe Generale and UniCredit. The data are in basis points. Period: 01.01.2007 – 31.12.2011.

5. Empirical Results

5.1 Marginal Probability of Default Results

This section presents the results for the individual probabilities of default for our sample of banks. In Figure 1, we present the minimum, median and maximum of the 5-year annualized CDS-implied probabilities of default of the 10 banks in our sample. The series reveal that the market considered the large and complex banking groups in the euro area relatively default-free before the start of the Subprime Crisis in August 2007 and experienced the first larger spike in marginal default risk around the bailout of Bear Stearns in the spring of 2008. We notice that the individual default risk was stable around 2% during the financial crisis and the following global recession. Two subperiods can be noticed in the PoD dynamics in the second half of the sample period: between the exacerbation of the sovereign debt crisis at the end of the first quarter of 2010, and the second quarter of 2011 and the period thereafter.

Figure 1 Minimum, Median and Maximum of 5-Year Annualized CDS-Implied Bootstrapped Probabilities of Default for 10 Banks



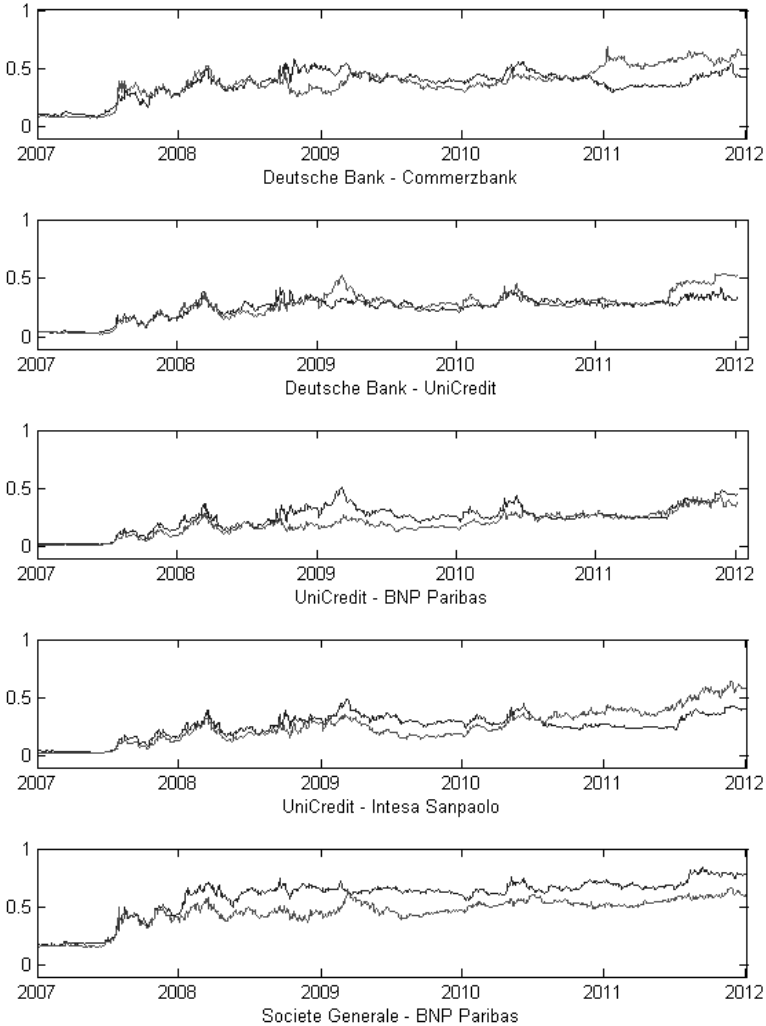
Notes: Minimum (dotted line), Median (solid line) and Maximum (dashed line) of 5-Year Annualized CDS-Implied Bootstrapped Probabilities of Default for BBVA, Banco Santander, BNP Paribas, Commerzbank, Credit Agricole, Deutsche Bank, ING, Intesa Sanpaolo, Societe Generale and UniCredit. Euro-denominated CDS spreads are used. Period: 01.01.2007 - 31.12.2011. Source: own calculations.

