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Dependence Structure of Volatility and Illiquidity on Vienna and Warsaw Stock Exchanges*

Henryk GURGUL – Faculty of Management, AGH University of Science and Technology in Krakow, Krakow, Poland (henryk.gurgul@gmail.com) *corresponding author*

Robert SYREK - Institute of Economics, Finance and Management, Jagiellonian University, Krakow, Poland

Abstract

In this paper, the results of an investigation into the relationship between the illiquidity and realized volatility of the time series of stocks listed on the Warsaw Stock Exchange (WSE) and Vienna Stock Exchange (VSE) are presented. The first measure of illiquidity is the well-known Amihud ratio (Amihud, 2002) called AMI*, and the second is a transformation of the Liquidity Index (Danyliv et al., 2014) called* $ILLIX$*. In the study, the results of the detection of the structural breaks (and their removal), the calculation of long memory, and finally the dependence structure of the illiquidity and realized volatility by the copulas are also demonstrated. Both types of series exhibit structural breaks and long-memory properties. Despite the similarities in the illiquidity measures, their associations with the realized volatility is different. The dependence structures described by the copulas for the* AMI*-realized volatility pairs show a dependence in the upper tails; i.e., the high values of illiquidity are related to the high volatility. However, in the case of the* $ILLIX$*-realized volatility pairs, the dependence was detected in the lower tail; i.e., the low* $ILLIX$ *is accompanied by the low realized volatility.*

1. Introduction

The liquidity of an asset is a property that is difficult to define or measure. However, a commonly accepted definition of liquidity assumes that an asset is liquid if large quantities of this asset can be traded in a short period of time without a significant decline in the price. Therefore, the price impact of trading is frequently used as a measure of liquidity. Accurate measures of liquidity are very important in empirical research. In the framework of a market's microstructure, liquidity is the most important measure of market quality.

Liquidity affects the price of an asset in the market. Many contributors stress that the liquidity of an asset determines its expected rate of return. The buyer of the stock is expected to have some future cost in case they sell the asset at a future point in time. Higher transaction costs result in lower prices for the assets; therefore, the expected returns must increase. This is important with respect to the cost of the capital of the issuer. The more-liquid asset will have a higher price for which it can be sold. This is true because investors will have a high rate of return. It should compensate for

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taking risks (including liquidity risks).

It is not possible to avoid all of the risks associated with an asset irrespective of the diversification opportunities.

A very important topic in corporate finance is the detection of channels determining the interrelations of liquidity and corporate decisions with respect to financing.

The common measures of liquidity are quoted as effective bid-ask spreads. To determine the spread intraday data, the bid and ask prices are necessary. In addition, the transaction prices must be known in order to estimate the effective spread. However, this data is often unavailable. It is necessary to underline that, even in the case of the availability of data, the direct estimation of the spread is not an easy task. The background has seen a huge increase in trading and quotation activity in recent years. This is the reason why contributors have developed and applied various methods for estimating the spread from daily data.

Investors in well-known stock exchanges are not as worried about stock liquidity as they are in small stock exchanges; hence, it is very important to also investigate smaller stock exchanges with respect to liquidity.

In the literature, contributors pay much attention to the different properties of new EU markets, especially taking intraday data into account (Hanousek et al., 2009; Gurgul and Wójtowicz, 2015).

Since stock liquidity is one of the major concerns of investors in both the Warsaw and Vienna stock exchanges, it is important to investigate this issue. Along with liquidity, we also used its opposite notion (called illiquidity). In our paper, we are concerned with the properties of the chosen illiquidity measures; i.e., Amihud illiquidity (Karolyi et al., 2012) and modified liquidity (defined by Danyliv et al., 2014). We check the statistical properties, long memory, and dependence structures of these measures with respect to the realized volatility. For comparison purposes, we used data from similar stock exchanges with respect to their size; the Warsaw Stock Exchange (WSE) and Vienna Stock Exchange (VSE).

The main goal of our paper is to investigate the relationship between illiquidity and realized volatility time series by copulas. Since it requires the transformation of an original series to a uniformly distributed series, it also includes the detection of structural breaks (and their removal) as well as the calculation and removal of their long memory properties (to obtain the stationary series used in Vector Autoregressive models).

