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Central Bank Digital Currency in Brazil

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Abstract

We calibrate to the Brazilian economy a model of means of payment choice, where households have different preferences over anonymity. The financial sector is monopolistically competitive and may break the link between borrowing and lending rates. A sufficiently attractive digital currency reduces holdings of both cash and bank deposits. Since cash use is costly, digital currency may increase welfare. However, if banks are liquidity constrained, the digital currency may result in fewer loans and output and reduce welfare. The digital currency interest remuneration can be set and adjusted over time to balance this trade-off optimally.

1. Introduction

Central banks have increasingly shown interest in developing central bank digital currencies (CBDCs) in response to the global rise of digital payment solutions, such as cryptocurrencies. CBDCs are digital representations of central bank-issued money denominated in the national unit of account that differs from balances in traditional reserve or settlement accounts. Notably, China has already launched real-world trials of its Digital Yuan in several cities. Other 44 countries are currently researching the topic, as well as exploring issuing their own CBDCs (BIS, 2021)). Despite CBDC's growing interest, there is still a question about its usefulness and if its benefits surpass its costs.

There is a long list of potential benefits proposed for CBDCs. Firstly, a CBDC offers a more efficient payment system where managing cash is inefficient due to security and transportation costs. Secondly, a CBDC could enhance financial inclusion if it helps users in accessing current digital payment tools at considerably lower costs than having banks bank accounts. Thirdly, a CBDC could lower barriers to entry for new firms in the payments sector, foster innovation among private players, and increase competition among banks in attracting deposits. Fourthly, a CBDC could compete with and counter the many current private digital currency initiatives, which could eventually undermine the Central Bank's control of monetary policy.

There is also a list of potential costs. Firstly, offering full-fledged CBDC requires central banks to be active along several steps of the payments value chain,

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potentially including interfacing with customers, building front-end wallets, picking, and maintaining technology, monitoring transactions, and being responsible for anti-money laundering and countering the financing of terrorism. Secondly, central banks could become direct competitors of payment service providers, thereby forcing banks to lose income. Thirdly, CBDC could reduce the consumer deposit demand and thus lower bank lending to the general economy and hurt economic growth. Fourthly, CBDCs can also increase the risks of system-wide bank runs.

This paper focuses on the CBDC potential benefit of eliminating cash versus the CBDC potential cost of financial disintermediation. We propose a model to quantitatively study this trade-off and show how, depending on its design, a CBDC may result in relevant welfare implications.

We calibrate our model to Brazil, where, as in other Emerging economies, urban violence is a pervasive problem and security is a costly factor. In such environments, costs from providing the necessary safety for storing and transporting cash are estimated to be around 0.5% of GDP[1,2]. If sufficiently attractive as a payment instrument, a CBDC could be a good substitute for cash and imply relevant welfare gains.

In the case of Brazil, households choose their payment instrument based on two main characteristics: anonymity and remuneration. Interestingly, security for storing cash at the individual level is not an essential issue for most of the population.[3] Research shows that most individuals are more concerned with the risks of payment fraud and identity theft that come with digital payments. Another particularity of Brazil, compared to Developed countries, is that confidentiality by itself is also not a relevant issue. Most individuals care about the anonymity provided by cash for tax evasion reasons, not because they value privacy. This is probably a consequence of the large informal sector, which employs about 40% of the population.

Cash and bank deposits coexist because they provide different amounts of each characteristic: deposits pay interest rates, and cash is anonymous. CBDC that either pays interests and/or is sufficiently anonymous can crowd out cash and thus provide welfare gains. However, by the same token, CBDC may crowd out bank deposits and cause financial disintermediation.

If consumers move money from bank accounts into CBDC, banks will have less funding for credit, which may reduce loans to firms. In particular, if the banking industry is assumed to be competitive[4], the pass-through from policy rates to bank lending rates is direct. In this case, the introduction of a CBDC that has the effect of imposing an interest rate floor in the deposit market necessarily discourages investment. In contrast, as Andolfatto (2020) shows, this may not happen in non-

[1] In Mexico, the indirect costs of cash (theft, corruption, security costs) are considered to eclipse the direct costs of managing cash balances. They are estimated to be higher than 1% of GDP (Chakravorti and Mazzotta, 2014).

[2] This logic is opposite to what is happening in some jurisdictions, such as Sweden, where cash is rapidly disappearing, and the central banks are analyzing a CBDC that could be made widely available to the general public and serve as an alternative payment instrument

[3] In contrast, Agur et. al (2020) assume households have preferences over the relative importance of anonymity and security.

[4] See for example Barrdear and Kumhof (2016) and Brunnermeier and Niepelt (2019).

competitive settings. In his model, introducing a CBDC in no way discourages bank lending since the opportunity cost of bank lending is the IOR (Interest on Reserves rate). Additionally, the presence of an interest-bearing CBDC forces the monopoly bank to increase the deposit rate, which in turn increases financial inclusion and bank deposits. The increase in deposits resulting from CBDC competition, in turn, induces an expansion in bank lending.

To capture the main characteristics of the Brazilian case, we propose a model of means of payment choice in which households have heterogeneous preferences for anonymity and the financial sector is monopolistically competitive. We obtain that when the CBDC is sufficiently attractive, either because it is anonymous enough or because it pays high enough interests, it crowds out both cash and bank deposits. If banks are not liquidity constrained, this implies in a reduction in the costs of using cash and does not reduce bank loans. However, if the CBDC is too attractive then the reduction in bank deposits eventually binds banks' regulatory liquidity constraints. Under this circumstance, when banks are liquidity constrained, the introduction of the CBDC implies a reduction of banks' loans, output, and welfare. Therefore, implementing the optimal CBDC amounts to choosing its interest rates and anonymity levels such that banks' liquidity constraint exactly binds.

Section 2 connects our model to usually discussed CBDC design choices. Section 3 presents a simple model which can be explicitly solved. Section 4 considers a model with banks' liquidity constraints, which is calibrated for the Brazilian economy and numerically solved. We then obtain the "liquidity frontier," the combination of CBDCs characteristics that exactly bind banks' liquidity constraints. We also obtain which position within this frontier yields maximum welfare. Finally, we discuss some practical challenges in reaching this optimum. Section 5 concludes.

2. Design Choices

In this section, we list CBDC's main design choices, relate them to the Brazilian case, and discuss how they are reflected in our modeling hypothesis.

There are two main types of CBDCs: wholesale CBDCs and retail (or general purpose) CBDCs. Wholesale CBDCs' access is limited to a predefined group of users, typically financial institutions. They intend to make financial systems faster and safer by helping streamline payments between central banks and private banks while enabling simpler cross-border transactions and reducing counterparty credit and liquidity risks. In contrast, retail CBDCs are essentially digital money meant for ordinary consumers' daily use. We are focusing here on retail CBDC, that is, one that the public can use for day-to-day payments.