5.2 Conditional Probabilities Results

Figure 2 and Figure 3 depict the univariate bank probabilities of default, given a particular bank defaults for the static (Figure 2) and the dynamic (Figure 3) cases.[7] In the static case in Figure 2, we notice interesting patterns in the banking measures. In most cases, the conditional PoDs within the couples trace each other narrowly, which means that they have very similar individual *unconditional* probabilities of default (the denominator in Equation 4). For instance, the respective conditional probabilities of default of Deutsche Bank and Commerzbank trace each other very narrowly until the beginning of 2011. Throughout 2011, however, the PoD of Deutsche Bank defaulting given Commerzbank defaults is higher than its mirror counterpart. This means that international investors considered Commerzbank a safer bank at the time and if it had defaulted, they would have expected that that it would have been more likely for the riskier Deutsche Bank to follow suit. What we also notice is that the PoDs across subplots differ to a large degree, both in level and dynamics, even when the plots include the same bank (e.g., compare the subplots for Deutsche Bank – Commerzbank and Deutsche Bank – UniCredit). This can be attributed to the different levels of dependence across couples.

[7] For the sake of brevity, we report the results only for several of the couples. The remaining results are available upon request.

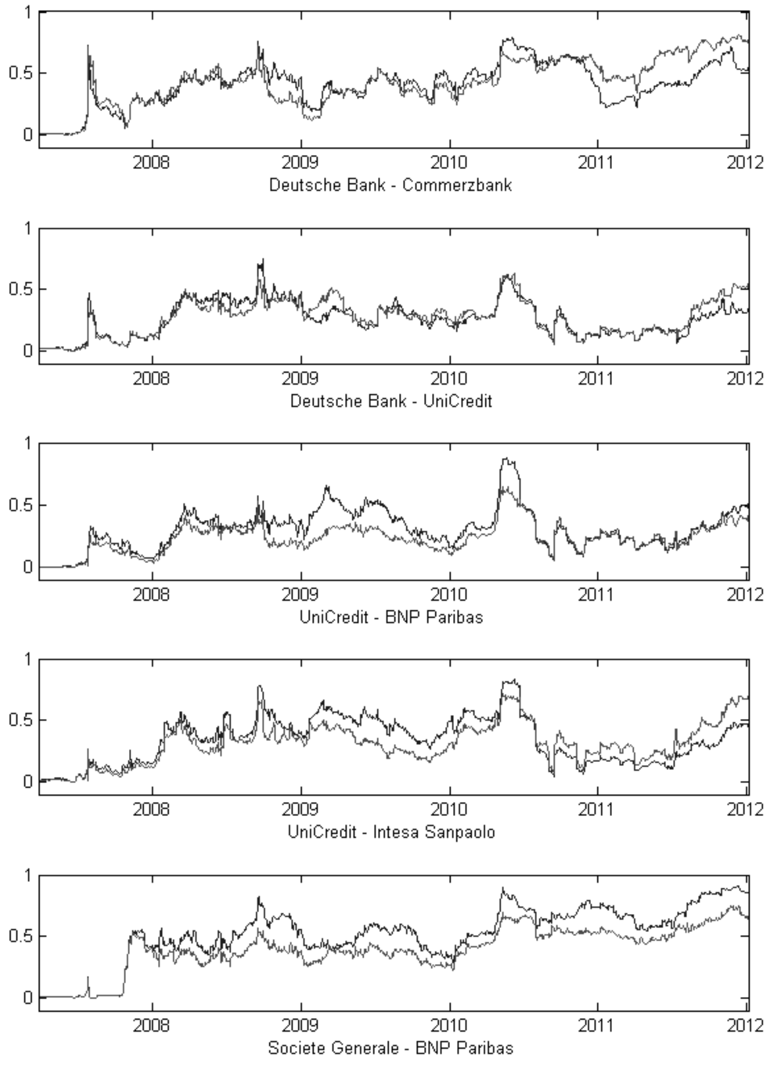
Figure 2 Banking Conditional Probability of Default Given a Particular Bank Defaults: Static Case



Notes: 5-year annualized conditional probabilities of default of selected complex banking groups in the period 01.01.2007 - 31.12.2011. The black (grey) line corresponds to the probability of default of the first (second) bank listed in the couple, given the second (first) bank defaults. E.g., the black line in the top plot represents the probability of default of Deutsche Bank given Commerzbank defaults, while the grey line corresponds to the probability of default of Commerzbank given Deutsche Bank defaults. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads.

Source: own calculations.

Figure 3 Banking Conditional Probability of Default Given a Particular Bank Defaults: Dynamic Case



Notes: 5-year annualized conditional probabilities of default of selected complex banking groups in the period 01.01.2007 - 31.12.2011. The black (grey) line corresponds to the probability of default of the first (second) bank listed in the couple, given the second (first) bank defaults. E.g., the black line in the top plot represents the probability of default of Deutsche Bank given Commerzbank defaults, while the grey line corresponds to the probability of default of Commerzbank given Deutsche Bank defaults. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads over a 3-month (60 business days) rolling window.

