# **Role Swap: When the Follower Leads and the Leader Follows**

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## Abstract

The game theoretic literature has commonly explored circumstances in which the players are identical. In the real world, strategic actors such as competing firms or political parties are however heterogeneous. Most importantly, their payoffs across the various possible outcomes generally differ. We consider payoff heterogeneity within a more general 'Stochastic leadership' framework. It allows for probabilistic revisions of each player's initial actions - upon observing what the others have done. The analysis shows that under Stochastic leadership it is the exact payoffs, not just their ranking, that affects the set of (subgame-perfect) equilibria. This is consistent with experimental studies that show payoff heterogeneity to hinder cooperation and aggravate conflict by moving the players away from the focal (symmetric/equitable) outcome. Furthermore, we demonstrate that if the payoffs are sufficiently asymmetric the players may essentially swap their roles in coordination and anti-coordination games. In particular, we derive circumstances within the Battle of the sexes, Stag hunt and Hawk and dove games under which the Stochastic follower (the more flexible player with a higher revision probability) starts behaving as the Stackelberg leader. Our main real-world example is from the area of climate change agreements between major countries.

# 1. Introduction

The paper attempts to offer novel insights into the interactions between leaders and followers. This relationship has been examined in a number of disciplines other than economics such as management, psychology, sociology, political science and biology. Our analysis focuses on the strategic aspect, which has been studied in the literature since Heinrich von Stackelberg's published his influential book Market Structure and Equilibrium. It utilizes a game-theoretic framework Stochastic leadership (developed in Libich and Nguyen, 2013), which is a natural generalization of the Stackelberg leadership concept.

Under Stackelberg leadership, the follower can make a decision with certainty when finding out about the leader's choice. In contrast, the Stackelberg leader is unable to alter his/her initial decision upon observing what the follower did. Under Stochastic leadership, both (all) players can change their mind (revise their actions)

https://doi.org/10.32065/CJEF.2021.04.02

The first two authors gratefully acknowledge financial support from the Czech Science Foundation GACR (project 19-19485S).

after they see what the opponent(s) did initially. However, such revision occurs with some probability, not necessarily certainty. Formally, each player *i* can alter their initial (simultaneous) move with probability  $p_i \in [0,1]$ , and this exogenous probability is known to all players in advance. The players cannot observe Nature's move at the time of the revision, i.e. they do not know whether the others are able to revise. Put differently, the revisions are simultaneous.

It is apparent that within the Stochastic leadership framework the more rigid player with a lower  $p_i$  becomes the Stochastic leader, and the more flexible player with a higher  $p_i$  becomes the Stochastic follower. It is also apparent that the framework nests the standard setups of simultaneous moves and Stackelberg leadership as special (polar) cases. Our focus lies in coordination and anti-coordination games such as the Battle of the sexes, Stag hunt and Hawk and dove (Game of chicken). To explore the strategic effects thoroughly, in addition to the timing of the game we also enlarge the strategy space. In particular, we extend the standard  $2 \times 2$ versions of these games to their  $3 \times 3$  versions, adding a compromise option into each.

The paper's main contribution is to highlight a potential role swap. We formally derive the circumstances under which the Stochastic follower (higher  $p_i$  player) starts acting as the Stackelberg leader, and the Stochastic leader (lower  $p_i$  player) starts acting as the Stackelberg follower. Such role swap is shown to occur if there exists sufficient asymmetry in the players' payoffs. Most studies however focus on games featuring symmetric payoffs and hence miss this possible scenario and the interesting logic behind it. The main reason for this is that within the conventional game-theoretic frameworks, the exact payoffs (and their asymmetry) do not generally make a difference. It is only the payoffs' ranking for each player that matters.<sup>1</sup>

To provide an example, consider for example the Battle of the sexes within the simultaneous move setting. In the standard  $2 \times 2$  version of the game the male (*M*) and female (*F*) choose between Soccer (*S*) and Ballet (*B*).<sup>2</sup> The set of Nash equilibria is unaffected by the exact payoffs obtained from (*S*, *S*) and (*B*, *B*). It suffices that these are the two best outcomes for both players, whereby the male prefers the former and the female prefers the latter. As such, even if (for instance) the female's payoff from (*B*, *B*) increases dramatically, the only equilibrium-related aspect that will be altered is the probability of the male randomizing between *S* and *B* within the mixed-strategy Nash equilibrium. However, the set of equilibria will not change; there will still be three Nash equilibria and no theoretic way to uniquely select between them. Similarly, under Stackelberg leadership there will be no change either. There will still be a unique subgame-perfect equilibrium; the outcome preferred by the leader.

Such theoretic 'irrelevance' of the exact payoffs and payoff asymmetry is in stark contrast to experimental studies. In terms of the former, there is a large literature showing that stake size matters in many contexts (see e.g. Kocher et al., 2008 or

<sup>&</sup>lt;sup>1</sup>A partial exception is the Pure coordination game, whereby a quantitative change in a payoff (within the game's payoff ranking) may lead to the Pareto-inferior Nash equilibrium becoming risk-dominant. In such case the class of game gets altered from Pure coordination to a Stag hunt. Nonetheless, even in this case the set of Nash equilibria is unaffected.

<sup>&</sup>lt;sup>2</sup>The extended  $3 \times 3$  version of the game also features a compromise option Concert (*C*).

Johansson-Stenman et al., 2005). In terms of the latter, Hausken (2005) and (2007) shows that in a repeated Battle of the sexes payoff heterogeneity tends to aggravate conflict. This is by moving the players away from the focal outcome. A similar result is reported for twelve simple coordination games by Lopez-Perez et al. (2015), whose experiments demonstrate that asymmetric payoffs may hinder the players' coordination on the equitable outcome.<sup>3</sup> In line with that, Agranov and Schotter (2012) document experimentally that it may be socially beneficial for communication to be vague so that the asymmetry in payoffs is concealed. In addition, Parravano and Poulsen (2015) show that for a greater stake size to improve the players' coordination on the focal outcome the payoffs need to be symmetric.

To offer novel insights on the effects of payoff heterogeneity in coordination and anti-coordination games, our Stochastic leadership framework extends the standard timing of moves by bringing in the heterogeneous revision probability  $p_i$ . It can capture technological, logistic, political, psychological, historical or environmental reasons behind the players' distinct flexibility to change their mind/actions. For instance, consider two firms in the process of putting forward a new technology such as a battery for electric cars. To ensure mass adoption, the firms need to coordinate on one technology, otherwise both fail. However, each firm would prefer its own technology to be coordinated on. This game has the structure of the Battle of the sexes. The firm that is more advanced in the R&D, production or marketing process will be more rigid in switching to the rival's technology, i.e., it will have a lower  $p_i$ . Put differently, it will be the Stochastic leader, whereas the more flexible firm with higher  $p_i$  will be the Stochastic follower.