To the best of our knowledge, this is the first contribution that explores the tail dependence between illiquidity and the realized volatility.

In the next chapter, a general literature review is conducted. In Section 3, the measures of the illiquidity and realized volatility is described based on specific literature methodology along with long memory and copulas.

In Chapter 4, the data and stock exchanges under study are presented. In Section 5, the empirical results are listed and discussed. Section 6 concludes the paper.

2. Literature Review

Different aspects of liquidity and liquidity risks are considered in Amihud and Mendelson (1986), Pastor and Stambaugh (2003), Goyenko et al. (2009), Holden

(2009), Corwin and Schultz (2012), Roll (1984), and Acharya and Pedersen (2005). Moreover, liquidity and liquidity risk are considered as factors in an asset pricing model in these studies. Amihud and Mendelson (1986) modeled the expected returns as a function of the bid-ask spreads and showed (not only theoretically but also empirically) that stocks with larger spreads outperform stocks with smaller spreads. This result is known in the financial literature as illiquidity premium.

It is widely accepted that liquidity has an effect on the expected returns and capital structure. Research by Giot et al. (2003) demonstrated that an important determinant of liquidity is volatility. This means that higher stock return volatility is associated with higher illiquidity.

In a more recent study, Blau and Whitby (2015) tested whether the volatility of bid-ask spreads is positively related to the expected returns. The authors found that the average risk-adjusted excess return for stocks in the highest spread volatility quintile amounts to 50 basis points per month. Applying multivariate tests, they found robust evidence of a return premium associated with spread volatility. This occurred to be statistically significant and economically meaningful. The results of Blau and Whitby (2015) were robust with respect to a variety of stock characteristics, different tick-size regimes, and other measures of liquidity volatility.

Akbas et al. (2010) established a positive relationship between illiquidity (the opposite notion of liquidity) and a stock's expected return. Lei et al. (2013b) found that company liquidity and market liquidity are directly related to stock excess returns.

Shieh et al. (2012) established that liquidity and the stock momentum effect were the most important factors of stock price changes. The contributors stated that stock market value and the book-to-market ratio do not play significant roles in stock price changes. Lei et al. (2013a) detected a positive relationship between capital gains and a stock's expected returns. In addition, they also found that there is a positive relationship between illiquidity and a stock's expected returns.

In a recent study, Amiram et al. (2016) determined the important factors with respect to the relationship between total volatility and illiquidity based on stocks listed on the NYSE and NASDAQ between 2002 and 2011. They decomposed the total volatility into its diffusive and jump components. They demonstrated that it is jump volatility that drives the positive relationship. On the other hand, diffusive volatility has a negative contribution. The authors found that the negative contribution of diffusive volatility is completely channeled through its impact on increased trading activity (turnover). This, in return, decreases the illiquidity. The contributors found that the volatility component maintains its type of impact on the liquidity risk and liquidity risk premiums. In this way, they found the determinants of liquidity, particularly with respect to volatility. The findings help us to understand the mechanisms that drive liquidity risk and liquidity risk premiums.

The paper by Valenzuela et al. (2015) checked the interrelation between two central concepts in the financial markets: liquidity and volatility. The authors stressed that understanding the effects of the liquidity provision on market dynamics had gained increased attention from regulators, market participants, and academics alike. Information on volatility and changes in trade prices is very important in assessing the risk-return trade-off for portfolio valuation and derivatives pricing models. In addition, knowledge of volatility is important for calibrating the execution probability of limit orders. The contributors proposed a new way of summarizing the distribution of liquidity in a limit order book. They investigated and checked its informativeness on future volatility.

Chung et al. (2009) tested day-trading activities for 540 stocks traded on the Korean Stock Exchange using transaction data for the period of 1999 through 2000. The cross-sectional analysis supports the opinion that day traders prefer lower-priced, more-liquid, and more-volatile stocks. Using bivariate VAR models with intraday data, the contributors found that greater day activity leads to greater return volatility. In addition, the impact of a day-trading shock dissipated gradually within one hour. Past return volatility also positively affected the future day-trading activity. We also found that past day-trading activity negatively affected the bid-ask spreads, while the past bid-ask spreads had a negative impact on the future day-trading activity. According to the authors, day traders used short-term contrarian strategies, and their ordered imbalance had a positive effect on the future returns. This result is in line with the cyclical behavior of day traders. The traders concentrated their buy or sell trades at the bottoms or peaks of short-term price cycles, respectively.