Additionally, it is common to distinguish between account-based and token-based digital currencies. The critical difference between token- and account-based money is the form of verification needed when exchanged. An account-based system requires verifying the payer's identity, while a token-based system requires verifying the validity of the object used to pay. With an account-based system, the key concern is identity theft, which allows perpetrators to transfer or withdraw money from accounts without permission. By contrast, in a token-based system, the worry is (electronic) counterfeiting, whether the token or "coin" is genuine or not, and whether it has already been spent. Following this definition, a crucial characteristic of a token is its anonymity vis-à-vis the central bank. As with bitcoin or old numbered Swiss accounts, for example, token transactions are publicly recorded using the payer's and the payee's public addresses. Still, these addresses do not reveal the identity of users.

Central Bank choice of the degree of anonymity in a CBDC must consider concerns relating to money laundering, financing of terrorism, and privacy. In most countries, as in Brazil, the general trend is to consider that the CBDC should have "controllable anonymity." Without third-party anonymity, the CBDC transactions may jeopardize personal data and privacy, but complete third-party anonymity may encourage criminal activities. A possible way to strike some balance between the two is to keep the degree of anonymity within a controllable range by making the transactions traceable. This would allow the central bank to keep track of necessary information to crack down on money laundering and other criminal offenses. In our model, we assume the CBDC degree of anonymity is a choice of the central bank, which affects how attractive it is for households use.

Another choice of the central bank is whether to make the CBDC interest-bearing and whether to limit its use by defining caps on individual holdings. As with other forms of digital central bank liabilities, it is technically feasible to pay interest (positive or negative) on CBDCs. The interest rate on CBDC can be set equal to an existing policy rate or be set at a different level to either encourage or discourage demand for CBDC. Moreover, rates could vary by the counterparty, amount held in the account, or some other characteristic. Similarly, different forms of quantitative limits or caps on the use or holdings of CBDC are often mentioned to control potentially undesirable implications or steer usage in a specific direction. In our model, we assume the Central Bank can optimally choose the CBDC interest but do not consider the possibility of quantitative caps.

A CBDC can also be designed as a one- or two-tier system. The two-tiered monetary system is where central banks supply cash and deposits to commercial banks. Commercial banks provide deposits through credit creation based on the central bank money they hold. In a one-tier system, individuals have an account at the Central Bank, which directly distributes the CBDC. This setup has clear scaling implications for the central bank, which would need a massive increase in size and scope since it will need to handle Know Your Customer (KYC) issues and disputes. Arguably, such a design would also pose the problem of who will supply credit and complementary financial services. Most countries, including Brazil, would probably keep the two-tier system. This assumption does not have any implication for our model.

There are a lot of technology considerations and choices for CBDCs, which we do not consider in our analysis. For example, a CBDC can be either technology decentralized (DLT) or centralized. DLT refers to a family of technologies that use a distributed group of participants to maintain a shared collectively, replicated, and synchronized record without reliance on a single central party or centralized data storage. Traditional centralized systems, by contrast, rely on a trusted central party to maintain the record. We assume that such technological choices are entirely orthogonal to the economic considerations we study in the model.

3. The Basic Model

Our model is a modified version of Agur, Ari, and Dell'Ariccia (2019), adapted to fit some Brazilian characteristics. In particular, rather than assume households prefer anonymity and security conflicting values, we consider that households care about anonymity. Additionally, as in Andolfatto (2018), the financial sector is modeled by a monopolistic competition of banks.

The economy is populated by households, banks, firms, and a central bank. There is only one period. Before the beginning of this period, the central bank decides on the characteristics of the CBDC to maximize welfare. Then, simultaneously, households choose between holding cash, bank deposits, and CBDC for their transactions, and banks collect deposits from households and extend loans to firms. Firms borrow resources from banks to produce consumption goods.

Households value two attributes in payment instruments: the rate of interest offered and anonymity, for which they have heterogeneous preferences. This modeling assumption reflects some research pools conducted in Brazil. In many countries, household choices are driven by interests and the trade-off between anonymity and security (cash offers anonymity but is less safe to keep; bank accounts are the opposite). In Brazil, security is not an essential factor driving household choices. Generally, most people perceive money as safe, sometimes even safer than electronic means of payment.

In fact, in Brazil, the primary motivation to choose the payment instrument is its degree of anonymity, which is associated with the possibility of avoiding taxes. The large size of the shadow economy probably explains this difference between developed countries and Brazil. By not revealing their income, informal workers manage to remain undercover, thus avoiding the Brazilian Internal Revenue Services' monitoring. Evidently, this motivation for tax evasion will affect how the central bank perceives welfare when choosing the optimal CBDC design. From the point of view of the Central Bank, tax evasion is not beneficial, even though it increases households' welfare in partial equilibrium.

When adopting a CBDC, the central bank can choose the level of its anonymity. A CBDC can approach the anonymity of cash if it takes the form of a token, such as a cryptocurrency, which is accessible through user accounts that are not independently verified or a nameless payment card that can be purchased at stores or online. On the other extreme, an account at the central bank that can be opened only using official identification would mimic bank deposits' security and traceability. However, a CBDC can also have intermediate amounts of anonymity. For example, transactions can be recorded but not accessed by the central bank unless there is suspicion of wrongdoing or breach of a transaction size limit.

To formalize these considerations, we assume households derive utility from anonymity according to the term jx. The parameter $j \geq 0$ denotes the household type, and the variable x^i denotes the place of each money type i in the anonymity-security scale. Deposits (denoted with D) are placed at the bottom of the scale at $x^d = 0$. Cash (denoted with M) is placed at the top of the scale at $x^m = 1$, and CBDC (denoted with C) is placed at $x^C = \theta$, where $\theta \in [0, 1]$ is a design parameter determined by the central bank. In addition, the central bank determines the interest rate offered on the CBDC, r^C, which is allowed to take any real value.

Below, we detail the maximization problem of households, banks, firms, and the central bank and then proceed to characterize the equilibria.

3.1 Households

There is a continuum with a unit mass of households denoted by $j \geq 0$ distributed over the preference for anonymity scale. All homes have identical endowments, which are normalized to 1, which are stored in a type of money i and used to purchase consumption at the end of the period. Importantly, we assume households cannot attain their preference for anonymity by mixing different forms of money in their transactions because a transaction is only as anonymous as the least anonymous payment instrument used. In other words, anonymity is undiversifiable.

Each household's j utility maximization problem can then be written as

$$MaxU^{i}_{\in\{D,C,M\}} = C^{i} + jx^{i} \tag{1}$$

subject to

$$C^{i} = x^{i} + \pi^{firms} + \pi^{banks} - T \tag{2}$$

where C^i denotes consumption, r^i are the interest earned on money holdings, π^{firms} represents the firm's profits, π^{banks} are bank's profits, and T is a lump-sum tax used to fund interest rates on CBDC and the cost of producing, storing, and keep safe cash. Notice that the single parameter J contains the information regarding the dispersion of households' (marginal) utility from anonymity already transformed into consumption units.

The solution to the household's problem yields the following cut-off conditions for a household with preferences j to choose (i) cash over CBDC: $j \geq r^C + j\theta$; (ii) cash over deposits: $j \geq r^D$; and (iii) CBDC over deposits: $r^C + j\theta > r^D$. These inequalities define, in turn, three threshold levels for j: (i) $r^C/(1 - \theta)$, (ii) r^D, and (iii) $(r^D - r^C)/\theta$.