Many other examples underlying heterogenous  $p_i$  come to mind. An established political party with a long tradition of views on a certain issue will be less flexible (the Stochastic leader) in changing its stance on this issue than a newly-emerged populist party. In the sports arena, the teammates that have long played in certain roles within the team will generally be less flexible to change their actions than the newcomers. In the macroeconomic policy game, the central bank (setting monetary policy) is arguably more flexible than the government (setting fiscal policy); see Hughes Hallett et al. (2014). This is because the central bank board/committee generally reconsiders its interest rate setting every month at its regular policy meetings, whereas the government puts together the budget only once a year. This implies the fiscal policy's greater degree of rigidity (lower  $p_i$ ), putting the government into the role of the Stochastic leader. Relatedly, financial heterogeneity within a currency union has also been found important for the transmission of monetary policy, see e.g. Fisera (2020). The revision probability  $p_i$  may also capture switching costs, which have been studied extensively (see e.g., Lipman and Wang, 2000). These may be monetary as well as psychological.

Within the stylized Battle of the sexes, a possible interpretation of  $p_i$  can relate to different means of transport available to the players. For example, assume that the female has a car to her disposal, whereas the male takes public transport. This makes her more flexible, i.e., her revision probability (denoted f) is much higher than the male's (denoted m), i.e., f > m. Intuitively, she has a much greater chance

<sup>&</sup>lt;sup>3</sup>For an analogous result in a transfer pricing Hawk and dove type situation see Ackelsberg and Yukl (1979).

of being able to go from one venue to the other in time. If, for example, she initially goes to the Ballet venue and does not find her partner there, the possession of the car makes her likely to make it to the Soccer venue before the start of the match. In contrast, the male is more rigid due to using public transport so he would be unlikely to move between the venues in time.

As rigidity puts the player into a leadership position, it is an advantage in the Battle of the sexes and Hawk and dove. As such, under symmetric payoffs the Stochastic leader has a greater chance of achieving its preferred outcome. However, our analysis shows that under heterogeneous payoffs the conventional wisdom may be qualified in an important way. We first derive each player *i's Dominance region*. It can be defined as *the set of payoffs and revision probabilities under which choosing the payoff-salient option throughout the whole game (both initially and in the revision) strictly dominates all other strategies available to i.* It is apparent that in coordination and anti-coordination games the Dominance region delivers a player's preferred outcome as the unique subgame perfect equilibrium.<sup>4</sup>

To broaden the insights the parameter values delivering each player's Dominance region are derived below for both the conventional  $2 \times 2$  and extended  $3 \times 3$  versions of the Battle of the sexes, Stag hunt and Hawk and dove games. The conditions show that under Stochastic leadership it is not just the ranking of the payoffs that matters. It is the exact payoffs that matter as they affect the size of each player's Dominance region, and thus the set of subgame-perfect equilibria. This also means that our analysis offers an avenue through which the above experimental findings on payoff asymmetry can be justified.

We then zoom in on how the leadership/followership roles may be affected by the exact payoffs. Under Stackelberg leadership, the leader always dominates in coordination and anti-coordination games, i.e. (s)he achieves its preferred outcome with certainty based on strict dominance. In contrast, under Stochastic leadership it may be the Stochastic follower who does so, and we derive the parameter values when it occurs. For example, in the Battle of the sexes, the female may surely secure her Dominance region and thus her preferred (*B*, *B*) outcome as the unique subgame perfect equilibrium even if the car makes her more flexible (f > m).

What is the intuition behind such paradoxical situation? It may occur because there exists substitutability between each player's level of rigidity (the degree of Stochastic leadership) and the player's desire to achieve his/her preferred outcome. For the sake of example, assume that the female has a much higher payoff from (B,B)than from (S,S), whereas the male only has a marginal preference for (S,S) relative to (B,B). In such case, the female will be focused firmly on achieving the (B,B)outcome, even if it means a greater risk of miscoordination (off-diagonal payoffs). Such preference gives her a strategic advantage, and it will enlarge her Dominance region while shrinking the male's Dominance region.

If such preference is sufficiently strong it outweighs the opposite effect of her higher revision probability that makes her the Stochastic follower. Formally, her Dominance region can occur even for values of the male's Stochastic leadership,

<sup>&</sup>lt;sup>4</sup>Parameter values that do not satisfy the conditions for a player's Dominance region fall into what we call a 'Multiplicity region'. Within this region more than one equilibrium outcome can obtain.

f > m. As a consequence, she can obtain her preferred outcome (B, B) with certainty even if the possession of the car makes her less rigid than the male. She then acts as the Stackelberg leader despite being the Stochastic follower. In such a situation the opposite is true for the male; he acts as the Stackelberg follower even if his rigidity makes him the Stochastic leader.

Needless to say that under symmetric payoffs such scenarios cannot occur. Also note that under the simultaneous move game a player's strong preference for her/his payoff-salient outcome has the opposite effect. In particular, taking our example of the female's increased payoff from (B, B), the male would start playing Ballet with a *lower* probability within the mixed-strategy Nash equilibrium. Hence conventional theory predicts that the (B, B) outcome becomes less frequently played, which seems counterintuitive. Our analysis offers the opposite prediction that is in line with both the intuition and existing experimental evidence.

To bring the theoretic analysis to life, we offer an application to the area of global climate change agreements. It makes clear that changes in the costs and benefits of climate action by individual countries may lead to a role swap. For example, availability of a cheaper mitigation technology and/or an increase in the expected damage from climate change may incentivize a country (such as China) to assume the leadership role, despite various characteristics that make it the Stochastic follower.

Our analysis also provides insights into leadership relevant to areas outside economics. Most notably, it suggests a formal channel that can underlie the leader-follower theory examined by management and psychology studies, e.g., Price and Vugt (2014), Hudson (2013) or Lord et al. (1996).

## 2. Benchmark Game: Battle of the Sexes

#### 2.1 Payoffs

In our exploration the main focus will lie in the Battle of the sexes game, but other classes of coordination and anti-coordination games will be explored also (see Section 6). In order to demonstrate the general nature of our findings, we will consider the conventional  $2 \times 2$  versions of each game, as well as their extended  $3 \times 3$  versions. It will be apparent that all our findings are robust and carry over to larger games.<sup>5</sup> Nevertheless, to keep our focus the main text will only provide proofs of our results for the conventional  $2 \times 2$  game, and those for the  $3 \times 3$  versions will be provided in an online appendix.

In the Battle of the sexes the set of actions in the  $2 \times 2$  game consists of Soccer (S) and Ballet (B), whereas the  $3 \times 3$  game also includes Concert (C). Such addition of a compromise (focal) option has been implemented in experimental studies, e.g., Jackson and Xing (2014) and He and Wu (2020). Both versions of the game can be summarized within one payoff matrix as follows

<sup>&</sup>lt;sup>5</sup>An additional motivation for examination of the  $3 \times 3$  games is that the compromise option brings a potential tension between the leadership and focal outcomes, so it seems worthwhile to explore its effects. It should also be mentioned that while the experimental literature examines  $3 \times 3$  coordination games and offers novel insights, the theoretic literature has lacked behind and predominantly focused on the  $2 \times 2$  games (and this includes Libich and Nguyen, 2013). Our analysis partly fills this gap.

|   |         |        | F       |        |
|---|---------|--------|---------|--------|
|   |         | Soccer | Concert | Ballet |
|   | Soccer  | a,t    | d, x    | e, v   |
| M | Concert | g, z   | b,s     | h, u   |
|   | Ballet  | i, y   | j, w    | c,r    |

where the pure-strategy Nash equilibria (indicated in bold) are implied by the following constraints on the payoffs

$$a > b > c > d \ge e \ge g \ge h \ge i \ge j$$
  
and  
$$r > s > t > u \ge v \ge w \ge x \ge y \ge z.$$
(2)