Long memory is important topic in theoretical and empirical research. The financial literature is especially focused on the memory length of return volatility expressed by the absolute values of the returns or, alternatively, by the squared returns and trading volume (e.g., Baillie, 1996; Barkoulas and Baum, 1996; Barkoulas et al., 1997; Bollerslev and Mikkelsen, 1996; McKenzie and Faff, 2003). Due to importance of this topic, many different estimators of the long-memory parameter that includes parametric and semi-parametric approaches (Phillips and Shimotsu, 2004, 2005, 2006; Shimotsu, 2010) have been developed.

Realized volatility (RV) is based on high-frequency data information. It has been demonstrated that this measure, calculated from the sum of intraday squared returns, is a better approximation of ex-post volatility than squared daily (or absolute) returns. Nowadays, many estimators of realized volatility are available, including the jump robust and microstructure noise robust methods. It is widely known that RV time series exhibit a strong serial dependence. This is documented in Andersen et al. (2001), Andersen et al. (2007), and Martens et al. (2009), for example. For this reason, long-memory models such as the ARFIMA model (autoregressive fractionally integrated moving average) are frequently applied to RV data.

In Andersen et al. (2001), the contributors checked the "realized" daily equity return volatilities listed in the DIJA index. The authors demonstrated that the unconditional distributions of the realized variances and covariances exhibited essential right-skewness. However, the realized logarithmic standard deviations and correlations underlined the Gaussian distribution. This observation concerns the distributions of the returns scaled by the realized standard deviations. The empirical findings are that realized volatilities and correlations not only exhibit strong temporal dependence but also show long-memory properties.

Rossi and Magistris (2013) showed the relationship between the volatility (measured by realized volatility) and trading volume of 25 NYSE stocks. They found that volume and volatility exhibit long memory but are not fractionally cointegrated in general. The contributors also found right-tail dependence in the volatility and volume innovations. Tail dependence is an important source of information on volatility and volume in the case when unexpected important news comes to the market. A fractionally integrated VAR model with shock distributions was described by a mixture of copula functions. Other papers about the dependencies between time series (e.g., return volatility and trading volume) include Bollerslev and Jubinski (1999), Lobato and Velasco (2000), and Gurgul and Syrek (2013).

In a recent contribution, Bhattacharya (2017) investigated the long-memory properties in the liquidity measures of the Indian Stock Market. It is clear that the long-range dependence structure indicates that these measures can be predicted. However, as the author of the study stressed, problems follow from this fact with respect to a number of issues, such as the linear modeling, forecasting, statistical testing of liquidity models based on standard statistical methods, and theoretical and econometric modeling of asset pricing involving liquidity. Based on the data from the Bombay Stock Exchange, the contributor tested the long-range dependence in the breadth, depth, resiliency, tightness, and immediacy characteristics of the market by means of a Hurst Estimate and Lo's Rescaled Range Statistics. In addition, semi-parametric GPH statistics were used as well as the modified GPH statistics of Robinson (1995). The empirical findings are not in favor of long memory in all of the liquidity parameters; this suggests that liquidity may not be predictable in the Bombay Stock Exchange.

In recent years, a lot of attention in the financial literature has been paid to the topic of the relationship between long memory and structural breaks. In the literature, it is well-known that the detection of long memory can be due to neglected structural breaks. On the contrary, long-memory processes may cause a spurious detection of these breaks. It is not easy to distinguish between long memory and structural breaks; see Diebold and Inoue (2001), Granger and Hyung (2004), and Smith (2005), among others. If a time series exhibits some structural breaks, then the series has the respective number of discontinuities in the data-generating process. In this case, a structural-break method will report the number of breaks that will divide the series into regimes that are of different subpopulations. The statistical properties of these subpopulations within the regimes will need to be estimated. One of the methods is the selection procedure of Bai and Perron (1998, 2003) based on a sequence of tests. There is a growing number of contributions on the tests to distinguish between true long memory and various spurious long-memory models. Berkes et al. (2006) and Shao (2011) elaborated a testing procedure to discriminate a stationary long-memory time series from a short-range-dependent time series with change points in the mean. In these contributions, a null hypothesis corresponds to changes in the mean; their alternative is that the series is stationary with long memory.