The hypothesis that $j \geq 0$ (since $j \epsilon [0, J]$) and $\theta \epsilon [0, 1]$ allow us to determine some properties of equilibrium:

> **Lemma 1.** If $r^C \leq 0$, then CBDC does not show up in equilibrium. If $r^C \geq r^{D,}$ then deposits do not show up in equilibrium
>
> **Proof.** If $r^C \leq 0$, then $j \geq r^C/(1 - \theta)$, and thus cash always dominates CBDC. When $r^C \geq r^{D,}$ then $j \geq (r^D - r^C)/\theta$, and thus CDBC always dominates deposits.

Lemma 1 determines cases that are not interesting. The central bank will not choose an r^C if it means that CBCD does not exist in equilibrium. And the inexistence of deposits would mean there is no supply of loans to firms. Thus, henceforth we focus on the case $0 < r^C \leq r^D$.

This hypothesis, in turn, implies the following possible orderings of the thresholds for j and the households' choices for payment:

Lemma 2. There are two possible ordering of thresholds for households' choices:

a) When $\theta < (r^D - r^C)/r^D$, then $r^C/(1 - \theta) < r^D < (r^D - r^C)/\theta$. Households with $j < r^D$ choose deposits, and households with $j > r^D$ choose cash. There is no CBDC in equilibrium

b) When $\theta > (r^D - r^C)/r^D$, then $(r^D - r^C)/\theta < r^D < r^C/(1 - \theta)$. Households *with* $j < (r^D - r^C)/\theta$ choose deposits, households with $(r^D - r^C)/\theta < j < r^C/(1 - \theta)$ hold CBDC, and households with $j > r^C/(1 - \theta)$ hold cash.

Proof. Other orderings imply that θ does not belong to the set [$0, 1$].

Lemma 2 helps reduce the focus of parameters to the case $\theta > (r^D - r^C)/r^D$ since, again, the Central Bank would only choose θ if it implies CBDC occurs in equilibrium.

3.2 Banks

Consider now the behavior of the banking sector, which we model as a consolidated bank sector, similarly to Andolfatto (2019). Assume initially that there is no CBDC and that the bank sector is a monopoly that takes the central bank policy rate, the IOR rate r^I as given. For a chosen deposit rate r^D, the monopoly bank attracts nominal deposits $s(r^D)$. A fraction τ of these deposits must stay at the Central Bank as non-remunerated reserves. The remaining fraction funds bank assets consisting of remunerated reserves and loans. For a chosen loan rate r^L the monopoly bank attracts a loan demand equal to $k(r^L)$.

Given r^D and r^L, the monopoly bank's demand for remunerated reserves g is implicitly given by its balance sheet constraint.

$$g + k(r^L) + \tau s(r^D) = s(r^D) \tag{3}$$

This balance sheet generates profit equal to

$$r^L k(r^L) + g\, r^I - r^D s(r^D) \tag{4}$$

Combining the last two expressions, the bank's maximization problem can be expressed as:

$$(r^D, r^L) = ArgMax\{(r^L - r^I)k(r^L) + \lfloor r^I(1 - \tau) - r^D \rfloor s(r^D)\} \tag{5}$$

Notice that the bank's maximization problem can be split into two parts. The first term represents the profit margin related to loans, and the second term represents the profit margin related to deposits. The banks' choice variables, $r^{L,}$ and r^D, are set to maximize each of the bank's businesses, the "lending to firms" business, and the "borrowing from households" business. These rates depend on the policy rate r^I but are otherwise set independently of each other.

The two first-order conditions of the problem can be expressed, as the usual Lerner's formula, according to:

$$\frac{(r^L - r^I)}{r^L} = -\frac{1}{\varepsilon} \ and \ \frac{r^I(1-\tau) - r^D}{r^D} = \frac{1}{\eta} \tag{6}$$

where

$$\varepsilon = \frac{\partial k(r^L)/\partial r^L}{r^L/k(r^L)} \ and \ \eta = \frac{\partial k(r^D)/\partial r^D}{r^D/k(r^D)} \tag{7}$$

are the elasticities of demand for loans and supply for deposits.

Consider now that rather than a monopoly, the banking sector is composed of *N* identical banks which compete for funds and deposits according to a Cournot model. The assumption of Cournot competition, where players choose quantity rather than prices, can be justified by Kreps and Sheinkman (1983). It should be understood as a convenient shortcut to a more complex environment, where banks monopolistically compete in an environment with differentiable products.

The Lerner's formulae are now modified to:

$$\frac{(r^L - r^I)}{r^L} = -\frac{1}{N\varepsilon} \ and \ \frac{r^I(1-\tau) - r^D}{r^D} = \frac{1}{N\eta} \tag{8}$$

where ε and η are defined as before, in (7).

Now let r^C denote the CBDC interest rate. The introduction of CBDC means households now have an option to hold an asset that is a direct claim on the Central Bank. In principle, the monopoly bank also has the option of lending to the Central Bank at the rate of r^C. However, by assuming that $r^C < r^I$, we make such a choice not optimal for the banks.

The only way r^C affects the "borrowing-from-households" business is by potentially affecting $s(r^D)$ and thus reducing the supply of deposits. Consequently, it may be optimal for the banks to choose a higher deposit rate rD to make profits in their deposit business.

To make matters simple, we assume the Central Bank refuses to lend to banks at r^C or, alternatively, liquidity facilities imply high stigma costs for the monopoly bank. Consequently, the "lending-to-firms business" is wholly insulated from r^C.

3.3 Firms

Firms are perfectly competitive and borrow funds from banks at a rate of r^L to produce a final good according to the technology *f(k)*, where *k* denotes the amount borrowed. We assume f is continuous and monotonically increasing. Firms' maximization problem can be written by:

$$k = ArgMax\{f(k) - r^L k\} \tag{9}$$

The first order condition to the maximization problem determines the demand for loans, which we denote

$$k^* = h(r^L) \tag{10}$$

where *h* is a monotonically decreasing function.

3.4 Central Bank

It may seem natural to assume that the Central Bank's objective is to maximize social welfare, defined as the sum of household utilities. However, part of the households' utilities come from anonymity, which is motivated by tax evasion. Therefore, we assume the Central Bank aim is to maximize only the consumption of households' utilities. In doing so, the central bank decides whether to introduce a CBDC and its design characteristics (θ, rC) if introduced. If a CBDC is introduced, the central bank's design problem is given by:

$$(\theta, r^L) = ArgMax\{s^D C^D + s^C C^C + s^M C^M\} \tag{11}$$

where s^D, s^C and s^M denote the share of households that choose the deposit, CBDC, and cash as means of payments, respectively, with $s^D + s^C + s^M = 1$. By plugging the consumption of each type of household, the Central Bank problem become:

$$(\theta, r^C) = ArgMax\{s^D r^D + s^C r^C + \pi^{firms} + \pi^{banks} - T\} \tag{12}$$

From the firms' and banks' problem we have, respectively

$$\pi^{firms} = f(k^*) - r^L k^* \tag{13}$$

$$\pi^{banks} = (r^L - r^I)k^* + [r^I(1 - \tau) - r^D]s^D \tag{14}$$

where we already plugged the market clearing condition for the resources that go from banks loans to firms ($k(r^L) = k^*$) and from households to banks deposits $s(r^D) = s^D$.