Using such general payoffs from (2) will enable us to better show the mechanics of Stochastic leadership under asymmetric preferences. Nevertheless, for clarity we will also postulate a specific game featuring symmetric payoffs from Jackson and Xing (2014).<sup>6</sup>

|   |         |                     | F                   |                     |
|---|---------|---------------------|---------------------|---------------------|
|   |         | Soccer              | Concert             | Ballet              |
|   | Soccer  | <b>5</b> , <b>2</b> | 0, 0                | 0, 0                |
| M | Concert | 0, 0                | <b>3</b> , <b>3</b> | 0, 0                |
|   | Ballet  | 0, 0                | 0, 0                | <b>2</b> , <b>5</b> |

(3)

#### 2.2 Equilibrium Outcomes

In the 2 × 2 version of the game there are two (efficient) pure-strategy Nash equilibria, (*S*, *S*) and (*B*, *B*), preferred by the male and female respectively. There is also an inefficient mixed-strategy Nash equilibrium; for example, under the specific payoffs in (3) each player chooses its preferred and its partner's preferred action with probabilities  $\frac{5}{7}$  and  $\frac{2}{7}$  respectively. The 3 × 3 game has an additional pure-strategy Nash equilibrium, (*C*, *C*), which constitutes the second-best outcome for both players, and can thus be thought of as a compromise. The extended game also features additional Nash equilibria in mixed-strategies.

Let us consider the outcomes of the one-shot simultaneous game. The main conclusion is that the multiplicity of equilibria prevents conventional economic theory to uniquely select the equilibrium outcome. Experimental studies do not offer clear guidance either, demonstrating that many factors play a role. Apart from the exact payoffs, these include culture, religion, gender, income and risk aversion (see e.g., Desjardins and Dubois, 2015, Jackson and Xing, 2014, Fung and Au, 2014, Levy and Razin, 2012, Shachat and Swarthout, 2004, or Lorenzi-Cioldi et al., 1995).

Notwithstanding that, in the (one-shot) simultaneous game Schelling's (1960)

<sup>&</sup>lt;sup>6</sup>We normalize the Jackson and Xing (2014) payoffs by dividing all of them by 10. In their experiment, the players choose (simultaneously) between three colours.

focal point argument is commonly used to select the equilibrium. A thorough discussion of this concept is beyond the scope of the paper; the reader can consult e.g., Als-Ferrer and Kuzmics (2013), Sugden (1995) or Crawford and Haller (1990). Let us just mention that empirical evidence for the use of focal points in coordination games appears in many papers, for instance, in Sitzia and Zheng (2019), Lopez-Perez et al. (2015), Parravano and Poulsen (2015), Pope et al. (2015), Jackson and Xing (2014), Isoni et al (2013), Pogrebna and Blavatskyy (2009) or Knittel and Stango (2003).

The focal outcome differs in the  $2 \times 2$  and  $3 \times 3$  games. In the  $2 \times 2$  game the mixed-strategy Nash equilibrium is considered the most likely outcome due to its symmetry.<sup>7</sup> In the  $3 \times 3$  game the compromise (*C*, *C*) is generally taken as the most probable outcome. This is because it is not only symmetric but also efficient - Pareto superior to the symmetric mixed-strategy Nash equilibrium.<sup>8</sup>

Turning from the simultaneous move to Stackelberg leadership, the set of equilibria is reduced and multiplicity is no longer an issue. In both the  $2 \times 2$  and  $3 \times 3$  versions of the Battle of the sexes, as well as in other coordination and anti-coordination games, Stackelberg leadership delivers a unique (subgame-perfect) equilibrium, preferred by the leader. Importantly, this is the case even if the payoffs are highly asymmetric. In the next section we put forward a framework featuring an alternative concept of leadership in order to revisit the robustness of these findings. We are interested in the effects of payoff asymmetry, and how it may impact the leadership/followership roles.

#### 3. Stochastic Leadership: Probabilistic Revisions of Actions

Our framework postulates probabilistic revision opportunities and in doing so it nests the conventional simultaneous moves and Stackelberg leadership setups as special (polar) cases. The revisions are reminiscent of the Calvo (1983) timing used extensively in macroeconomics in relation to price setting behaviour.<sup>9</sup>

For comparability with the conventional settings, we retain most of the standard assumptions. The players are rational, have common knowledge of rationality as well as complete information about all aspects of the game. Our framework also adopts the simultaneous move of the players at the start of the game. Players are assumed to observe this move of all opponents, and then, still at time t = 0, they may change their mind and revise the opening action. However, such change can only be made with a certain player-specific probability  $p_i$ : in the Battle of the sexes the female's and male's revision probabilities are denoted f and m respectively. Put differently, there is a move of Nature, which dictates the remaining moves in the game. The revision

<sup>&</sup>lt;sup>7</sup>Experimental studies have provided evidence that people do use mixed-strategy Nash equilibria in many circumstances, for example Oprea et al. (2011), Azar and Bar-Eli (2011), Pogrebna and Blavatskyy (2009), Chiappori et al. (2002) and O'Neill (1987). In the Battle of the sexes, experiments indicate that the Pareto-inferior (off-diagonal) outcomes obtain with more than 50% probability. Players played the payoff-salient action 48-72% of the time, making the probability of coordinating on (*S*, *S*) and (*B*, *B*) only 20-25% each; see Martinangeli et al. (2017), Chuah et al. (2016), Parravano and Poulsen (2015), Colman and Stirk (1998) and Cooper et al. (1989, 1993).

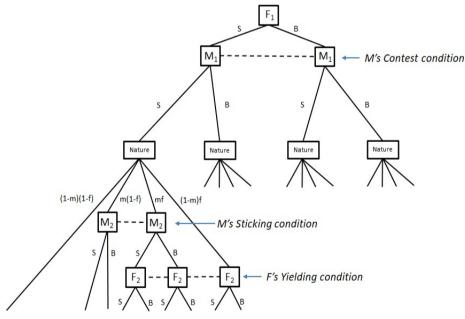
<sup>&</sup>lt;sup>8</sup>For example, in the experiments of He and Wu (2020) the implied probability of the players coordinating on the focal outcome in the  $3 \times 3$  game was 69%, and the same number was found by Jackson and Xing (2014) for U.S. subjects.

<sup>&</sup>lt;sup>9</sup>For more on Stochastic leadership see Libich and Nguyen (2013). For alternative leadership frameworks see e.g. Hughes Hallett et al. (2014), Wen (2002) or Maskin and Tirole (1988).

opportunities are simultaneous in the sense that at the time of the revision the players do not know whether the other(s) have been given a revision opportunity. These assumptions imply that while the payoffs are not affected by the pre-revision actions, the expected payoffs are.<sup>10</sup>

The  $2 \times 2$  version of the resulting extensive-form game is summarized in Figure 1. With probability mf both players get a revision opportunity, i.e., the game is equivalent to a two-shot simultaneous game. Conversely, with probability (1 - m)(1 - f) we have a one-shot simultaneous game. With probability f(1 - m) only the female can revise her initial action, and with probability m(1 - f) the opposite situation arises. In Figure 1 and throughout the paper, the number in the subscript will express the players' action. Number 1 denotes the opening move and 2 denotes the post-revision action (regardless of whether a player actually had a revision chance). The superscript will indicate the selected action; e.g.,  $M_1^S$  is the male's initial choice of Soccer. To avoid cluttering Figure 1, it only shows one of the four subgames starting with the move of Nature, the others are analogous.