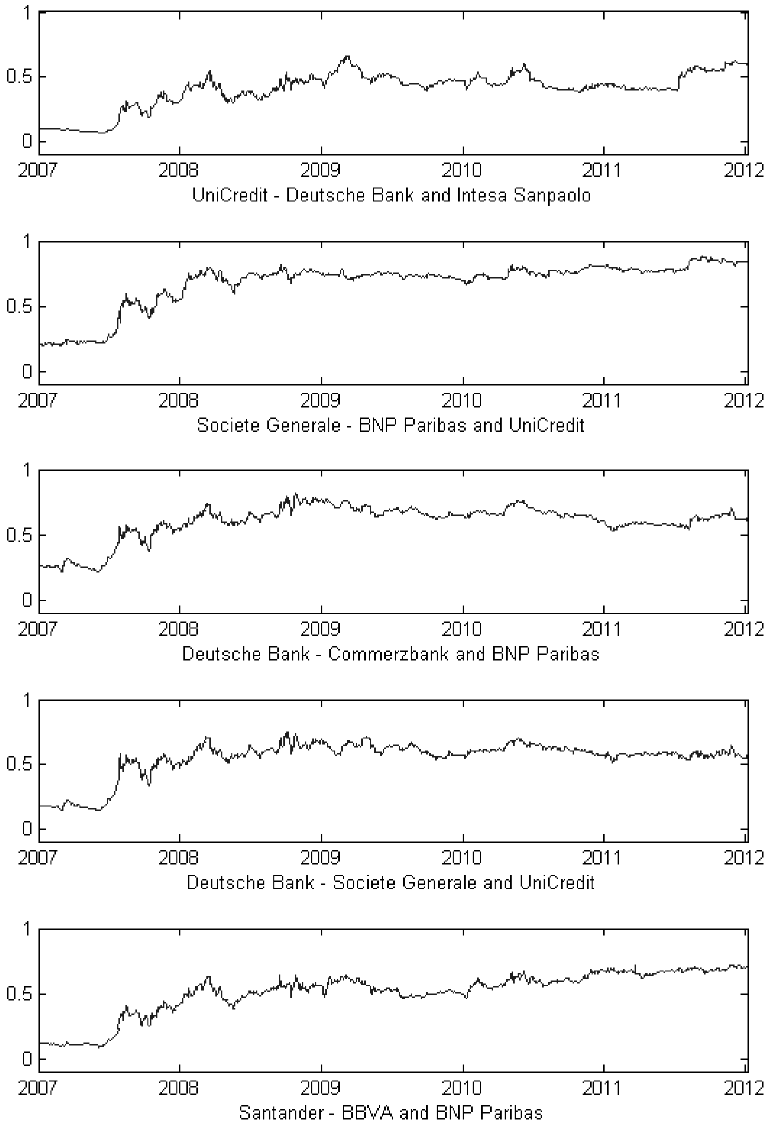
Source: own calculations.

The dynamic case in Figure 3 reveals substantial differences to the static case in Figure 2. We notice an increase in the level of default risk and much more pronounced spikes, compared to the static plots. Since the input data is the same across figures, we can attribute the change in dynamics exclusively to the application of dynamic correlation, which captures better the changing patterns of dependence during crisis and tranquil times. We also notice that the PoDs using the averaged-out static correlations are rarely above the measures using a rolling window, which shows that in the former case, we lose a lot of nonlinear dynamics in the joint distribution of bank assets.

In Figure 4 and Figure 5, we present univariate bank probability results, conditional on two banks defaulting, for the static and dynamic case, respectively. The top subplot in Figure 4 reveals that a joint default of Deutsche Bank and Intesa Sanpaolo has a substantial effect on the default perceptions with regard to UniCredit. We observe similar patterns for all triplets, indicating that a joint default of any two banks would have a large detrimental effect on a third bank.

Incorporating dynamic correlation in Figure 5 again enriches our understanding of joint default risk. The spikes are again much more pronounced, especially around the bailout of Greece in May 2010. The dynamics is very different, compared to the static correlation case, especially for the first and the last subplots: UniCredit – Deutsche Bank and Intesa Sanpaolo, and Santander – BBVA and BNP Paribas, respectively. In the former case, we observe a notable drop in the conditional default risk of UniCredit after the First Greek Bailout, which is not that pronounced in the static version. In the latter case, we document a very dynamic pattern of the conditional default risk of Santander, ranging from as low as 25% to as high as 99% default probability in case of BBVA and BNP Paribas default. These values are again driven by the very dynamic dependence patterns among these banks, which our approach manages to capture.

