Abstract

We examine daily USD returns for Bitcoin, Ethereum and Litecoin between October 2013 and September 2019 at six separate exchanges employing wavelet methodology. This approach, as compared to the standard time domain analysis, is superior because it tests the existence of cyclical persistencies at different investment horizons. We identify significant but temporal cyclical movements and coherence between the markets at high frequencies which is broadly consistent with market inefficiency given liquidity constraints of cryptocurrencies. Moreover, we identify temporal arbitrage opportunities between the selected exchanges.

1. Introduction

Since their creation more than a decade ago, cryptocurrencies are confronted with a deep skepticism. Several researchers have suggested that cryptocurrencies cannot be treated as regular currencies and their exchange rates are often described as a speculative bubble (Garcia et al., 2014). The exponential rise as well as sharp decline of the value of cryptocurrencies attracted significant attention in the financial world and stir up a debate about cryptocurrency markets efficiency. The weak form of efficient market hypothesis is traditionally tested by various unit root tests (e.g. Urquhart, 2017) or time-frequency domain analysis (Omane-Adjepong et al., 2019) focusing on an ability to use their past information in order to predict future returns. Most of the recent studies conclude that cryptocurrency markets are not weakly efficient (Urquhart, 2017, Alvarez-Ramirez et al., 2017), especially due to the time-scale persistency (Omame-Adjepong et al., 2019), long memory (Phillip et al., 2018), permanent linkages between the cryptocurrencies (Bouri et al., 2019), and cross-correlation with Dow Jones Industrial Average (Zhang et al., 2018). Price inconsistency of Bitcoin between popular marketplaces was identified by Pieters and Vivanco (2017) or Kliber and Wlosik (2019).

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On the contrary, Aslan and Sensoy (2019) and Sensoy (2019) provide a strong evidence on weak-form informational efficiency for high-frequency data (5-minutes and 10-minutes series) of Bitcoin, Ethereum, Ripple and Litecoin. Kristoufek (2015) concludes that the Bitcoin market follows underlying yearly cycles and is driven by trade, money supply and price level in the long run. Li and Wang (2017) show that technology and the public perceptions are also important drivers of bitcoin prices. Such general factors are expected to influence all markets in a similar way and thus to contribute to price similarities.

Additionally, there is growing body of literature on triangular arbitrage opportunity identification in the foreign exchange market (Drożdż et al., 2010; Cui et al., 2019; Gębarowski et al., 2019). Nevertheless, to the best of our knowledge, this paper represents the first contribution applying wavelet coherence and phase shift for identification price arbitrage opportunities between the cryptocurrency exchanges.

Despite increasing research on cryptocurrencies, there are only a few contributions employing wavelet analysis to cryptocurrencies. The standard random walk approach tests only the mean-reverting property of aggregate time series. In turn, wavelet analysis of cryptocurrency returns can prove persistent cycles at different frequencies which may contradict the efficient market hypothesis. It also provides empirical evidence of the fractal market hypothesis (Peters, 1994; Kristoufek, 2013) about a dominance of specific investment horizons during turbulent times holds. As far as different frequencies of cyclical movements represent specific investment horizons and provide important implications for portfolio management. From this perspective, we emphasize short investment horizons domination in 2018 because buying and selling orders were not efficiently cleared during the price fall.

We cover a period from end of 2013 to September 2019 which includes not only the fall of cryptocurrency prices in 2018, but also their revival in the first half of 2019 and price correction at the end of the sample. Reflecting the ambiguity in the previous literature, we make two main contributions to this growing stream of literature. First, we explore temporal cyclical behavior at high frequencies which confirm cryptocurrency market inefficiency at short investment horizon from 2017 to mid-2018. Second, we analyze coherence and phase shift to explore co-movements between the selected cryptocurrency markets and market arbitrage opportunities identified by leads and lags between the price cycles.

The paper is organized as follows. Section 2 contains the literature review. A detailed overview of methods and data is provided in Section 3. Section 4 presents the results of the continuous wavelet transform and wavelet coherence and Section 5 concludes.