Figure 1 The  $2 \times 2$  Extensive-Form Battle of the Sexes Game with Revision Probabilities m and f Capturing Stochastic Leadership



Notes: For parsimony, only the first move of nature is fully drawn.

#### 4. Equilibrium Effects of Heterogeneous Payoffs

Let us now consider the effects of payoff asymmetry within the Stochastic leadership framework. While both our main results are first formulated for the Battle

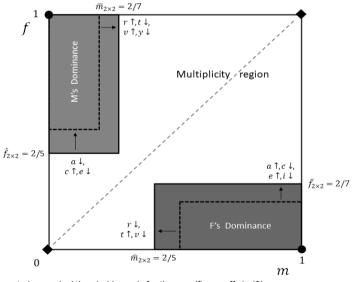
<sup>&</sup>lt;sup>10</sup>The only exception is the case of the revisions arriving with certainty for both players (m = f = 1), which is the special case of a two-shot simultaneous game.

of the sexes, we later prove them for other classes of games such as the Stag hunt and the Hawk and dove (Game of chicken).

**Proposition 1.** In the  $2 \times 2$  and  $3 \times 3$  Battle of the sexes postulated in (1)-(2), under Stochastic leadership the exact values of the payoffs - not just their ranking - determine the size of the Dominance and Multiplicity regions. As such, the exact payoffs determine the set of equilibrium outcomes.

*Proof.* In order to best present the intuition, the main text will provide the proof for the standard  $2 \times 2$  version of the game, including all the accompanying intuition. The conditions for the extended  $3 \times 3$  version are analogous, and will be relegated to online Appendix A. The Multiplicity and Dominance regions are plotted in Figure 2 as functions of the players' revision probabilities. The figure also shows the comparative statics, i.e., how each payoff affects the thresholds separating the Dominance and Multiplicity regions. Note that in the  $2 \times 2$  game every single payoff  $\{a, c, e, i, r, t, v, y\}$  affects the size of at least one Dominance region. As such, its change can alter the set of equilibrium outcomes.

Figure 2 Equilibrium Regions in the  $2\times 2$  Battle of the Sexes under Stochastic Leadership, Featuring Revision Probabilities m and f



## The 2x2 game

Notes: The reported numerical thresholds apply for the specific payoffs in (3).

There are three types of conditions for each player's Dominance region, which we refer to as Yielding, Sticking and Contest conditions. They are indicated in Figure 1. Unless all three are satisfied the game ends up in a Multiplicity region, which features more than one equilibrium outcomes. Let us first durive the circumstances for the male's Dominance region. Therein,  $(M_1^S M_2^S, F_1^S F_2^S)$  uniquely obtains on the equilibrium path, i.e., Soccer is surely played by both players throughout the whole

game. The reason is that for the male playing S in both moves is a strictly dominant strategy.

Solving by backwards induction, consider the female's Yielding condition. It guarantees that if she observes her partner to have gone to Soccer initially  $(M_1^S)$ , she would choose her static best response and join him there in her revision  $(F_2^S)$ . She would be incentivized to do so even in the worst-case scenario of knowing with certainty that he would like to switch to Ballet in his possible revision  $(M_2^B)$ . When will the female be willing to 'yield' in such way? This will be the case if the male's revision probability is sufficiently low, i.e., he is unlikely to be able to switch to  $M_2^B$ . Using our earlier interpretation, if the bus runs infrequently there is only a small chance of the male to make a move between the Soccer and Ballet venues in time. Knowing this, the more flexible female would use the car and switch from  $F_1^B$  to  $F_2^S$  (if given a revision opportunity). Her yielding condition that ensures this is the following

$$\underbrace{\frac{F_2^S}{\underbrace{(1-m)t}_{M_2^S \text{ as can't revise}} + \underbrace{my}_{\text{Switch to } M_2^B}}_{\text{Switch to } M_2^B} > \underbrace{\underbrace{(1-m)v}_{M_2^S} + \underbrace{mr}_{\text{Switch to } M_2^B}}_{M_2^S}.$$
(4)

The inequality in (4) requires that the expected payoff of the Stochastic follower F from  $F_2^S$  (the left-hand side) exceeds that from  $F_2^B$  (the right-hand side). As explained above, the condition in (4) assumes the worst-case scenario from the female's point of view, in which the male played Soccer initially  $(M_1^S)$  but plans to choose Ballet  $(M_2^B)$  in his potential revision. Upon rearranging, we obtain the Yielding condition for the female.<sup>11</sup>

$$m < \bar{m}_{2\times 2} = \frac{t - v}{t - v + r - y} \stackrel{(3)}{=} \frac{2}{7}.$$
(5)

Moving backwards, assume the Yielding condition in (5) holds and consider M's revision. To ensure equilibrium uniqueness of the Soccer outcome a Sticking condition for the male is required, whereby he surely chooses to stick to his initial choice of Soccer rather than to switch to Ballet. This must be the case even in the worse scenario of him observing the female to have gone to Ballet initially  $(F_1^B)$ , which is ensured by the following

$$\underbrace{\frac{M_2^S}{\underbrace{(1-f)e}_{F_2^B \text{ as can't revise}}^M + \underbrace{fa}_{\text{switch to } F_2^S}}_{F_2^B \text{ as can't revise}} \times \underbrace{\frac{M_2^B}{\underbrace{(1-f)c}_{F_2^B \text{ as can't revise}}^M + \underbrace{fi}_{\text{switch to } F_2^S}}_{\text{switch to } F_2^S}.$$
(6)

The left- and right-hand sides of this condition report M's expected payoffs from  $M_2^S$  and  $M_2^B$  respectively. Both assume  $F_1^B$  has been played, as well as female's

<sup>&</sup>lt;sup>11</sup>To provide greater clarity using a quantitative example, the right-hand side of the  $\stackrel{(3)}{=}$  notation will report the conditions for the specific payoffs postulated in (3) - in addition to the general conditions on the left-hand side.

intended switch to  $F_2^S$  if she is given a revision opportunity (implied by the Yielding condition). Rearranging (6) we obtain *M*'s Sticking condition

$$f > \frac{c - e}{c - e + a - i} \stackrel{(3)}{=} \frac{2}{7}.$$
 (7)

Continuing the backwards induction solution, assume the Yielding and Sticking conditions are satisfied and consider the male's opening move. For the Soccer outcome to be surely played it must be the case that the male would open with  $M_1^S$  even if he knew with certainty that the female would go to Ballet initially  $(F_1^B)$ . Formally, his expected payoff from  $(M_1^S M_2^S, F_1^B F_2^S)$  must be higher than that from  $(M_1^B M_2^B, F_1^B F_2^B)$ . This is ensured by the following Contest condition

$$\underbrace{\frac{M_1^1 M_2^S}{(1-f)e} + \underbrace{fa}_{\text{Switch to } F_2^S}}_{F_1^B \text{ as can't revise}} > \underbrace{\frac{M_1^B M_2^B}{\widehat{C}}}_{F_1^B}.$$
(8)

Upon rearranging, we obtain

$$f > \hat{f}_{2\times 2} = \frac{c - e}{a - e} \stackrel{(3)}{=} \frac{2}{5}.$$
(9)

It is straightforward to verify that this Contest condition is stronger than the Sticking condition in (7) for all general parameter values. As such, the necessary and sufficient conditions for *M*'s Dominance region are jointly (5) and (9).