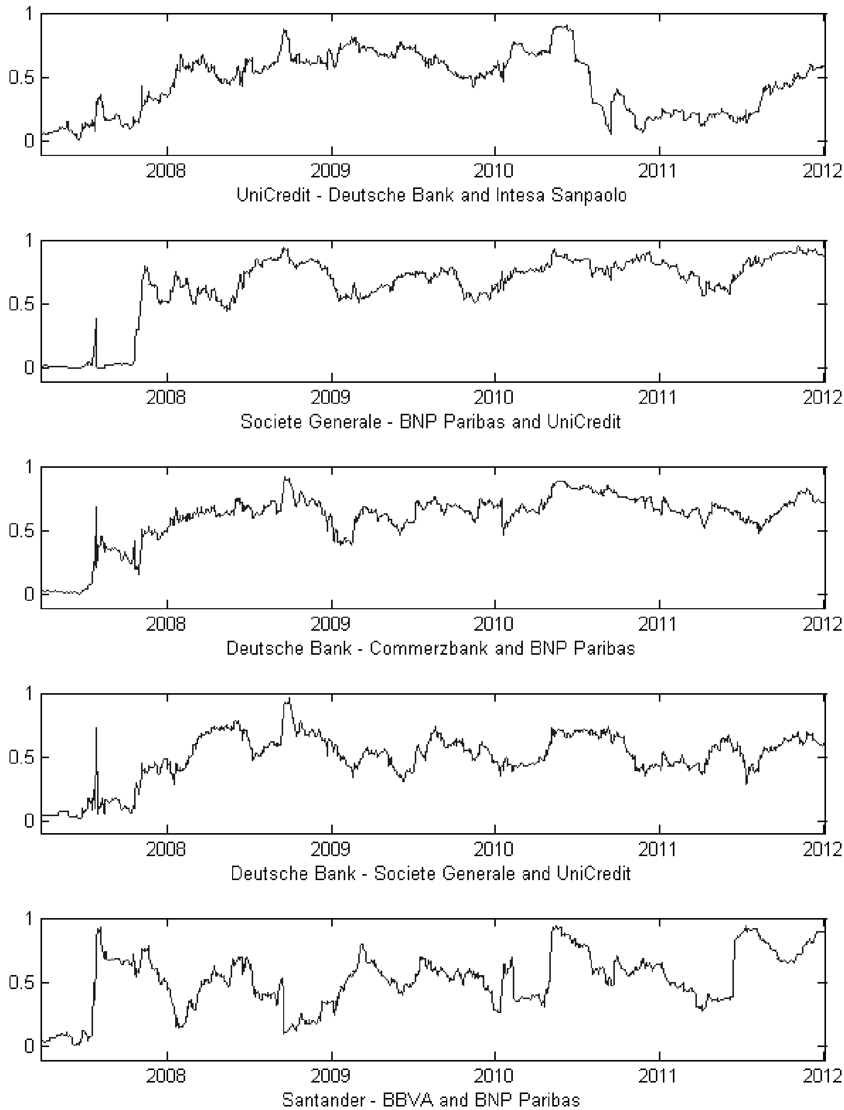
Figure 4 Banking Conditional Probability of Default Given Two Banks Default: Static Case



Notes: 5-year annualized conditional probabilities of default of selected complex banking groups in the period 01.01.2007 - 31.12.2011. The black line corresponds to the probability of default of the first bank listed in the couple, given the remaining two listed banks default simultaneously. E.g., the line in the top plot represents the probability of default of UniCredit given Deutsche Bank and Intesa Sanpaolo default. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads.

Source: own calculations.

Figure 5 Banking Conditional Probability of Default Given Two Banks Default: Dynamic Case



Notes: 5-year annualized conditional probabilities of default of selected complex banking groups in the period 01.01.2007 - 31.12.2011. The black line corresponds to the probability of default of the first bank listed in the couple, given the remaining two listed banks default simultaneously. E.g., the line in the top plot represents the probability of default of UniCredit given Deutsche Bank and Intesa Sanpaolo default. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads over a 3-month (60 business days) rolling window.

Source: own calculations.