2. Literature Review

The efficient market hypothesis states that exchange rates or prices of any financial assets manifest all available information at any time. The previous literature focuses especially on the weak form efficiency, which implies that the current prices fully reflect the information obtained in the past prices. This version of efficient market hypothesis is traditionally tested by various unit root tests. The random walk hypothesis implies that new information is immediately reflected in asset prices,
concluding that future price changes will reflect only future news and will be independent of the lagged price changes.

In addition to the weak version, the literature (Fama, 1970 and 1991) discusses the semi-strong and strong versions of the efficient market hypothesis. The semi-strong efficiency states that current exchange rates fully react to all publicly available information, which not only includes the past prices but also any other variables like inflation, interest rates, etc. In the case of the semi-strong efficiency, multivariate analysis cannot ensure better performance than the naïve random walk strategy either. Finally, the strong form efficiency implies that current prices fully reflect all existing information both public and private. Under this hypothesis, no investors should be able to generate profits even if they trade with the knowledge not yet publicly available at the time (e.g. in the case of insider information).

Price inconsistencies across multiple exchanges of traditional financial assets have been in the spotlight of research in the last decades (Fama, 1991, Malkiel, 2003, Lo, 2007). While random walk approach and unit root or cointegration tests dominate the empirical literature on efficient market hypothesis, several authors employ alternative cross-spectral analysis based on the method described by Granger (1969). Granger and Morgenstern (1979) use the cross-spectral methods in order to characterize the long-run relationships between the non-stationary time series of the stock prices. They discover that the price dynamics of stock markets is not only based on supply and demand, but it rather reflects a single market. In sum, the earlier literature shows that besides regular economic factors, asynchronies in production, transaction costs, speculative movements and market-specific constraints can have an impact on the adjustment speed of different markets. Not all markets are significantly cointegrated and only geographical close markets have a stable equilibrium relationship. Moreover, the earlier literature identified frequent and important financial market anomalies (e.g. excess volatility) which have become the base for behavioral finance (Shiller, 2003).

The standard random walk approach tests only the mean-reverting property of aggregate time series. In turn, wavelet analysis of cryptocurrency returns can prove persistent cycles at different frequencies which may contradict the efficient market hypothesis. It also provides empirical evidence of the fractal market hypothesis (Peters, 1994; Rachev et al., 1999; Weron and Weron, 2000; Kristoufek, 2013) because different frequencies of cyclical movements represent specific investment horizons which poses important implications for portfolio management. The fractal market hypothesis, in comparison with traditional Efficient Market Hypothesis, assumes that different market participants analyse past events and news with different time horizons (Weron and Weron, 2000). With respect to fractal market hypothesis the wavelet analysis explores time-varying investment horizons, especially situations when the long-term investors start to panic and sell during the price fall.

Additionally, Baumöhl (2018) confirms that a cross-spectral approach and can provide more information on the dependence structure of different frequencies than a detrended moving-average cross-correlation analysis. Moreover, Kristoufek and Vosvrda (2013) propose a new measure of market efficiency based on the fractal dimension and entropy and measure long-range dependence. They also employ time-
frequency analysis and test dominance of specific investment horizons during the Financial Crisis after 2007 (Kristoufek, 2018; Kristoufek and Vosvrda, 2019).

Following general discussion, we use the terms cryptocurrency and exchange rate between cryptocurrencies, although the decentralised nature of cryptocurrencies and high volatility have led to extensive discussion whether they can be classified as currency in the sense of a transaction medium (Li and Wang, 2017). Especially the high volatility makes cryptocurrencies less attractive for regular transactions than fiat money. Alvarez-Ramirez et al. (2018) document the risks for users to engage with cryptocurrencies as a transaction medium with such a significant price fluctuation. Urquhart (2017) shows that Bitcoin exchanges do not fulfil the property of efficient markets. Thus, several scholars argue that cryptocurrencies hardly fulfil the traditional characteristics of an exchange tool we commonly refer to as currency (Bariviera et al., 2017).