Lastly, the Central Bank budget constraint is such that lump-sum taxes must cover the expenditure in CBDC remuneration and bank reserve remuneration:

$$T = s^C r^C + r^I[k^* - (1-\tau)s^D] \tag{15}$$

After plugging in taxes, firms' profits, and banks' profits, the Central Bank problem becomes simply:

$$(\theta, r^c) = ArgMax\{f(k^*)\} \tag{16}$$

where, according to (10), $k^* = h(r^L)$ and r^L is in turn determined by the expressions in (6) and (7). Notice that these expressions do not depend on the variables the Central Bank chooses to maximize welfare. In fact, these expressions imply that r^L is determined by the equation:

$$r^L - r^I = \frac{-h(r^l)}{N\partial h(r^L)/\partial r^L} \tag{17}$$

The conclusion, in this initial case, is stated in the following:

> **Lemma 3.** When cash use is costless, and banks' problem is not constrained, welfare is independent of the CBDC design

3.5 Equilibrium in a Simple Example

To make the equilibrium characterization concrete, restrict to the case where households' anonymity parameter j is uniformly distributed with support $[0, J]$, there is only one bank, and that firm's production technology is given by $f(k) = Ak^\alpha - \delta k$. In this formulation, α can be thought of as the capital share, and δ plays the role of depreciation rate since the model economy collapsed to fit in only one period.

From lemma 2, the restriction to the case $\theta > (r^D - r^C)/r^D$, and the hypothesis that $j \in [0, J]$ is uniformly distributed, we can determine the supply for each means of payment as:

$$s^D = \frac{(r^D - r^C)}{J\theta} \tag{18}$$

$$s^C = \frac{r^C - r^D(1-\theta)}{J\theta(1-\theta)} \tag{19}$$

$$s^M = \frac{1 - r^C}{J(1-\theta)} \tag{20}$$

where s^D, s^C, and s^M are respectively the households' choices for deposits, CBDC and cash.

The elasticity for deposits supply can be easily obtained by:

$$\eta = \frac{\partial s(r^D)/\partial r^D}{r^D/s(r^D)} = \frac{r^D}{r^D - r^C -} \tag{21}$$

From the firm's problem we can obtain the demand for loans as

$$k = \left[\frac{\alpha A}{\delta + r^L}\right]^{\frac{1}{1-\alpha}} \tag{22}$$

This, in turn determines the elasticity of loan demands as:

$$\varepsilon = \frac{\partial k(r^L)/\partial r^L}{r^L/k(r^l)} = \frac{-r^l}{(1-\alpha)(\delta + r^L)} \tag{23}$$

By plugging these elasticities into the bank first order conditions, one obtains:

$$r^D = \frac{N r^I (1-\tau) + r^C}{1+N} \tag{24}$$

$$r^L = \frac{N r^I + (1-\alpha)\delta}{N - 1 + \alpha} \tag{25}$$

Plugging (24) and (24) into (18), (19), and (20) determine the allocation amongst payment means. Plugging (25) into (22) determines the amount of capital in equilibrium,

4. Costly Use of Money and Liquidity Constrained Banks

The basic version of the model presented already has some frictions. In particular, we assumed that households and firms need the financial sector for intermediate resources and that the financial sector is not perfectly competitive. However, because we also assumed that the Central Banks use an interest rate policy rule, a CBDS does not affect the bank's optimal lending decision since this would continue to be determined at the margin by the IOR rate. Since bank lending determines a firm's production, which determines welfare, the CBDC is not a relevant tool.

Below we add two additional frictions to the model, modifying the assumptions regarding the banks' maximization and the Central Bank problem. These two frictions make the model more realistic and add both a role and an obstacle to CBDCs.

4.1 Liquidity Regulated Banks

Consider now banks face a liquidity regulation in the form of a minimum reserve requirement. In Brazil, as in most economies, banks are subject to the Basel III liquidity-coverage-ratio (LCR) requirement that increases the regulatory demand for reserves (and other high-quality liquid assets). The analysis so far assumed that such liquidity constraints are either absent or do not bind. But when they bind, banks must consider the deposit amount to decide how much to lend.

In the context of our model above, an LCR-like restriction can be modeled by

$$g \geq \lambda s(r^D) \tag{26}$$

where λ ϵ [0, 1] is a policy parameter specifying the minimum reserve-to-deposit ratio. Combining (27) with the balance sheet constraint (4) permits us to rewrite this liquidity constraint as,

$$(1-\lambda)(r^D) \geq k(r^L) \tag{27}$$

The banks' problem is now to maximize profit (17) subject to the balance sheet constraint (4) and the reserve requirement (27). If we denote the Lagrange multiplier for reserve requirements by $\xi \geq 0$, Lerner's formulae now become

$$\frac{[r^L-(r^I+\xi)]}{r^L} = -\frac{1}{N\varepsilon} \; and \; \frac{[r^L(1-\tau)+\xi(1-\lambda)-r^D]}{r^D} = -\frac{1}{N\eta} \tag{28}$$

Thus, when the liquidity constraint is binding, and $\xi > 0$, both the lending and the deposit rate increase. That is, banks end up borrowing more deposits and reducing lending to satisfy the constraint.

We can anticipate that a CBDC will now affect banks' lending and firms' production. We know that from the point of view of the household, the CBDC is a potential substitute for deposits. By making CBDC more attractive, the Central Bank may affect households' supply for deposits and thus make banks' liquidity constraints (more) binding. In turn, banks will react by increasing both the deposit and the lending rates to offset this effect. Consequently, firms will face a more expensive loan supply and will reduce the production of final goods.

4.2 The Cost of Cash

Consider now that cash use is costly. As we discussed before, the costs related to cash are considered very small for Brazilian households. Instead, Brazilians typically consider that debit and credit cards are costly since they are mainly concerned with payment fraud and identity theft.

However, from the point of view of the Government and the Business sector, cash carries a substantial amount of costs. For the government, the cost to print currency is a federal expense. But the size of that expenditure is vastly smaller than the financial consequences of cash's indirect costs, such as tax evasion and money laundering. The private sector similarly faces both direct and indirect costs of cash. The direct cash expenses include security systems such as safes, alarms, vaults, and armored car systems and the time required to account for and audit cash processes. Where businesses transact in cash and salaries are paid in cash, additional controls are necessary to audit every payment within the enterprise since the payments are made by hand. Cash also creates indirect costs to the enterprise, such as opportunities for fraud, theft, and embezzlement that are far more difficult with electronic payment systems.