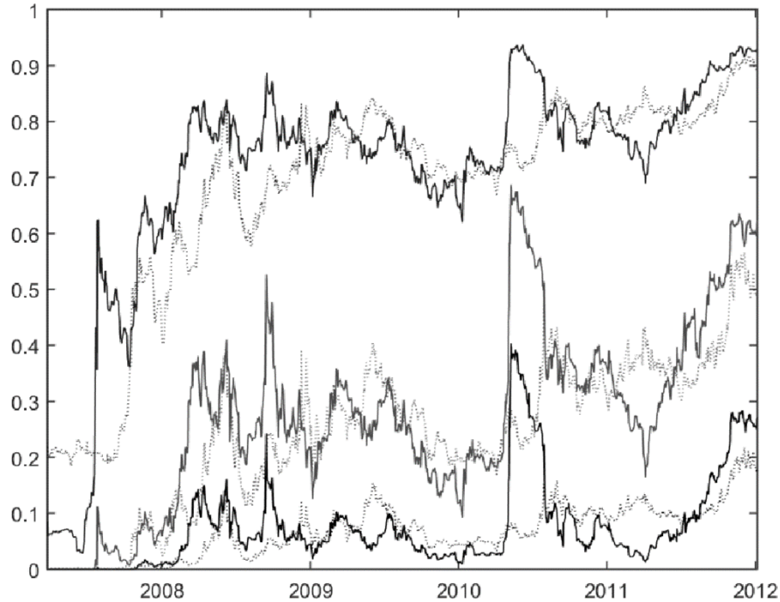
Our last measure, the probability of at least n additional banks defaulting given a particular bank defaults is presented in Figure 6. As mentioned before, for convenience, we call the measure PAN. The solid lines present the dynamic case, while the dotted lines present the static case. For presentation purposes, each curve in Figure 6 presents the cross-sectional median values of the respective probability.[8] Keeping the dimensions of the banking system fixed at 10 banks, we would expect that the likelihood that an additional bank defaults drops when we require more banks from the system to default. That is, in our 10-dimensional banking system, the conditional probability of 8 additional banks defaulting is strictly lower than the conditional probability of additional 7 banks defaulting. In Figure 6, the conditional probability of 1 additional bank defaulting, presented by the top two lines, is the highest in the static and dynamic cases compared to the cases where at least 5 additional banks default (middle two lines) and at least 9 additional banks default (bottom two lines).

The results confirm the impression from all our previous measures that the distress in the banking system started already in mid-2007. Our dynamic measures are much more volatile again, with significant spikes in August 2007 (the outbreak of the Subprime crisis) and in May 2010 (the First Greek Bailout). What can be also noted is that the conditional probability of at *least one bank* defaulting rises very fast to the unity limit of the probability domain (the unreported maximum values are even closer to 1 than depicted), making the dynamics of this measure (introduced in Segoviano and Goodhart (2009), and often referred to as the probability of spill-over effects) relatively uninformative. Therefore, we believe that our generalization, which investigates different number of defaulting banks, provides a richer picture of the depth of penetration of default spill-over effects within the financial system.

Interestingly enough, the static case of PAN has higher values than the dynamic case for prolonged periods of time. The reason for that is that in the static case we use fixed correlation matrices calculated over the full sample, while in the dynamic case, we calculate the matrices using a rolling window of 60 days. The latter approach allows us to adjust for increases or decreases in dependence which typically occur during crises and tranquil times (see, e.g., Forbes and Rigobon, 2002; and Radev, 2022e). We believe that dynamic conditional measures present a more accurate picture of the level and direction of changes in systemic risk.

[8] We calculate the respective probabilities for each of the 10 banks and take the median across the cases where at least 1, at least 5 and at least 9 additional banks from the remaining in the system default, respectively. That is, for each of the depicted lines, we calculate the median value across 10 conditional probabilities.

Figure 6 Probability of at Least n Additional Banks Defaulting Given a Particular Bank Defaults: Dynamic Case (solid line) and Static Case (dotted line)



Notes: involving 10 large and complex banking groups in the period 01.01.2007 - 31.12.2011. The median values across the cross-section of the respective probabilities are reported. The top two lines correspond to the probability of at least 1 additional bank defaulting for the dynamic case (solid line) and static case (dotted line). The middle two lines correspond to the probability of at least 5 additional banks defaulting for the dynamic case (solid line) and static case (dotted line). The bottom two lines correspond to the probability of at least 9 additional banks defaulting for the dynamic case (solid line) and static case (dotted line). The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads over a 3-month (60 business days) rolling window (solid line) and between changes of the respective banks' 5-year CDS spreads (dotted line).

Source: own calculations.

5.3 Extension of Time Period and Alternative Modelling of Correlation

In this section, we use additional data for some of the banks to extend the period of estimation to June 30, 2017. We also perform a sensitivity analysis regarding the choice of proxy for asset correlation by comparing static and dynamic correlations based on equity data to their respective CDS-based counterparts.