From the perspective of the market efficiency hypothesis, one important aspect of cryptocurrency pricing is that the cryptocurrencies are traded on many different exchanges. Briere et al. (2015) show that Bitcoin prices can vary widely across different exchanges at the same time, which contradicts market efficiency. It is relevant to point out that cryptocurrency exchanges are not directly based on the blockchain, so they are not a decentralised ledger and do not suffer from the long transaction time of system. They rather operate like traditional asset exchanges with matching algorithms bringing buyers and sellers together.

Brandvold et. al (2015) focus on the price discovery on several exchanges, which were popular with market participants at the time. They point out, that due to the missing regulation framework the deposition/withdrawal process and fees of the different exchanges can vary drastically. Brandvold et. al (2015) notice that these characteristics can lead to diverging prices on the cryptocurrency exchanges, which has already been mentioned by Briere et. al (2015). Their results show that exchanges, which were struggling at the time due to hacking attacks, such as Mt. Gox (McMillan, 2014), are not cointegrated with other markets. These exchanges either traded with a discount or a premium on the Bitcoin prices. Another significant aspect discovered by Brandvold et al. (2015) is the change in trading activity, based on the location and currency of the exchange.

De Jong et al. (2001) analyse the multivariate time series data by using short intervals to capture the high frequency aspect while trying to avoid possible noise in the time series. They conclude that the most popular exchanges, measured by their trading volume, are the most important price leaders and smaller exchanges follow the market with a lag.

Several recent studies challenge the efficiency of cryptocurrency markets with a particular focus on Bitcoin (Alvarez-Ramirez et al., 2017), showing that the market is not uniformly efficient and may exhibit cyclic behaviour in price returns. Yermack (2013) supports this claim in his analysis, by comparing the high value of certain cryptocurrencies, such as Bitcoin to the volumes traded on the exchanges. He concludes that Bitcoin is not a currency but a speculative investment opportunity, due to long transaction times and a limited number of places where the currency is accepted. This is supported by the study of Halaburda and Gandal (2016), claiming that the value of cryptocurrencies primarily derived by its relation to other cryptocurrencies, not its underlying economic values. According to several statistics,
almost a third of respondents held cryptocurrencies solely for investment purposes (OECD, 2019). If cryptocurrencies are held as an asset, they are significantly effective as a supplement to a portfolio, due to their low correlation to regular assets, such as gold (Brandvold et al., 2015).

Li and Wang (2017) also point out that most of the earlier studies do not focus on price determination in cryptocurrency markets, but rather on discovering potential bubbles, which have occurred several times since the creation of Bitcoin in 2008. Supply and demand are important determinants for the price of Bitcoin (Buchholz et al., 2012), but several authors, such as Kristoufek (2015) and Yermack (2013), have argued that the price formation of a Bitcoin cannot be explained completely by regular economic theories. They have stated that influencing factors over these markets are difficult to identify, due to the unique properties. The decentralised system excludes governments, laws, and other legal entities as factors of direct influence on the pricing of cryptocurrencies. This includes the taxation of such financial instruments because governments currently don’t have the technical capabilities to track the possession of cryptocurrency assets to their citizens (Naidu, 2016).

Kristoufek (2013, 2015, 2018) presents, to the best of our knowledge, so far the only contribution, employing wavelet coherence analysis, which identifies possible drivers of the Bitcoin price over short-term and long-term perspectives. His study does focus on the correlation with traditional financial assets and explicitly excludes transactions, which were conducted on an exchange, hence using the information directly contained in the blockchain. He analyses the influence of the technical aspects of Bitcoin, such as the amount of the coin produced per second and the mining difficulty on the Bitcoin/USD exchange rate, with time series on a one-minute basis. The Fourier spectral analysis points out several spectral peaks in lower frequencies, showing cycles of one year and half a year and revealing the phenomenon that Chinese volume leads the USD prices. Kristoufek (2015) concludes that the Bitcoin market follows underlying yearly cycles and is driven by macroeconomic factors in the long run.