One can derive the conditions for F's Dominance region by symmetry. In particular, M's Yielding and F's Contest conditions are the following

$$f < \bar{f}_{2\times 2} = \frac{c - e}{c - e + a - i} \stackrel{(3)}{=} \frac{2}{7} \quad \text{and} \quad m > \hat{m}_{2\times 2} = \frac{t - v}{r - v} \stackrel{(3)}{=} \frac{2}{5}.$$
 (10)

Noting that the sizes of the players' Dominance and Multiplicity regions (and hence equilibrium outcomes) are functions of the exact playoffs completes the proof.

In summary, when the conditions for either player's Dominance region are satisfied equilibrium uniqueness and efficiency are guaranteed. If this is not the case we obtain the Multiplicity region featuring more than one equilibrium outcome, giving rise to potential miscoordination that is costly to both players.

In particular, if we change the references in the direction of either increasing M's successful contest payoff a, reducing his yielding payoff c, and/or reducing his compromise payoff b, then M's Dominance region is enlarged. If we however change payoff asymmetry by lowering a, increasing c and/or increasing b then M's Dominance region shrinks. These payoffs also affect the size of F's Dominance region. All these changes therefore may, for any given revision probabilities m and f, lead to a transition between the Dominance and Multiplicity regions. As such, they may alter the set of equilibrium outcomes.

#### 5. Leadership/Followership Role Swap

Let us now focus on the roles of the leader and follower. The proof of Proposition 1 implies that under symmetric payoffs there is never any role swap, i.e., the intuition of Stochastic leadership is analogous to Stackelberg leadership. In both setups, if a Dominance region obtains in the Battle of the sexes the unique equilibrium is always the Stochastic leader's preferred outcome. However, if the payoffs are asymmetric this may no longer be the case in the presence of stochastic revisions, and a role swap may occur. This can be summarized as follows.

**Proposition 2.** In the  $2 \times 2$  and  $3 \times 3$  Battle of the sexes postulated in (1)-(2), there exist parameter values under which the less rigid player secures its Dominance region. In such case, the **Stochastic follower behaves as the Stackelberg leader**, and ensures his/her highest payoff.

*Proof.* Let us again focus on the 2 × 2 game here; the proof for the 3 × 3 version is reported in online Appendix B. The proof of Proposition 1 implies that there exist three main equilibrium regions. In particular: 1) the Soccer outcome uniquely obtains if both  $m < \bar{m}_{2\times 2}$  and  $f > \hat{f}_{2\times 2}$  (*M*'s Dominance region); 2) the Ballet outcome uniquely obtains if both  $m > \hat{m}_{2\times 2}$  and  $f < \bar{f}_{2\times 2}$  (*F*'s Dominance region); and 3) multiple equilibria occur under all remaining circumstances (the Multiplicity region).

The parameter space within which the role swap occurs is indicated in Figure 3 in blue. The left panel shows the  $2 \times 2$  game, the right panel shows the  $3 \times 3$  game. In particular, it is the triangle within one player's Dominance region that stretches across the 45-degree line. This means that the dominating player has a higher revision probability, which makes him/her the Stochastic follower. Despite this, s/he obtains its preferred outcome with certainty, like the Stackelberg follower.

In the  $2 \times 2$  Battle of the sexes the condition for *M* to dominate despite being the Stochastic follower is

$$\hat{f}_{2\times 2} = \frac{c-e}{a-e} < \bar{m}_{2\times 2} = \frac{t-v}{t-v+r-y}$$

whereas for F it is

$$\hat{m}_{2\times 2} = \frac{t-v}{r-v} < \bar{f}_{2\times 2} = \frac{c-e}{c-e+a-i}$$

To offer a numerical example, consider the specific payoffs in (3), but change the female's payoff r from 5 to some value r > 11. That is, the female would care very strongly about going to Ballet with her partner. In such case  $\hat{m}_{2\times 2}$  decreases to a value below  $\frac{2}{7}$ . We may uniquely obtain F's preferred Ballet outcome even if she has car to her disposal, which makes her more flexible, f > m.

Conversely, if the payoff *a* is changed from 5 to a > 11 while the other payoffs in (3) are unchanged, then  $\hat{f}_{2\times 2}$  decreases to a value below  $\frac{2}{7}$ . The Soccer equilibrium can then uniquely occur even if f < m, i.e., the male can be the

dominating player even if he is more flexible (has the car).

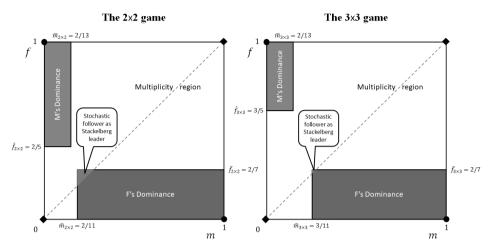


Figure 3 Equilibrium Regions in the  $2\times 2\,$  (Left Panel) and  $\,3\times 3\,$  (Right Panel) Battle of the Sexes

The intuition behind the result lies in the substitutability between a player's rigidity, which puts the player into a leadership position, and the strength of his/her preference to achieve the preferred outcome. If the latter is sufficiently strong it can outweigh the former, and lead to what seems like a role swap between the leader and the follower.

Let us note that theoretic methods do not tell us what exactly happens outside the Dominance region; multiple equilibria arise there. However, it is plausible that the closer we get to a player's Dominance region the higher the probability that in an experimental/real-world setting the player's behaviour would resemble that within his/her Dominance region. This means that a role swap *may* occur also for other parameter values in the vicinity of the blue triangles (across the 45-degree line). Nonetheless, it arises *with certainty* only within the blue triangles in Figure 3.

## 6. Other Coordination and Anti-Coordination Games

This section shows that the above results are general, i.e., they obtain in other classes of coordination and anti-coordination games. As a practical application, we will examine the Stag hunt game that occurs in the climate change area, and also consider the Hawk and dove game.

## 6.1 The Stag Hunt

Many possible applications of this class of game come to mind. For instance, the bank-run situation is commonly modeled as a Stag hunt, see Diamond and Dybvig (1983). We will focus on the climate change area given its immense importance for the future.

Notes. Like in Figure 2, the thresholds are plotted for the specific payoffs in (3); except for the value of r, which has been increased from 5 to 11. The blue triangles across the 45-degree line show the parameter values of the role swap.

There exists an overwhelming consensus that human activities cause global warming (see e.g., the meta-study by Powell, 2016).<sup>12</sup> There is also mounting evidence that this presents a major threat to both the environment and the economy (see e.g., IPCC, 2019). Global climate negotiations have traditionally been represented as a  $2 \times 2$  version of the Prisoner's dilemma game. In this game a country considers two options, *Action* (*A*, reduce greenhouse gas emissions substantially) and *No-Action* (*N*, do nothing).<sup>13</sup> Due to a free-riding problem, *N* is the strictly dominant strategy in the Prisoner's dilemma and hence the game's unique equilibrium (*N*, *N*) is inefficient.