To incorporate cash costs in our model in a straightforward way, we assume that its production is costly to Central Bank. We thus rewrite the Central Bank budget constraint (15) as

$$T = s^C r^C + r^I[k^* - (1-\tau)s^D] - \gamma s^M \qquad (15)$$

where $\gamma \geq 0$ is a parameter. In addition, to be consistent with market clearing, we also assume that part of the goods produced by firms is wasted due to the use of cash so that it cannot be consumed by households (In other words, the Central Bank "buys" goods from the firms to produce cash).

4.3 Calibration

To proceed with the analysis, we need to resort to numerical simulations, which in turn require that we calibrate our economy. We focus on Brazilian data before the pandemic of 2020, as it had greatly distorted liquidity and cash hoarding. We begin by assuming j is distributed according to a log-normal probability function with parameters μ (mean) and σ (standard deviation). Since two parameters determine this distribution, we can adjust it to fit both the level and the sensitivity of cash (and deposits) to interest rates. Using monetary aggregates data, we obtain that during 2019 the fraction allocated to deposits corresponds to 87% of the resources allocated to both cash and deposits. That is, $s^D = 0.87$. Since there is no CBDC so far, $s^M = 0.13$.

We use monthly data for a fraction of resources allocated to deposits from 2004 to 2013 for the Central Bank's primary rate (the Selic rate) to obtain the elasticity of deposit holdings to the deposit rate. This time horizon corresponds to a period of relative tranquility in the Brazilian macroeconomic environment, followed by a very sharp recession. Before estimating elasticity, we filter the data using a Hodrick Prescott filter to clean for low-frequency changes in household behavior. We implicitly assume that deposit rates move proportionally to the Selic rate so that movements in Selic correspond to movements in the deposit rate. Since households need time to adjust their portfolio, we search for time lags that provide the best fitting and obtain a lag of 4 months from changes in Selic rate to changes in deposit allocation. Figure 1 shows this relationship. A simple ordinary least square estimation implies an elasticity of $\eta = 0.30$ with a standard deviation of 0.03.

We calibrate the level of deposit rate using the average data for 2019, implying $r^D = 0.046$. That, in turn, allows us to calculate $\partial s^D/\partial r^D$ and, with the functional forms of the lognormal distributions, obtain $\mu = -4.00$ and $\sigma = 0.81$.

We calibrate $r^I = 0.065$ and $\tau = 20\%$ using the average Selic rate and the fraction of non-remunerated reserve requirements over 2019. By applying the Banks' first order condition for $r^{D,}$ we can obtain the number of banks $N = 23.33$.

To obtain the usual shape of capital demand, we use the Brazilian capital share to calibrate $\alpha = 0.35$. We set $r^L = 12\%$ using the average loan rate for non-revolving credit to firms in 2019. Then, by applying the banks' first-order condition with respect to $r^{L,}$ we can calibrate $\delta = 1.85$.

We set λ = 30% using the total amount of reserve requirements over 2019 (both remunerated and non-remunerated). To calibrate the amount of loans (and thus production capital), we note that the banks operate with the LCR non-binding by holding an excess of about 30% high-quality liquid assets. Thus, we set $k = (1 - \lambda) s(r^D) / 1.3 = 0.47$. That, in turn, allows us to calibrate the technology parameter A = 3.45 by using the firms' first order condition.

Figure 1 Elasticity of Deposits to Interest Rate

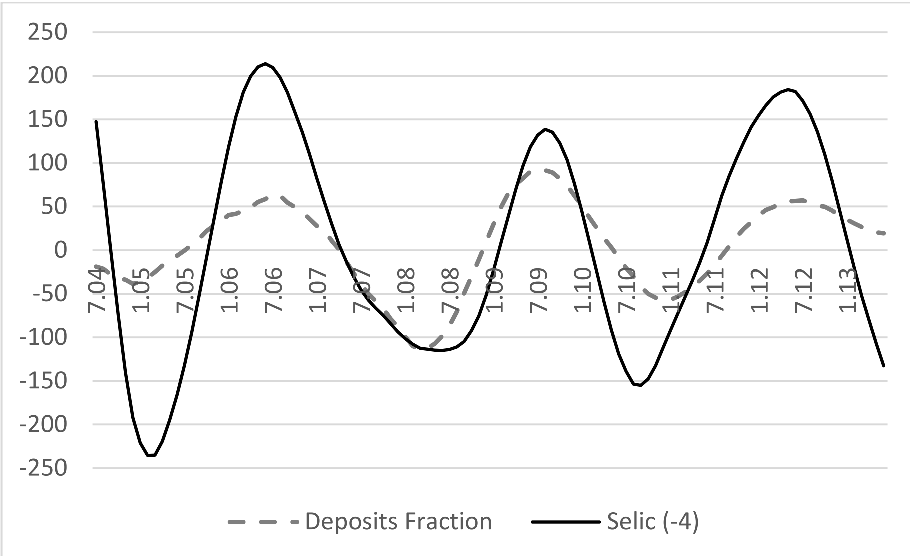
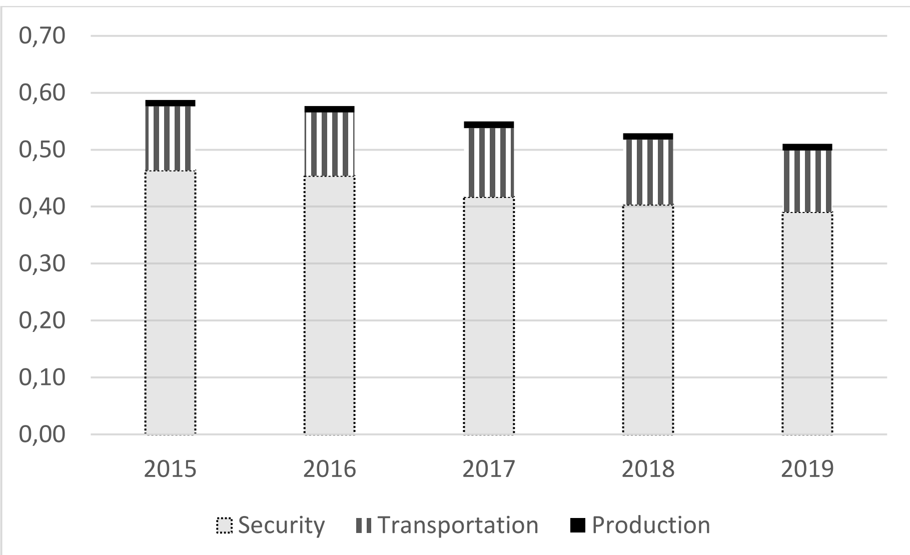


Figure 2 Costs of Cash (% GDP)