However, frequency analysis is widely used for investment horizon identification at different capital markets. Baruník et al. (2016) employ wavelet analysis to investigate relationship at different investment horizons between gold, oil and stocks. Baruník and Kočenda (2019) find asymmetric connectedness between oil and forex markets and Njegić et al. (2019) confirm stock-bond interactions or Đuraškovic et al. (2019) employ wavelet analysis for two-asset portfolio optimization.

As mentioned before, most research exclusively focusses on Bitcoin as the most popular cryptocurrency on the market but ignores other significant cryptocurrencies with similar market volumes. A small number of essays address this issue, thus including multiple cryptocurrencies in their analysis. Corbet et al. (2018) examine the return and the volatility transmission of the three most popular cryptocurrencies, by using the approach of generalized variance decomposition to measure possible spillover effects on different markets. The authors employ a frequency approach to estimate unconditional connectedness relations and a time domain approach to analyse directional connectedness. The paper claims that the Bitcoin price has a strong effect on the price dynamics of the other analysed cryptocurrencies without sufficient explanation of their volatility spillovers.
conclude that the cryptocurrency markets are strongly interconnected but could be potentially isolated from other markets over the short term. Similar to Li and Wang (2017), they argue that main drivers of prices are technology and the perception of cryptocurrencies by the public.

3. Data and Methods

Previous research in the field of cryptocurrency pricing has been mainly focused on the oldest and the most prominent cryptocurrency, Bitcoin, while only a few authors have considered other blockchain-based currencies (see e.g. Corbet et al., 2018). Filling this gap in the literature, we analyze fluctuations in log returns of the main three cryptocurrencies (Bitcoin, Ethereum, and Litecoin). We use daily data (closing prices at midnight, synchronized with the UTC time zone) from the six largest cryptocurrency exchanges: Bitfinex, Bitstamp, Bittrex, Coinbase, Kraken and Poloniex, between October 6, 2013 (August 7, 2015, for Ethereum and October 24, 2013, for Litecoin) and September 21, 2019.

Our data set covers the period of unprecedented rise in the value of all analyzed cryptocurrencies as well as its subsequent correction and corresponding portfolio rebalancing (Figure 1). Interestingly, the markets recovered again during 2019, but experienced a new correction at the end of the sample. According to the available data, the value of Bitcoin started at slightly more than USD 100 (available only from Kraken) in 2013. The highest closing price of Bitcoin nearly USD 20,000 per token, was achieved on December 16, 2017. It is interesting to note that the crashes of 2018 and 2019 occurred simultaneously on all major crypto-exchanges.

The time-frequency domain analysis can be applied to test the weak form of efficient market hypothesis, especially the presence of cyclical persistencies at different frequencies. The presence of any persistencies means that past information can be used to predict future returns for given investment horizons given by identified frequency scales.

In the first step, following Torrence and Webster (1999) we employ continuous wavelet transformation using Morlet wavelet,

\[ \Psi(t) = \frac{1}{\sqrt{2\pi}} e^{-i\omega_0 t} e^{-\frac{t^2}{2}}, \tag{1} \]

which provides an optimal trade-off between both time and frequency localization in financial time series (Crowley, 2007; Rua, 2010; Poměnková et al., 2014; Kapounek and Kučerová, 2019). The oscillation is regulated by the parameter \(\omega_0\), leading to improved scale localization but decreased time localization and vice-versa.

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1 Due to low liquidity and long transaction periods, daily data are likely to be more appropriate for the presented analysis than high frequency (e.g. minute) data.
2 The data are obtained from the free database http://www.CryptoDataDownload.com.
3 Some smaller crypto-markets (e.g. quadrigacx and coinfloor, which are not analysed here) experienced the crash up to two days later in 2018. However, the price levels on these markets were lower than on the main markets.
Figure 1 Time Domain Representation of the Analyzed Time Series
Following Rua (2010), we choose $\omega_0 = 6$ as it exhibits strong similarities to Fourier period leading to a better interpretation of the result.

In the second step, we test the law of one price between the analyzed crypto-markets, because earlier research suggests that cryptocurrency prices are highly different in various crypto-exchanges. Therefore, we analyze cyclical co-movement employing squared coherency which is common time-localized oscillation in analyzed time series. The coherency is interpreted as a co-movement or time-frequency varying correlation. The results of the coherency are normalized and can range from 0 to 1, where a small value suggest a very weak correlation and values close to 1 indicate a strong correlation between the signals.