5.3.1 Sample Extension

Thomson Reuters provide in-house CDS data for some of the banks in our sample, which allows us to update several of our probability measures. In particular, we managed to recover CDS data for Banco Santander, ING Groep and UniCredit until June 30, 2017.[9]

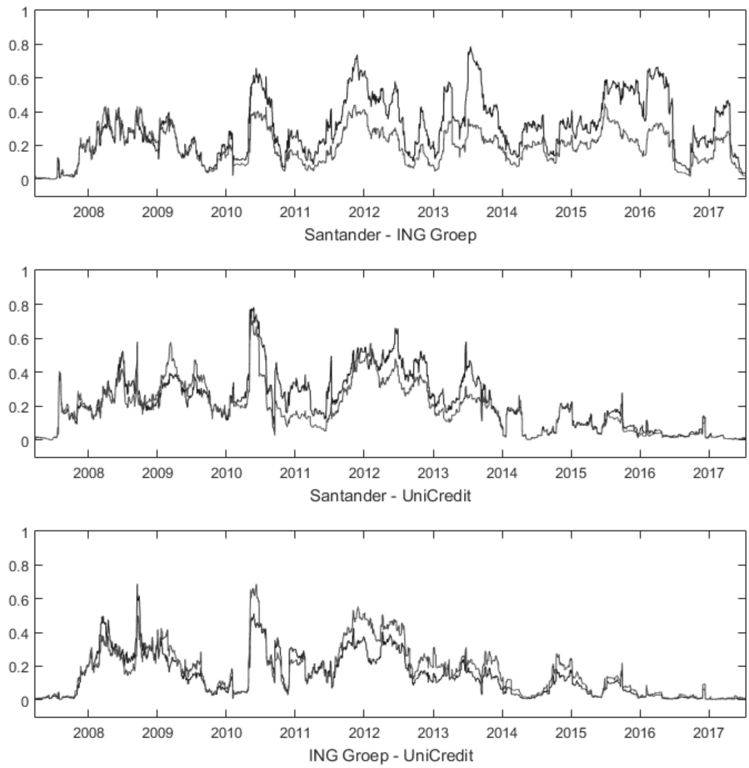
While for only a limited sample and time period, that data allows us to trace beyond December 2011 the dynamics of our lower-dimensional measures, such as the probability of a bank defaulting, given another bank defaults (Figure 7), and the probability of a bank defaulting, given two other banks default (Figure 8). In the upper subfigure of Figure 7, we observe that Banco Santander is more vulnerable to the default of ING Groep than the reverse case. Interestingly, the conditional PoDs of Banco Santander and UniCredit (second subplot) trace each other very narrowly, indicating that both banks have had very similar individual unconditional probabilities. In the third subplot, the conditional probabilities of ING Groep and UniCredit trace each other more narrowly, but overall, Banco Santander and UniCredit appear to be riskier than ING Groep and are more sensitive to the hypothetical default of the latter than vice versa.

In Figure 8, we observe that Santander is more sensitive to the joint default of ING Group and UniCredit than any of the other two constellations and that UniCredit is the least sensitive to the joint default of the remaining two banks.

Observing the overall dynamics, the riskiness of the conditional probabilities is influenced by major events in the euro area throughout the extended period, with major spikes around the Private Sector Involvement agreement in late 2011 and early 2012, which indicated the de facto default of Greece on its government debt; the "whatever-it-takes" speech of Mario Draghi in mid-2012, which calmed down the markets and was a de facto promise that the ECB would bail out the euro area to save the euro; and the Cypriot Banking Crisis of late 2012 and early 2013. In all cases, the conditional probabilities drop by the end of the time period, especially after mid-2016.

[9] While Thomson Reuters reported data beyond June 2017, for two of the banks, ING Groep and UniCredit, the values do not vary at all, which does not allow us to calculate dynamic correlations based on changes of CDS spreads of these banks.