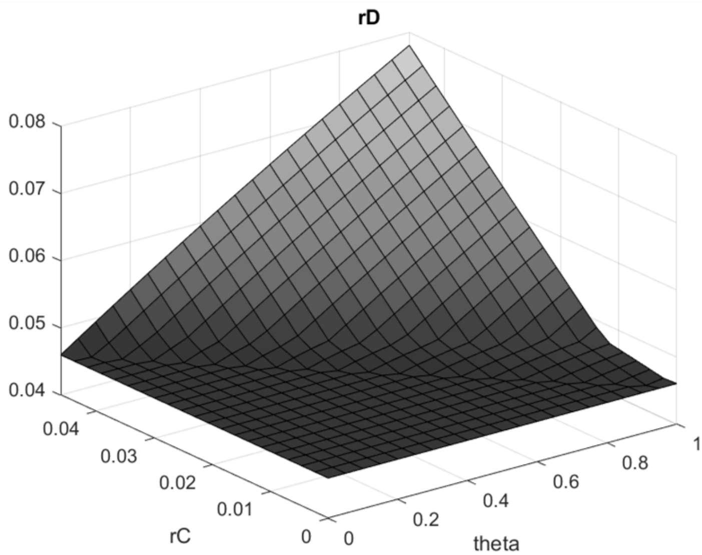
The last parameter to calibrate is the cost of money γ. In Brazil, if one sums up the cost of producing the numerary with the private sector costs related to security and transportation of cash, one obtains estimations close to 0.5% of GDP (figure 2). That seems very large, but it is similar to estimates for other countries (Krüger and Franz, (2014)). If one considers that the amount of cash in circulation in Brazil during 2019 was about 2.7% of GDP, that implies a considerable benefit of reducing cash.

Assuming that all costs are variable and proportional to the amount of cash, these numbers would imply $\gamma = 0.5/2.7 = 18\%$. This is larger than the return of a marginal increase in productive capital, calibrated to $r^L = 12\%$. In other words, a reduction in money circulation that causes the same amount of reduction in productive capital would imply welfare gains. We assume that a decrease in cash holdings would imply only half of the proportional savings costs because part of the costs is fixed. That is, in our basic simulations, we assume $\gamma = 9.0\%$. Of course, this choice is fairly arbitrary, but it is in line with our belief that the return of productive capital is higher than the return of cash reduction.

4.4 Simulations

Figures 3 to 10 show the results of our simulations with the calibrated model. For each pair of CBDC design choices (θ, r^C), they plot the equilibrium borrowing and lending rates, the holdings of cash, deposit and CBDC, and the stock of capital and welfare.

Figure 3 Borrowing Rate



We start by looking at the deposit side. Figures 3, 4, 5, and 6 show the borrowing rates and the deposit, cash, and CBDC holdings, respectively. Notice that the borrowing rate stays at the 4.6% level for a large part of the (θ, r^C) set. This happens because the CBDC does not show up in equilibrium unless $\theta < (r^D - r^C)/r^D$ (Lemma 2). This inequality defines a triangle in the (θ, r^C) space for which the CBDC is not attractive enough for households. This, in turn, results in the same equilibrium outcomes: banks maximize profits by setting $r^D = 4.6\%$, and the holdings of cash, deposits, and CBDC are constant.

Figure 4 Deposits Holdings

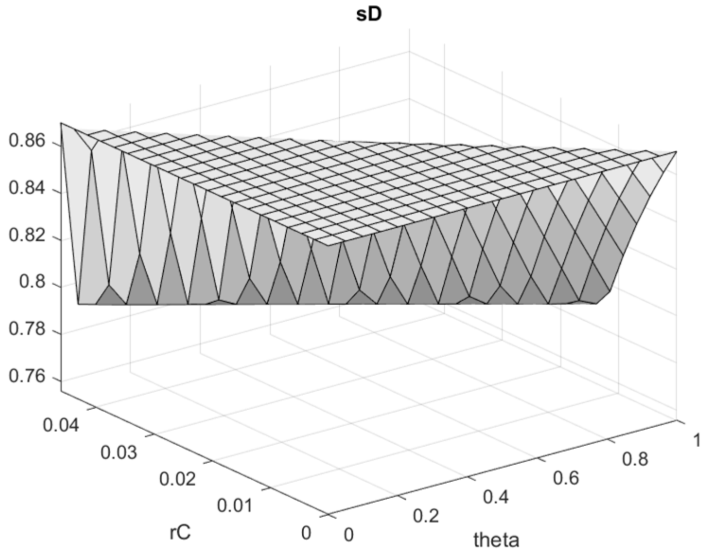


Figure 5 Cash Holdings

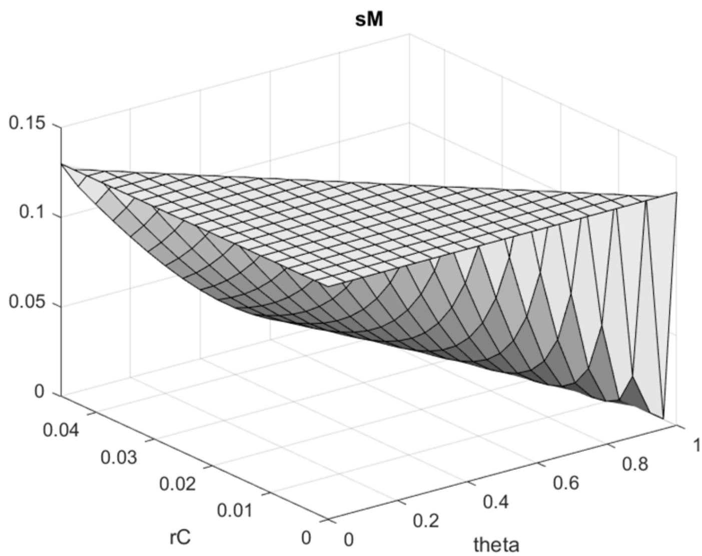
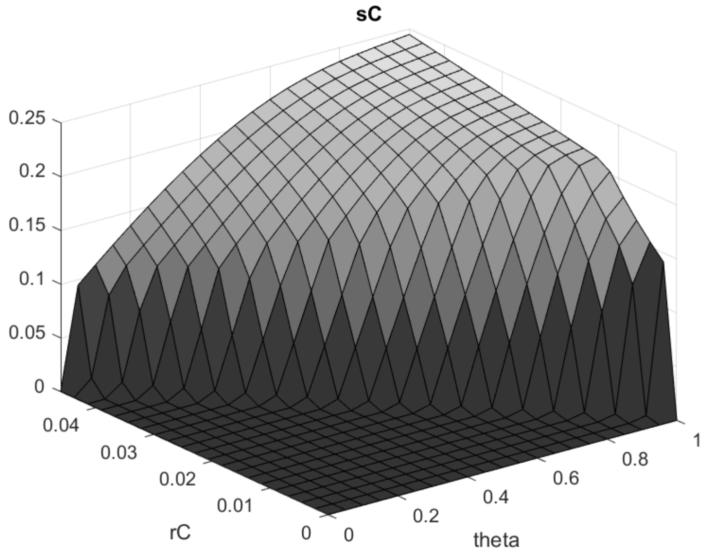


Figure 6 CBDC Holdings



When CBDC becomes sufficiently attractive, that is, $\theta + r^C/0.046 > 1$, it begins to affect households' allocation and bank maximizations. As $(\theta + r^C/0.046)$ increases, the deposit rate increases, the holdings of deposits and cash decrease, and the holdings of CBDC increase. One can see that the changes in these allocations are monotonic but not linear. They depend on the properties of the j distribution, which determines how elastic rates are means of payment choices. Some results, most notably the CBDC holdings, also show a kink, an abrupt slope change, which we will discuss next.