The significance of the wavelet coherence analysis at the five percent level is achieved by comparison to a random distribution generated from a Monte Carlo Simulation as proposed by Grinsted et al. (2004). Moreover, we apply phase shifts in order to obtain delays between the cycles of two signals. In our analysis shifts of up to $180^\circ$ with a change in polarity are computed. The arrows indicate the direction of lag between the signals in presented figures. The methodology is widely used in wavelet literature (see details in Grinsted et al., 2004).

4. Market Efficiency and Arbitrage Results

4.1 Continuous Wavelet Transform and Market Efficiency

We analyze the individual exchange rates of cryptocurrencies using the CWT to identify optically cyclical persistency in specific periods and frequencies. For efficient markets, we would expect no significant cycles in value of cryptocurrencies because these regularities could be exploited by market participants. This approach allows us to identify periods with significant regularities and thus with possibilities for profitable trading.

Figure 2 presents the returns of analyzed cryptocurrencies and crypto-exchanges in the frequency domain. The areas surrounded by black lines display the significance of the cycles on a five percent level as tested against red noise using Monte Carlo simulations. The coned line in the bottom half of the graphic displays the regions which are influenced by the so called ‘edge effects.’ These effects occur when the wavelet is centered near the beginning or the end of the time series, which can potentially disturb the results for these periods. Data outside of this line cannot be statistically inferred in the analysis. The affected areas are defined as the cone of influence (Torrence and Compo, 1998). The color intensity represents the power spectrum of the results. This spectrum can range from dark blue for low-power areas up to bright yellow for high-power spectra.

In the case of Bitcoin (Figure 2a), we cannot reject the efficient market hypotheses before 2017 for any crypto-exchange as there are nearly no or only very short and nearly randomly distributed areas of significant cycles. However, the picture has completely changed from the beginning of 2017 to the mid of 2018 and again in 2019, when significant arrears are visible especially around the investment horizon of 60 to 120 days. Moreover, the results confirm the fractal market hypothesis. The cryptocurrencies have seen significant demand from investors from the beginning of 2017. On the contrary, the cryptocurrency exchanges suffered massive selloffs in 2018, when supply exceeded the demand. These counteracting
developments resulted in significant cycles over a broad range of frequencies, which can correspond to prevailing investment horizons during the year 2017 and 2018. Finally, we can see high similarities between the all crypto-exchanges and cryptocurrencies.

This pattern is slightly different from the cycles found for the other two alternative cryptocurrencies. Both Ethereum (Figure 2b) and Litecoin (Figure 2c) are characterized by significant occasional cycles nearly during the whole analyzed period. In general, mainly shorter cycles (below 60 days) are significant for these currencies. However, the periods with significant cycles at specific frequencies are relatively short, usually not much longer than one or two full cycles of these frequencies, which makes it more difficult for investors to exploit them for profitable trading and creates mostly selling signals as a result of increasing uncertainty.

The importance of these regularities has also declined towards the end of the sample as the importance of trading in these currencies lost on importance as well. Finally, we can see somewhat larger differences between patterns found for different crypto-exchanges.

**Figure 2a Continuous Wavelet Transform of Bitcoin at Selected Exchanges**

Notes: Colour scales represent wavelet power using Morlet wavelet, the areas surrounded by black lines denote the results of Monte Carlo significance test, and light shading shows region influenced by edge effects.

Source: Own Estimations.
4.2 Coherence and Market Arbitrage

The previous analysis showed that the wavelet test has largely failed to reject the efficient market hypothesis with the exception of Bitcoin since 2017. However, the investors are likely to interact between different markets. Usually, this is done through arbitrage trading as investors buy assets at underpriced and/or sell them at over-priced markets. In the case of cryptocurrencies, low market liquidity, high fees...
and long transaction times limit the possibilities of the direct arbitrage. Nevertheless, investors may use information revealed on alternative markets and leave the markets if they see downward developments, or vice versa. This channel could be as efficient as direct arbitrage in achieving highly similar developments across different crypto-exchanges.