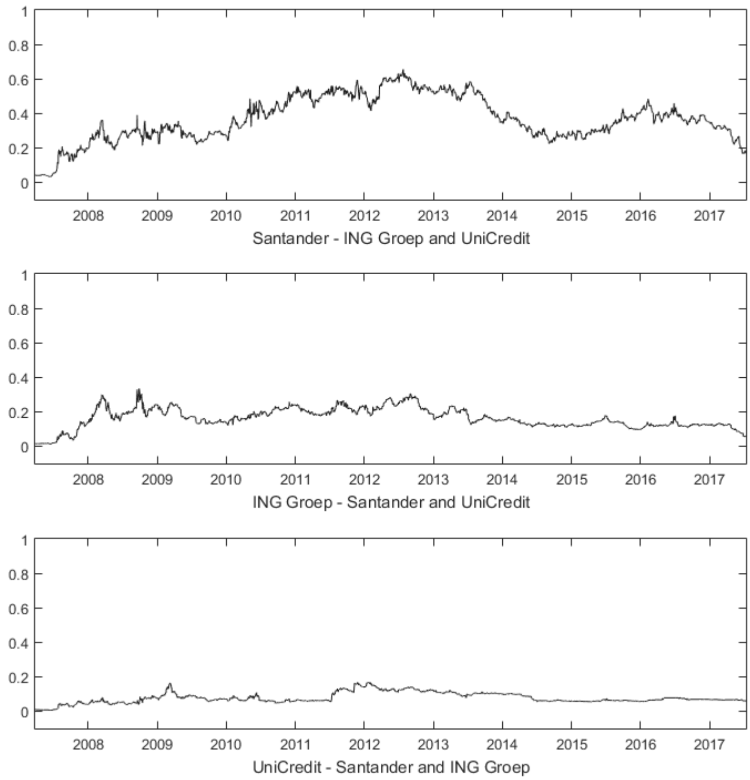
Figure 7 Banking Conditional Probability of Default Given a Particular Bank Defaults: Dynamic Case.



Notes: 5-year annualized conditional probabilities of default of selected complex banking groups in the period 01.01.2007 - 30.06.2017. The black (grey) line corresponds to the probability of default of the first (second) bank listed in the couple, given the second (first) bank defaults. E.g., the black line in the top plot represents the probability of default of Santander given ING Groep defaults, while the grey line corresponds to the probability of default of ING Groep given Santander defaults. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads over a 3-month (60 business days) rolling window.

Source: own calculations.

Figure 8 Banking Conditional Probability of Default Given Two Banks Default: Dynamic Case.



Notes: 5-year annualized conditional probabilities of default of selected complex banking groups in the period 01.01.2007 - 30.06.2017. The black line corresponds to the probability of default of the first bank listed in the couple, given the remaining two listed banks default simultaneously. E.g., the line in the top plot represents the probability of default of Santander given ING Groep and UniCredit jointly default. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads over a 3-month (60 business days) rolling window.

Source: own calculations.

5.3.2 Alternative Correlation Modelling

Throughout the study, we follow Gorea and Radev (2014) and Radev (2022b) in using changes in CDS 5-year spreads to calculate static and dynamic correlations among the assets of the banks in our sample. An alternative proxy for asset dynamics could be the growth of the respective bank's traded equity. Gorea and Radev (2014) explain that CDS spreads are not only useful to calculate individual probabilities of default, but also to capture the dynamics of bank assets in the default region of the asset distribution. The alternative proxy, equity price changes, reflects asset dynamics for the whole distribution. Nonetheless, in this subsection we provide comparisons between correlations calculated using both CDS spreads and equity prices sourced from Thomson Reuters for the three banks in Section 5.3.1.

In Table 3, we report static correlation estimates based on changes in 5-year CDS spread series (Panel A) and equity prices (Panel B) of Santander, ING Groep and UniCredit. The values are not sufficiently different to lead to markedly different static estimates of our conditional probability measures. In Figure 9, we also depict dynamic correlations based on a 60-day rolling window using both approaches. We notice that the lines follow each other very narrowly, indicating that for that particular couple, the correlation in the default region of the joint asset distribution is similar to the overall correlation.

Table 3 CDS-Derived vs Equity-Derived Correlations: Static Case

Panel A: Static correlations based on daily CDS spread changes

	Santander	ING Groep	UniCredit
Santander	1		
ING Groep	0,53901152	1	
UniCredit	0,53396711	0,53287199	1