Figures 7 and 8 show the lending rate and firms' capital stock, respectively. Note that they are constant for a large region of the (θ, r^C) set but that this region is not the same as in figures 3 to 6. The lending rate only increases (and the capital stock only decreases) when the combination of variables θ and r^C are greater than what is necessary for the CBDC to become attractive. This happens because the bank's choice of r^L is independent of the CBDC. After all, banks are not liquidity constrained (Lemma 3). However, banks become liquidity constrained when CBDC becomes increasingly attractive and the deposit amount becomes small enough. After this point, banks choose to raise r^L to reduce the loan amount to ameliorate their constraint (Section 4.1).

Figure 9 reports the "liquidity frontier," the combination of points (θ, r^C) where banks' liquidity constraint strictly binds, in black. To compare, we also plot, dashed line, the locus where CBDC is just attractive enough to be held. The two lines define three regions in the (θ, r^C) space. In the lower region, the CBDC does not exist in equilibrium. Banks are liquidity constrained in the area at the top, and there is a reduction in capital stock and output. Between the two lines, the CBDC is sufficiently attractive to provide gains from the reduction of cash use but not too attractive to make banks liquidity constrained and result in output losses.

Returning to Figure 6, we can now identify that the kink position in CBDC holdings coincides with the locus defined by the liquidity frontier. Those combinations of (θ, r^C) define the condition where banks begin to change not only r^D but also r^L to maximize profits. Consequently, the elasticity of r^D to the attractiveness of the CBDC changes abruptly.

Figure 7 Lending Rate

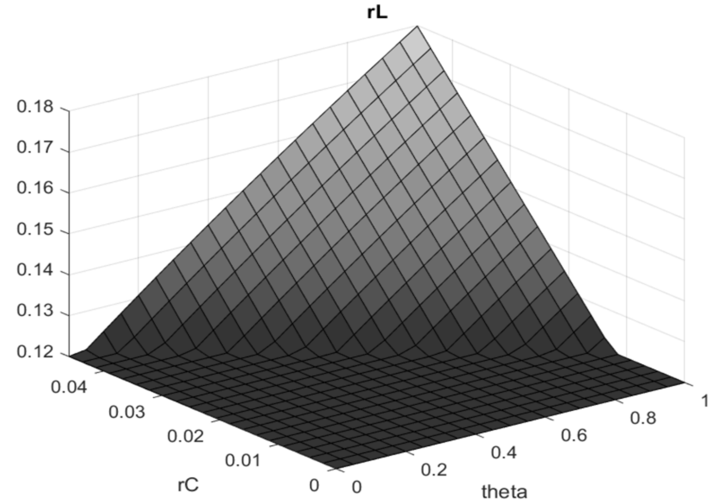


Figure 8 Capital Stock

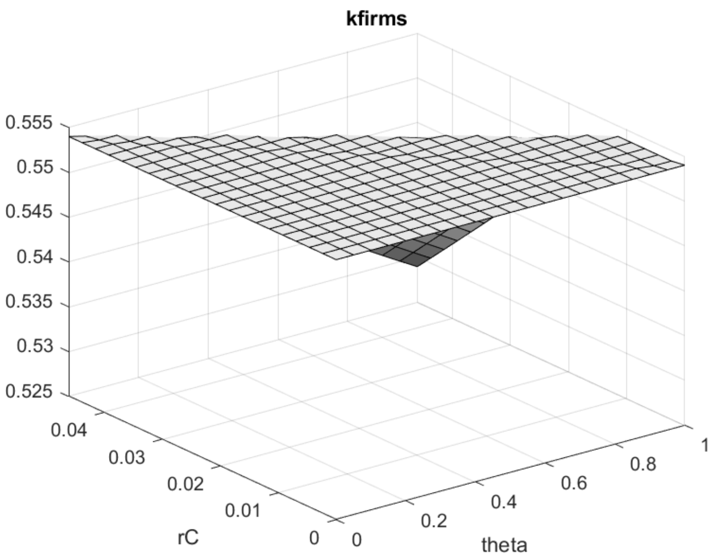
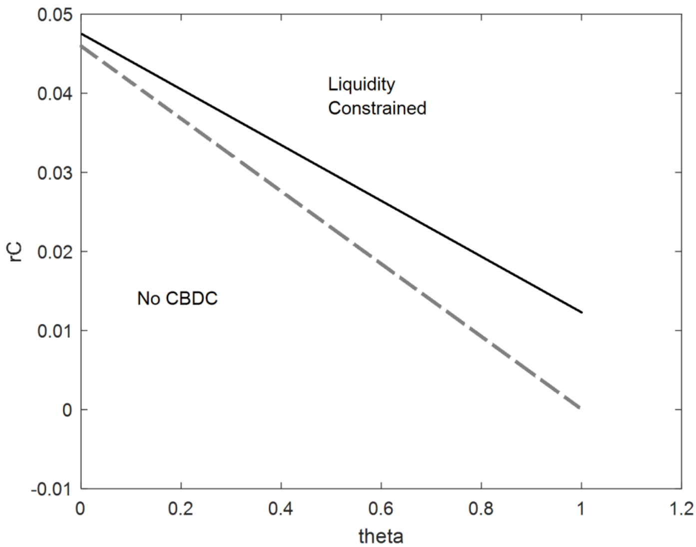
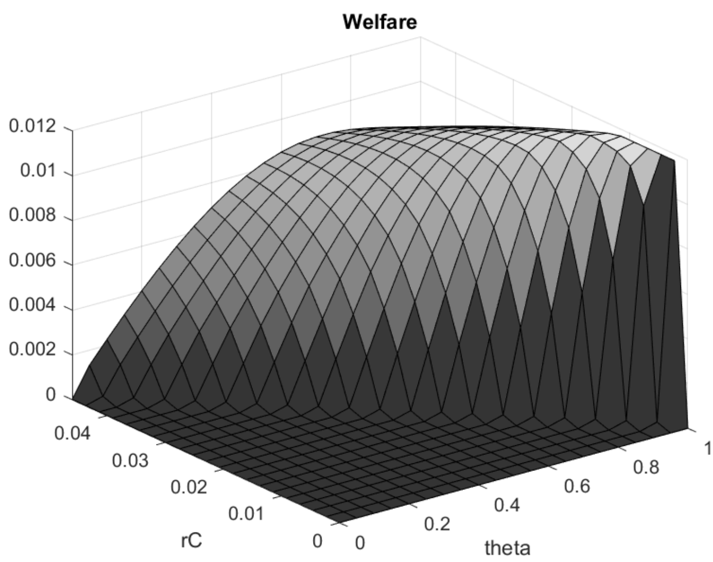


Figure 9 Liquidity Frontier



Finally, Figure 10 reports the welfare implications of (θ, r^C) choices. As previously suggested, the best CBDC is such that it reduces costly cash use but doesn't reduce capital stock. Since we assumed that the return of capital is higher than the return of reducing cash, capital reduction causes welfare losses. Thus, the optimal CBDC choice must lie on the "liquidity frontier" locus.