From this perspective, it is interesting to analyze co-movements between the crypto-exchanges as the arbitrage hypothesis implies a high degree of synchronization of these markets. In particular, we analyze the movements of individual cryptocurrencies in the time-frequency domain using wavelet coherence. Figure 3 illustrates the co-movements of cryptocurrencies at kraken and the other selected markets. As before, we conduct a significance test, which is displayed by the black lines in the plot with five percent significance interval. Moreover, the figures show the phase shifts between different crypto-exchanges: The black arrow pointing to the right indicates that the first named market generates buying and selling signals for the second market, while an arrow pointing to the left is an indication for negative correlation. If an arrow is pointing downwards, it indicates that the second named market in the plot is leading the first one. These references must be treated carefully, as a lead of 90 degrees can also represent a lag of 270 degrees in relation to the anti-phase.

Similarly, to the previous results, we can see that the movements of the Bitcoin exchange rates were largely similar between the markets, however, there have been several exceptions especially for short cycles (up to about 30 days) before 2017, specifically for the pairs including Bitrex and Bitfinex. This pattern is even stronger for the Ethereum and Litecoin (see blocks B and C of Figure 3).

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4 For simplicity, we will use the term ‘arbitrage’ also for using information from other markets.
5 Results comparing all markets each to other are available in the Appendix, see Figure A1.
Figure 3 Wavelet Coherence of Kraken Exchange with Selected Crypto-Exchanges

A. Bitcoin
B. Ethereum
C. Litecoin

Notes: The color scales represent wavelet squared coherencies, the black contours denote insignificance at 5% against red noise, and the light shading shows regions probably influenced by edge effects. The direction of the causal relationship is represented by arrows (a left arrow denotes anti-phase (180°) while a right arrow denotes in-phase (0° or 360°)). Phase arrows pointing the first market as a leading market are turned up by 90° (between 0° and 180°).
5. Conclusion

Cryptocurrencies are still novel in the financial world, but they have attracted already strong research interest. We use a longer period of daily data than earlier studies, including a highly dynamic period between October 2013 and September 2019.

Our research makes two main contributions to the existing discussion on cryptocurrencies. First, we employ wavelet analysis, which identifies time-frequency varying persistency of cycles related to specific investment horizons. We use a broad range of wavelet indicators, starting from CWT analysis, to coherence analysis and phase shift, and interpret them from the perspective of the market efficiency.

Second, despite significant low frequency cycles of Bitcoin, Ethereum and Litecoin returns at different exchanges, the markets do not show significantly persistent cycles over sufficiently long periods of time. The importance of these cycles increased during the crash of the cryptocurrencies at the end of 2017, but they disappeared again more recently. Thus, this implies that short investment horizons dominated because the long-term investors were selling during the price fall.

Contrary to recent studies (e.g. Bariviera et al., 2017 or Kristoufek and Vosvrda, 2019), we show that cryptocurrencies tend to be most inefficient during turbulent periods (both in Winter 2017/2018 and in Fall 2019). A possible explanation is that the beginning of 2017 was determined mainly by demand factors, while excess supply was more important in 2018. This resulted in more significant cycles during this period.

Moreover, we identified market arbitrage opportunities between the different exchanges in Summer 2018. However, we show signs of higher interdependence between different market places during the period of increased volatility. Our results also pose important implications for portfolio management. The identified phase shift between the selected cryptocurrency exchanges could serve as the leading indicator at different investment horizons. Moreover, the identification and subsequent realization of price arbitrage eliminates market efficiency violation in the long run.
Figure A1 Wavelet Coherence of the All Selected Crypto-exchanges
Notes: The colour scales represent wavelet squared coherencies, the black contours denote insignificance at 5% against red noise, and the light shading shows regions probably influenced by edge effects. The direction of the causal relationship is represented by arrows (a left arrow denotes anti-phase (180°) while a right arrow denotes in-phase (0° or 360°)).

Source: Own Estimations.
REFERENCES