It has however been put forward that the Prisoner's dilemma may no longer provide an accurate description of the global climate game.<sup>14</sup> There are two main reasons for the departure from the Prisoner's dilemma. First, the large environmental and economic costs of unmitigated warming have become more apparent over time, as well as the relatively low estimates of the cost of climate policy action (see e.g., IPCC, 2019). Second, some countries (especially China) realize more clearly that climate action presents a major business opportunity for them to become the driving force in inventing and producing clean technology (see e.g., Perdana and Tyers, 2018). As argued by Skyrms (2003) and others, the Stag hunt is an appropriate depiction of the 21<sup>st</sup> century realities, and the 2015 Paris agreement seems in line with this argument.

To formalize this discussion, and examine how the classes of games depend on the underlying parameters, assume the following objective function

$$U_{i} = \overbrace{\beta_{i}\left(\sum_{i=1}^{n}D\right)^{2}}^{\text{benefit of policy action}} - \overbrace{\widetilde{\gamma_{i}D_{i}^{2}}}^{\text{cost of policy action}}, \beta > 0, \gamma > 0, \qquad (11)$$

where i denotes a country, D is the degree of climate change mitigation (greenhouse gas emissions reduction). The quadratic terms express that: (i) the benefits of mitigation are amplified if more countries join in the efforts, and (ii) the costs of mitigation rise more than proportionately. Naturally, many alternative specifications are possible, but it will be apparent that (11) enables us to capture the key insights.

For clarity, let us make several simplifications. First, we will normalize  $\beta_i = 1, \forall i$ , in which case  $\gamma_i$  expresses country *i*'s climate action's *cost relative to its benefit*. Second, we will assume this relative cost to be the same for all countries,  $\gamma_i = \gamma, \forall i$ . The payoffs can then be written as functions of a single parameter  $\gamma$ . To obtain the conventional  $2 \times 2$  game we can further normalize the continuous *D* variable into two levels: A = 2 and N = 0. In such case the payoff matrix of the two-country game can be written as follows

<sup>&</sup>lt;sup>12</sup>Using five surveys of the peer-reviewed literature from 1991 to 2015 (combining 54,195 articles), Powel estimates the scientific consensus during this period to have been 99.94%.

<sup>&</sup>lt;sup>13</sup>Many other labels have been used in the literature, for example mitigate or abate for option A, and pollute or exploit for N.

<sup>&</sup>lt;sup>14</sup>DeCanio and Fremstad (2013) show that 'of the 144 distinct  $2 \times 2$  games in which the players have ordinally ranked utilities, 25 are potentially relevant to the climate problem'.

|   |                 | F  |                           |      |
|---|-----------------|--|---------------------------|------|
|   |                 | Action $(A)$                                     | No-Action $(N)$           | (12) |
| M | Action $(A)$    | $4\left(4-\gamma\right), 4\left(4-\gamma\right)$ | $4\left(1-\gamma ight),4$ | (12) |
|   | No-Action $(N)$ | $4,4\left(1-\gamma\right)$                       | 0,0                       |      |

We will follow the convention of the two-country analysis and interpret it as an interaction between the two largest greenhouse gas emitters, China and the United States.

The matrix in (12) makes apparent that several classes of games can arise, depending on the relative cost of climate action  $\gamma$ . If the cost is too high,  $\gamma > 3$ , then climate *Action* becomes strictly dominated by *No-Action*. This means that (N, N) is the unique outcome by strict dominance, and even Stochastic leadership cannot bring about global climate coordination and efficiency in the game. In particular, within the  $\gamma > 3$  range the Prisoners' dilemma scenario obtains if  $\gamma \in (3,4)$ , because in such case the unique Nash equilibrium (N, N) is Pareto-inferior to the non-equilibrium (A, A) outcome. This is apparent in the following payoff matrix, which uses  $\gamma = 3.5$  for illustration.

|    |           | Chi                            | na   |
|----|-----------|--------------------------------|--|
|    |           | Action                         | No-Action  |
| US | Action    | Pareto-superior non-Nash $2,2$ | -10, 4   |
|    | No-Action | 4, -10                         | $egin{array}{c} { m Pareto-inferior Nash} \ 0,0 \end{array}$ |

The global climate game as the Prisoner's dilemma ( $\gamma = 3.5$ )

If, however, the cost of mitigation decreases and/or its benefit increases such that  $\gamma \in (1,3)$ , then the climate game has the structure of the Stag hunt game. For example, under  $\gamma = 2$  we obtain:

|    |           | Chi                                | na  |      |
|----|-----------|------------------------------------|---|------|
|    |           | Action                             | No-Action   |      |
| US | Action    | Payoff-dominant Nash ${f 8},{f 8}$ | -4, 4   | (14) |
|    | No-Action | 4, -4                              | $\begin{array}{c} {\rm Risk-dominant\ Nash}\\ {\bf 0}, {\bf 0} \end{array}$ |      |

The global climate game as the Stag hunt  $(\gamma = 2)$ 

The Stag hunt game has no dominated strategies and it features two pure-strategy Nash equilibria (indicated in bold), namely (A, A) and (N, N). The former is payoff-dominant whereas the latter is inefficient but risk-dominant. The multiplicity complicates equilibrium selection in the standard one-shot simultaneous game, whereby the latter outcome may often obtain (see e.g., Van Huyck et al., 1990 and Harsanyi, 1995).<sup>15</sup>

Our earlier analysis of the Battle of the sexes shows that under Stochastic

<sup>&</sup>lt;sup>15</sup>Even the mixed-strategy equilibrium, in which the players choose each option with 50% probability (receiving a payoff of 2), is Pareto-superior to the risk-dominant equilibrium. Experimental evidence by Belloc et al. (2019), Chuah et al. (2016) and Colman and Stirk (1998) implies that in the Stag hunt players choose Stag somewhere between 52% and 79% of the time. As such, they achieve the efficient outcome between 27% and 62% of the time.

leadership the threat of inefficient outcomes can be alleviated even if the countries are highly risk averse. In line with Proposition 1, the payoff-dominant (A, A) outcome gets played on the equilibrium path of the unique subgame perfect equilibrium in the Stag hunt when one player's revision probability is sufficiently low and the other's sufficiently high. Hence such heterogeneity in revision probabilities is beneficial in enabling implicit coordination between the players.

It can be argued that due to its one party governing structure and the central planning process China can be more flexible in some areas of government decision making than the US (or the European Union).<sup>16</sup> Because of that, China is likely to assume the role of the Stochastic follower in the global climate game, and enable the more rigid US to lead (i.e. to pre-commit) more effectively. Such situation benefits both countries as it guarantees the (A, A) outcome within the Dominance region of the U.S.