Figure 10 Welfare



The specific position over the liquidity frontier becomes a matter of calibration details. Depending on the shape of the j distribution, different combinations of the pair (θ, r^C) that lie on the liquidity frontier imply different amounts of cash holdings. That is, the threshold that defines the amount of cash holdings, $r^C/(1-\theta)$, varies with increases of θ and rC, which depend on how elastic the j distribution is at each point. For our calibration, it turns out that the optimal CBDC is obtained for high θ and low r^C, but one should not put much weight on this result.

4.5 Implementation

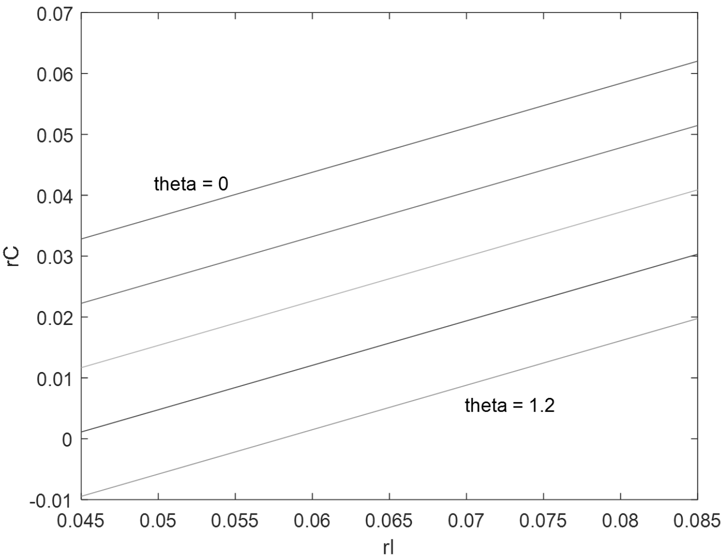
Given the results above, how should the Central Bank implement its CBDC? As discussed, to implement the optimum CBDC, the Central Bank should choose the parameters (θ, r^C) that lie on the liquidity frontier. But there are a few challenges to implementing this policy recommendation practically.

A first challenge is that the parameter θ does not correspond to any precise instrument chosen by the Central Bank. The Central Bank can undoubtedly choose some characteristics of the CBDC that affect its degree of anonymity but does not know exactly which would be the change in θ caused by some hypothetical choice. Moreover, given the concerns related to anti-money laundering and combating the financing of terrorism, Central Banks are constrained to design CBDCs with a very low level of anonymity.

A second challenge relates to changes in culture and technological attributes. Due to its nature, mainly if it adopts the Distributed Ledger Technology, Digital Currencies incorporate new features, such as programable money, smart contracts, and, more generally, “the internet of things.” That is, besides anonymity and remuneration, a CBDC will likely be attractive for other reasons. To put it in our model language, one can think that CBDCs anonymity level θ can reach higher levels even though its anonymity was set at a low value.

A third challenge concerns changes to the liquidity frontier caused by macroeconomic fluctuations at the business cycle frequency. One example includes changes in the Interest of Reserves rate, r^I, that reflect changes in monetary policy. To evaluate this possibility, Figure 11 reports how the CBDC rate r^C should be set for different values of r^I. Each curve reports, for a given $\theta \, \varepsilon$ {0, 0.3, 0.6. 0.9, 1.2}, the pair (r^I, r^C) that lies in the liquidity frontier. Notice that is a clear (linear) correspondence of r^I and $r^{C;}$ that is, as the monetary policy changes, the CBDC rate should be adjusted accordingly. Notice also that high values for θ and low values of r^I imply that r^C should be set at negative values.

Figure 11 Frontier Implementation



Although we are not modeling the dynamics of a CBDC implementation, we believe that a natural approach would be for the Central Bank to start with CBDC that is not anonymous at all (that is, with an account based CBDC) and that pays no interest rates. By doing so, it is less likely to incur the problem of having a CBDC that is so attractive (for having other technology features) that it drives banks' liquidity down. Then, as the demand for the CBDC is observed, the Central Bank can gradually increase the CBDC rate making sure that banks' liquidity ratios still have some cushion.

Even after reaching a situation where the CBDC interest seems adjusted to make the economy lie in the liquidity frontier, the Central Bank should be prepared to change it with technology, culture transformations, and monetary policy shifts.

5. Conclusions

This paper proposes a model to study Central Bank currencies for emerging markets. We advance a model of means of payment choice, where households have different preferences over anonymity. The financial sector is monopolistically competitive and may break the link between borrowing and lending rates. We calibrate the model to fit the Brazilian economy. We show that a sufficiently attractive digital currency reduces cash and bank deposit holdings. The digital currency may increase welfare as the use of cash is costly. However, if banks are liquidity constrained, which is likely the case in Brazil, the digital currency may result in fewer loans and output, thereby reducing welfare. We show, however, that the digital currency interest remuneration can be set and adjusted over time to balance this trade-off optimally.

APPENDIX

Table A1 Calibration

Parameters	***Notation***	***Value***
Deposit fraction	s^D	0.87
Cash fraction	s^M	0.13
Elasticity of deposits	η	0.30
Borrowing rate	r^D	0.046
Interest on reserves	r^J	0.065
Lending rate	r^L	0.12
Total loans	k	0.47
Mean of log(j) distribution	μ	-4.0
Standard deviation of log(j)	σ	0.81
Number of banks	N	23
Non-remunerated requirements	τ	0.20
Capital share	α	0.35
Depreciation rate	δ	1.85
Reserve requirement	λ	0.30
Productivity	A	3.45
Cost of cash	γ	0.090

REFERENCES

Agur I, Ari A, Dell'Ariccia G (2019): Designing Central Bank Digital Currencies. *IMF Working Papers*, 19/252.

Andolfatto D (2018): Assessing the Impact of Central Bank Digital Currency on Private Banks. *FRB St. Louis Working Paper*, (2018-25).

Barrdear J, Kumhof M (2016): The Macroeconomics of Central Bank Issued Digital Currencies. *Bank of England Staff Working Paper,* No. 605.

Bank for International Settlement (2020): Central Bank Digital Currencies: Foundational Principles and Core Features. Report No. 1 in a series of collaborations from a group of Central Banks.

Bank for International Settlement (2021): Ready, Steady, Go? Results of the Third BIS Survey on Central Bank Digital Currency. *BIS paper,* 114, Monetary and Economic Department.

Brunnermeier M, Niepelt D (2019): On the Equivalence of Private and Public Money. *Journal of Monetary Economics,* 106(C):27-41.

Chakravorti B, Mazzotta BD (2014): The Cost of Cash in Mexico. The Institute for Business in the Global Context, Working Paper, The Fletcher School, Tufts University.

Group of Thirty (2020): Digital Currencies and Stablecoins: Risks, Opportunities, and Challenges Ahead. Washington, D.C., July 2020.

Kreps D, Scheinkman J (1983): Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes. *Bell Journal of Economics,* 1(2):326-37.

Krüger M, Franz S (2014): Costs and Benefits of Cash and Cashless Payment Instruments. *Deutsche Bundesbank Working Paper*.