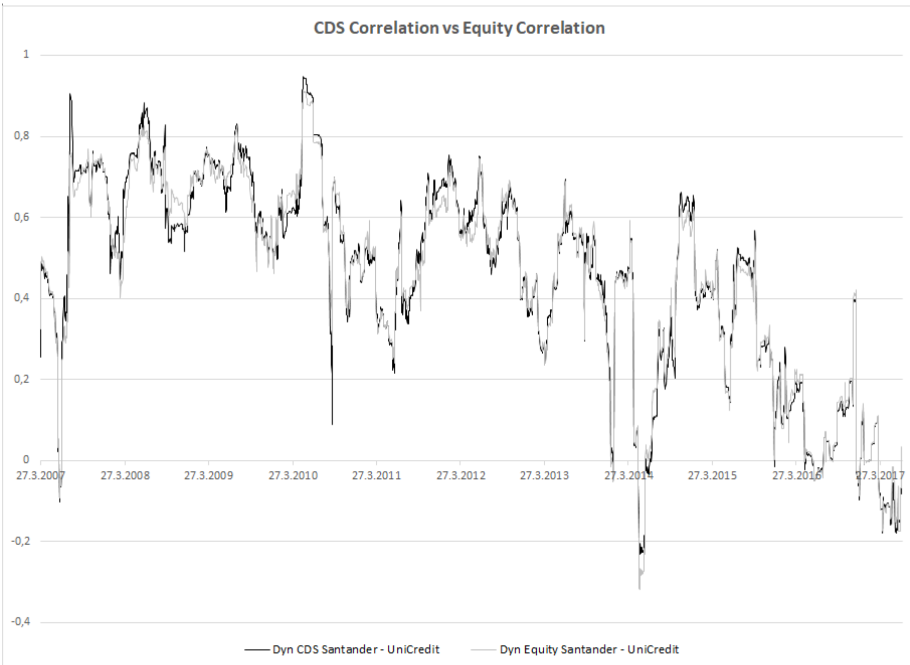
Panel B: Static correlations based on daily equity growth

	Santander	ING Groep	UniCredit
Santander	1		
ING Groep	0,53047988	1	
UniCredit	0,57531303	0,45749751	1

Notes: Correlation estimates based on changes in 5-year CDS spread series (Panel A) and equity prices (Panel B) of selected complex banking groups in the period 27.03.2007 - 30.06.2017.

Our results suggest that both types of correlations may be used interchangeably for our sample and the results are robust to that alternative modelling of correlation. A possible application of equity-based correlations may be to extend the calculations of the probability measures in cases where the CDS spreads do not vary much or at all in large parts of the time period and therefore the changes in CDS spreads are equal or close to 0. This is a very common case for data sourced from Thomson Reuters. In our case, using equity correlations would not have helped to extend the calculations, because the equity data after June 2017 for the three banks in this section is missing in Thomson Reuters as well. Alternative data sources may be used to circumvent the problems with that particular data provider.

Figure 9 CDS-Derived vs Equity-Derived Correlation: Dynamic Case



Notes: CDS-derived vs equity-derived correlations between Santander and UniCredit in the period 27.03.2007 - 30.06.2017. The black line corresponds to dynamic correlations between Santander and UniCredit calculated using changes of the respective banks' 5-year CDS spreads over a 3-month (60 business days) rolling window. The grey line corresponds to dynamic correlations between Santander and UniCredit calculated using equity price growth of the respective banks over a 3-month (60 business days) rolling window.

Source: own calculations.

6. Conclusions

This paper improves several existing measures of default risk of European banks using a procedure for a consistent estimation of individual and joint default risk. On the methodological side, we introduce dynamic dependence to the minimum cross-entropy approach used in previous literature (see Segoviano and Goodhart, 2009; Gorea and Radev, 2014 and Radev, 2022b). This helps us to capture more adequately the changes in dependence that occur between crisis and tranquil times on financial markets.

Our analysis documents a rise in banking default risk from the onset of the Subprime Crisis on, later exacerbated by the events surrounding the First Greek Bailout in May 2010. Our measures also capture other significant events in the euro area, such as Mario Draghi's "whatever-it-takes" speech in mid-2012 and the Cypriot Banking Crisis of 2012-2013. The dynamic dependence versions of our measures provide a richer depiction of conditional default risk in the European banking system and in many cases show very different dynamics to their static counterparts. Our results are robust to different approaches for calculating correlations.

The project contributes to the ongoing debate on joint default risk measures and should improve our understanding of market default risk perceptions and the effects of regulatory interventions and economic reforms. The dynamic dependence approach and the measures that we propose extend the policy makers' toolkit for analysis of systemic default risk. Our approach can be employed to capture the effect on systemic risk of reforms in the financial system, such as the introduction of bank resolution regimes and Basel III, as well as major European and global crisis events such as Brexit, the Global Pandemic due to Covid-19 and the War in Ukraine. These are all topics for future research.

Another issue that should be addressed is the use of the CIMDO approach to approximate the joint asset distribution of euro area banks. The CIMDO method is only one of the approaches to arrive at joint probabilities of default. An alternative could be a multivariate t-distribution with varying default thresholds a-la Merton (1974), instead of fixed ones as in Segoviano (2006), Gorea and Radev (2014), Radev (2022b) and Radev (2022c). A beneficial side product from CIMDO that does not exist with the Merton approach is the calculation of the Lagrange multipliers μ, λ_1, λ_2 to λ_n, which may be interpreted as the shadow prices of individual and joint default. The analysis of these multipliers, albeit interesting, is beyond the scope of this work.

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