In order to consider additional insights under asymmetric payoffs, let us add a compromise *Partial-Action*, *P*. It can be continuous and cover everything in between *A* and *N*, but we will for parsimony only consider a single *P* action. In such  $3 \times 3$  game we have  $D \in \{Action, No-Action, Partial-Action\}$ , and we will think of *P* as the most compromise-prone level available to the countries, a focal point. For illustration, let us assume *P* to be the degree of mitigation half way between *A* and *N*, i.e. in our specific example that means P = 1. The payoffs of the extended game implied by (11) under  $\gamma = 2$  are as follows:

|    |                |                                    | China   |  | ]    |
|----|----------------|------------------------------------|---|--|------|
|    |                | Action                             | Partial-Action  | No- $Action$   | ]    |
|    | Action         | Payoff-dominant Nash ${f 8},{f 8}$ | 1,7   | -4, 4  |      |
| US | Partial-Action | 7,1                                | $egin{array}{c} { m Risk-dominant \ Nash}\ 2,2 \end{array}$ | -1, 1  | (15) |
|    | No-Action      | 4, -4                              | 1, -1   | $egin{array}{c} { m Minimax} \ { m Nash} \ {f 0}, {f 0} \end{array}$ |      |

Global climate Stag hunt featuring a compromise option  $(\gamma = 2)$ 

Note that in the 3 × 3 stage game (*P*, *P*) emerges as another pure-strategy Nash equilibrium. It becomes (under a range of parameter values including  $\gamma = 2$ ) the risk-dominant equilibrium, while the risk-dominant Nash equilibrium of the original 2 × 2 game only remains the minimax play.<sup>17</sup> In order to provide a general result we will consider the following payoffs

|   |                |                            | F                          |                            |      |
|---|----------------|----------------------------|----------------------------|----------------------------|------|
|   |                | Action                     | Partial-Action             | No-Action                  |      |
|   | Action         | $\mathcal{A}, \mathcal{R}$ | $\mathcal{H}, \mathcal{S}$ | $\mathcal{J}, \mathcal{T}$ | (16) |
| M | Partial-Action | $\mathcal{B}, \mathcal{X}$ | $\mathcal{D},\mathcal{U}$  | $\mathcal{I},\mathcal{V}$  |      |
|   | No-Action      | $\mathcal{C}, \mathcal{Z}$ | $\mathcal{E},\mathcal{Y}$  | $\mathcal{G},\mathcal{W}$  |      |

<sup>&</sup>lt;sup>16</sup>Obviously, this does not in any way imply overall superiority of the Chinese political system.

<sup>&</sup>lt;sup>17</sup>The game in (15) also features two mixed-strategy Nash equilibria. In one of them both players choose A and P with 50% probability and achieve a payoff of 4.5 each, in the other they choose N and P with 50% probability and achieve a payoff of 0.5.

where the constraints satisfy

$$\mathcal{A} > \mathcal{B} > \mathcal{C} > \mathcal{D} > \mathcal{E} \ge \max \{ \mathcal{G}, \mathcal{H} \} \ge \min \{ \mathcal{G}, \mathcal{H} \} > \mathcal{I} > \mathcal{J},$$
  
$$\mathcal{R} > \mathcal{S} > \mathcal{T} > \mathcal{U} > \mathcal{V} \ge \max \{ \mathcal{W}, \mathcal{X} \} \ge \min \{ \mathcal{W}, \mathcal{X} \} > \mathcal{Y} > \mathcal{Z}.$$
 (17)

Using this general version of the game, we can revisit our earlier results obtained for the Battle of the sexes.

**Proposition 3.** In the standard  $2 \times 2$  and extended  $3 \times 3$  versions of the Stag hunt game in (16)-(17) Propositions 1 and 2 apply too.

*Proof.* Let us again focus on the  $2 \times 2$  game and follow the steps of the proof of Proposition 1. The Yielding, Sticking and Contest conditions for *M*'s Dominance region in the  $2 \times 2$  game are, respectively

$$m < \bar{m}_{2 \times 2}^{SH} = \frac{\mathcal{R} - \mathcal{T}}{\mathcal{R} - \mathcal{T} + \mathcal{W} - \mathcal{Z}}, \quad f > \hat{f}_{2 \times 2}^{SH} = \frac{\mathcal{G} - \mathcal{J}}{\mathcal{G} - \mathcal{J} + \mathcal{A} - \mathcal{C}} \quad \text{and} \quad f > \frac{\mathcal{G} - \mathcal{J}}{\mathcal{A} - \mathcal{J}}.$$
 (18)

For F's dominance the three respective conditions are

$$f < \bar{f}_{2\times 2}^{SH} = \frac{\mathcal{A} - \mathcal{C}}{\mathcal{A} - \mathcal{C} + \mathcal{G} - \mathcal{J}}, \quad m > \hat{m}_{2\times 2}^{SH} = \frac{\mathcal{W} - \mathcal{Z}}{\mathcal{W} - \mathcal{Z} + \mathcal{R} - \mathcal{T}} \quad \text{and} \quad m > \frac{\mathcal{W} - \mathcal{Z}}{\mathcal{R} - \mathcal{Z}}.$$
 (19)

The analogous conditions for both Dominance regions of the  $3 \times 3$  game are provided in online Appendix C. Note that while the intuition in both versions of the Stag hunt game is analogous to the Battle of the sexes, there is a difference. In the Stag hunt the Sticking conditions in both the  $2 \times 2$  and  $3 \times 3$  games are stronger than the respective Contest conditions. This is because for M (or F) it may be beneficial to unilaterally deviate from the Risk-dominant Nash equilibrium (and ). As a consequence, it is more challenging for the dominating player to stick with their initial action in their revision.

In terms of the claim of Proposition 2 in relation to the Stag hunt game, we can derive the role-swap conditions using (18)-(19). For F to achieve her Dominance region even from the position of the Stochastic follower it must hold that

$$\hat{m}_{2\times 2}^{SH} = \frac{\mathcal{W} - \mathcal{Z}}{\mathcal{W} - \mathcal{Z} + \mathcal{R} - \mathcal{T}} < \bar{f}_{2\times 2}^{SH} = \frac{\mathcal{A} - \mathcal{C}}{\mathcal{A} - \mathcal{C} + \mathcal{G} - \mathcal{J}},$$
(20)

whereas for M this is ensured by

$$\hat{f}_{2\times 2}^{SH} = \frac{\mathcal{G} - \mathcal{J}}{\mathcal{G} - \mathcal{J} + \mathcal{A} - \mathcal{C}} < \bar{m}_{2\times 2}^{SH} = \frac{\mathcal{R} - \mathcal{T}}{\mathcal{R} - \mathcal{T} + \mathcal{W} - \mathcal{Z}}$$

This completes the proof of Proposition 3 for the  $2 \times 2$  game.

Intuitively, our analysis shows that a change in the costs and/or benefits of

climate action for certain countries does affect the set of equilibria, and hence the chances of global coordination on climate change. Furthermore, it indicates that if payoff heterogeneity if sufficiently large than a role swap may occur, and a follower may step into a leadership role. China seems to be a case in point.<sup>18</sup>

### 6.2 Hawk and Dove (Game of Chicken)

This class of game occurs in many settings. One example is the macroeconomic policy game featuring the government (fiscal policy) and the central bank (monetary policy).<sup>19</sup> In the conventional  $2 \times 2$  Hawk and dove game each player chooses between *Hawk* (*H*) and *Dove* (*D*). We will also consider an extended  $3 \times 3$  version featuring a compromise option *Owl* (*O*). If played jointly, (*O*, *O*) delivers the second-best payoff for both players. Using general payoffs, both versions of the game can be written as

|   |      |                          | F                       |                          |
|---|------|--------------------------|-------------------------|--------------------------|
|   |      | Dove                     | Owl                     | Hawk                     |
|   | Hawk | $\mathbf{A}, \mathbf{W}$ | H, Y                    | J, Z                     |
| M | Owl  | B, V                     | $\mathbf{C},\mathbf{T}$ | I, X                     |
|   | Dove | D, U                     | E, S                    | $\mathbf{G}, \mathbf{R}$ |

such that

 $A > B > C > D > E \ge G \ge H \ge I > J \text{ and } R > S > T > U > V \ge W \ge X \ge Y > Z.$ (22)

To offer the intuition, we will use the following specific payoffs

|   |      |                     | F                   |            |
|---|------|---------------------|---------------------|------------|
|   |      | Dove                | Owl                 | Hawk       |
|   | Hawk | <b>5</b> , <b>0</b> | -2, -5              | -4, -4     |
| M | Owl  | 4, 1                | <b>3</b> , <b>3</b> | -5, -2     |
|   | Dove | 2, 2                | 1, 4                | <b>0,5</b> |

The game features three pure-strategy Nash equilibria, (Hawk, Dove), (Dove, Hawk) and (Owl, Owl). It also has some mixed-strategy Nash equilibria, e.g., under the specific payoffs in (23) there are four of them. We can now show that the above findings for the Battle of the sexes and Stag hunt carry over to this class of game as well.