Yang and Chen (2014) investigated the properties of the realized volatility of the Shanghai Stock Exchange Composite Index and four individual stocks from the Shanghai and Shenzhen stock exchanges. Among other things, they found that the realized volatility exhibited long-term memory and structural breaks in the mean. The structural breaks explain the long memory only to some extent. In a time series with a long-memory property, the autocorrelation function tails off hyperbolically. The long-memory property is related to the nonlinearities in the financial data (Gurgul and Syrek, 2013) and is not easy to detect.

A number of empirical studies have argued for the importance of tail dependence (not only in financial applications). The paper of Poulin et al. (2007) emphasized the importance of taking tail dependence into account in the context of bivariate frequency analyses based on copulas. Rossi and Magistris (2013) presented the importance of modeling both long memory and tail dependence to capture extreme events based on simulation and forecasting exercises.

The property of long memory in realized volatility is a stylized fact of financial econometrics nowadays. In many contributions, the occurrence of structural breaks in the realized volatility is analyzed. Taking these features into account is an important issue when forecasting realized volatility (Yang and Chen, 2014; Martens et al., 2009).

Since volume and volatility exhibit long memory, we may expect that (il)liquidity measures also have long-memory properties based on them. Wang (2010) used a modified version of the Amihud illiquidity measure for 12 markets. Applied to the daily data, he found that the null hypothesis of no long memory was strongly rejected in all of them.

Tail dependence is an important issue in finance due to its practical implications. Some examples are diversification and portfolio selection, hedging, instrument pricing, and value at risk forecasting. Therefore, the literature about modeling extreme dependencies using copulas is very extensive. Fortin and Kuzmics (2002) analyzed a set of European stock indices and found that asymmetric tail-dependent distributions are favored instead of normal or Student-t dependence. Patton (2002) found asymmetry in the conditional dependence between the exchange rate returns of the Deutsche Mark-U.S. dollar and Yen-U.S. dollar pairs. Lee and Long (2009) compared their new C-MGARCH models with the corresponding conventional MGARCH models using exchange rates and found that the results from the in-sample and out-of-sample analysis outperforms standard MGARCH models. Okimoto (2008) combined the copula theory and Markov switching model to model the asymmetric dependence for the U.S.-UK market and G7 countries. Chollete et al. (2009) constructed a multivariate regime-switching model of vine copulas to model the asymmetric dependence in the international financial returns from the G5 and Latin American regions. Rossi and de Magistris (2012) and Gurgul and Syrek (2013) found asymmetric tail dependence in the realized volatility and trading volume.

The last two contributions contain descriptions of the econometric tools used in the computations (which are briefly reiterated in the following chapter).

3. Methodology

In the following three subsections of this chapter, we review the basic methodology used in our computations (which are highlighted in the fourth chapter).

3.1 Measures of Illiquidity and Realized Volatility

The realized volatility is computed from the high-frequency prices using the method presented in Ait-Sahalia et al. (2005). We use a microstructure noise robust average subsampled realized variance estimator aligned at one-minute returns at five subgrids. The detailed description given in this seminal paper is quite complex. Therefore, we advise the reader to study it from the original source. Finally, we take the logarithm of the realized volatility and denote the results for day t as RV_t.

Among the many definitions of the (il)liquidity measure, we apply two: the Amihud illiquidity ratio and Liquidity Index. In most cases, the Amihud illiquidity ratio (Amihud, 2002) is obtained using averaging over a month or year. We follow Karolyi et al. (2012) and define the daily Amihud illiquidity measure as follows:

$$AMI_t = \ln\left(1 + \frac{|R_t|}{P_t \cdot V_t}\right), \qquad (1)$$

where R_t, P_t, and V_t are the return, closing price, and volumes on day t, respectively. The added constant and taken logs reduce the impact of outliers.