**Proposition 4.** In the standard  $2 \times 2$  and extended  $3 \times 3$  versions of the Hawk and dove game in (21)-(22) Propositions 1 and 2 apply too.

<sup>&</sup>lt;sup>18</sup>China's surprising decision at the 2021 COP26 climate summit in Glasgow declaring close climate action cooperation with the U.S. is in line with this interpretation.

<sup>&</sup>lt;sup>19</sup>For more details see Hughes Hallett et al. (2014) or Sargent and Wallace (1981).

*Proof.* The necessary and sufficient conditions for the  $3 \times 3$  game are provided in online Appendix D. The conditions therein imply those for the  $2 \times 2$  game, which are nested within. For *M*'s Dominance region they are

$$m < \bar{m}_{2\times 2}^{HD} = \frac{W - Z}{W - Z + R - U}, f > \hat{f}_{2\times 2}^{HD} = \frac{G - J}{G - J + A - D} \text{ and } f > \frac{G - J}{A - J}.$$
 (24)

Conversely, for F's Dominance region they are

$$f < \bar{f}_{2\times 2}^{HD} = \frac{G - J}{G - J + A - D}, m > \hat{m}_{2\times 2}^{HD} = \frac{W - Z}{W - Z + R - U} \text{ and } m > \frac{W - Z}{R - Z}.$$
 (25)

In the Hawk and dove game, like in the Battle of the sexes, the Sticking conditions are always stronger than the Contest conditions. The implied circumstances of the role swap, under which the Stochastic follower F surely achieves its Stackelberg leadership payoff, are

$$\widehat{m}_{2\times 2}^{HD} = \frac{W - Z}{W - Z + R - U} < \overline{f}_{2\times 2}^{HD} = \frac{G - J}{G - J + A - D}.$$
(26)

The analogous conditions for F behaving as the Stackelberg leader despite being the Stochastic follower are

$$\hat{f}_{2\times 2}^{HD} = \frac{G-J}{G-J+A-D} < \bar{m}_{2\times 2}^{HD} = \frac{W-Z}{W-Z+R-U}.$$
(27)

This completes the proof of Proposition 4 for the  $2 \times 2$  game.

These results suggest that it may not always be the leader who gets an upper hand. For example, in the monetary-fiscal policy game, the government is generally more rigid than the central bank, and hence the Stochastic leader. As suggested by Sargent and Wallace (1981), this could lead to an unpleasant monetarist arithmetic and force debt monetization. Our analysis shows that this undesirable outcome may occur even if the central bank is the Stochastic leader. If the government is highly averse to fiscal consolidation than it may prevail in the Game of chicken even from the role of the Stochastic follower. Another possible example is the videotape-format war between Video Home System (VHS) and Betamax in the late 1970s and the 1980s. The swap in the leadership position from Betamax to VHS does not seem to be a matter of technological superiority, in fact it was widely considered to be the inferior format.

The examples extend beyond economics. For instance, one popular stream of literature within psychopathy is based on the premise that psychopathy represents an alternative evolutionary strategy consisting primarily of cheating behaviors (e.g., Glenn and Raine, 2009, Crawford and Salmon, 2002, and Mealey, 1995). In particular, according to the Cheater-Hawk hypothesis (Brook and Quinsey, 2004), psychopathy is an evolved strategy of exploiting other people and resources through deception

(Cheaters) and aggression (Hawks); see Visser et al. (2020). Empirical studies within this literature show that in the Game of Chicken psychopaths tend to behave like Hawks, as their psychopathic traits are considered by others to be quite rigid, putting them into a (Stochastic) leadership position. However, studies also show that psychopaths are at least to some extent sensitive to the threat of sanctions and reputational risks. When they face a substantially severe threat of punishment by an otherwise more flexible individual (e.g., their partner leaving the relationship), they may change their behaviour into Dove. In essence, they may act as the Stackelberg follower. One possible interpretation is the channel we propose, namely a sufficiently strong payoff asymmetry that more than outweighs the degree of Stochastic leadership.

## 7. Summary and Conclusions

This paper offers two main messages regarding leadership and strategic interactions between individuals, firms, political parties and countries. It does so by extending the game-theoretic timing of these interactions.

The broader message is that once we move away from the polar cases of Stackelberg leadership and simultaneous move game, the exact payoffs and their asymmetry will generally matter, not just the payoffs' ranking. This is in line with experimental evidence, e.g., Parravano and Poulsen (2015), Lopez-Perez et al. (2015), Agranov and Schotter (2012), Kocher et al. (2008), Johansson-Stenman et al. (2005), and Hausken (2005). This result cannot be captured in the most widely used conventional game-theoretic setups, and our framework featuring stochastic revision opportunities provides a simple avenue to enable it.

In particular, our analysis conducted within coordination and anti-coordination games shows how the exact payoff of each strategy profile affects the relative sizes of the players' Dominance and Multiplicity regions. As such, each payoff determines the set of (subgame-perfect) equilibria. Most notably, if a player has a much stronger preference for its preferred outcome than the opponent in coordination and anti-coordination games, then s/he gains a strategic advantage over the opponent. Such strong preference can then compensate its lack of Stochastic leadership, because these two features act as substitutes in enforcing a player's preferred outcome.

Our second key insight is that if such payoff asymmetry across the players is substantial, their behaviour may change to the point that they seemingly swap their roles. The more flexible Stochastic follower starts behaving as the Stackelberg leader, and conversely, the more rigid Stochastic leader starts behaving as the Stackelberg follower. Put differently, it is the Stochastic follower, not the leader, who ensures her/his preferred outcome as the unique equilibrium in coordination and anti-coordination such as the Battle of the sexes and Hawk and dove.

The conclusion is that payoff heterogeneity matters for coordination, and ignoring it may lead to erroneous predictions/conclusions. Our findings can potentially help explain various results in the literature and phenomena observed in the real world. We discussed above examples related e.g., to global climate change agreements, monetary-fiscal interactions, or the behaviour of psychopaths. It is however apparent that not all changes in leadership can be explained by the channel we postulate. Many market leaders lose their supreme position to their competitors for other reasons related to technological innovation or legal restrictions. An example of the former is the rise of Apple vis-a-vis its competitors, an example of the latter is the impact of regulatory interventions on Microsoft (see Maci and Zigic, 2008).

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