The second measure we use is the LIX liquidity measure proposed in Danyliv et al. (2014). We multiply the original definition by -1 to get the illiquidity measure, and we obtain the following:

$$ILLIX_t = -\log_{10}\left(\frac{V_t \cdot P_t}{P_{H,t} - P_{L,t}}\right), \qquad (2)$$

where $P_{H,t}$ is the highest price on day t and $P_{L,t}$ is the lowest. Danyliv et al. (2014) list some differences of LIX and the Amihud illiquidity ratio (from the original definition). First, $(IL)LIX$ uses "high minus low" as a measure of volatility instead of absolute return. It eliminates the currency from the calculations and is easy to calculate. It decays almost linearly from the most-illiquid stocks (with values at around 5) to the most-liquid (with values at around 10). Some of these differences disappear when comparing AMI and $ILLIX$. Both measures are defined on daily data and do not use averaging. Taking the minus logs in the original definition of LIX has one important advantage: considering illiquidity instead of liquidity allows us to calculate the tail dependence coefficients directly from the estimated copulas.

In the empirical part, we conduct causality testing based on the VAR models that are defined for stationary time series. The $VAR(k)$ model (Vector Autoregressive) for vector $\boldsymbol{P_t} = (\boldsymbol{X_t}\ \boldsymbol{Y_t})$ is given by

$$\boldsymbol{P_t} = \boldsymbol{\Phi_0} + \sum_{i=1}^{k} \boldsymbol{\Phi_i} \boldsymbol{P_{t-i}} + \boldsymbol{\varepsilon_t}, \qquad (3)$$

where $\boldsymbol{\Phi_0}$ is vector of intercepts, $\boldsymbol{\Phi_i} = \begin{bmatrix} \phi_{11,i} & \phi_{12,i} \\ \phi_{21,i} & \phi_{22,i} \end{bmatrix}$ is the matrix of the parameters for lags $i = 1, \dots, k$ and $\boldsymbol{\varepsilon_t}$ is vector of error terms.

3.2 Long Memory

Process X_t has degree of fractional integration d (we write $I(d)$) when:

$$(1 - L)^d X_t = u_t, \qquad (4)$$

Here, L is a lag operator ($LX_t = X_{t-1}$), and u_t is a process with short memory. Expression $(1 - L)^d$ can be written in the form of a series expansion.

For parameter $0 < |d| < 0.5$, this process is stationary and invertible, and the autocorrelation function exhibits a hyperbolic decay. In addition, if $d \in (0; 0.5)$, the process has long memory, and if $d \in (-0.5; 0)$, the process is antipersistent and has intermediate memory.

For $d \in [0.5; 1)$, the variance of X_t is infinite. In this case, the process is covariance nonstationary but still mean-reverting.

In the empirical part, we use the exact local Whittle estimator (Phillips and Shimotsu, 2005) of long-memory parameter d.

Simplifying – if d_0 is the value of the true parameter of long-memory parameter d and the assumed m is such that $\frac{1}{m}+\frac{m(\log m)^{1/2}}{n}+\frac{\log n}{m^{\gamma}}\to 0$ for any $\gamma>0$ as $n\to\infty$, then the ELW estimator is consistent and the following holds true:

$$\sqrt{m}(\hat{d}_{ELW}-d_0)\xrightarrow{d} N\left(0,\frac{1}{4}\right). \tag{5}$$

3.3 Copulas and Tail Dependence

Assume that random variables X and Y have continuous distribution functions F and G, respectively. Let their joint distribution function be $H(x,y)$. According to Sklar's theorem (1959) (comp. Nelson, 2006), function C (called the copula) exists such that $H(x,y)=C(u,v)$ with $u=F(x)$ and $v=G(y)$. The copulas are then multivariate cumulative distribution functions with univariate margins uniformly distributed on interval [0, 1]. Sklar's theorem allows us to separately model the marginal distribution and dependence structure described by the copula (which joins the marginal distributions). Given the many families of univariate distributions and copulas, the latter are very useful in modeling the dependence structures between random variables. One of the properties of the copulas is their invariance for strictly monotone transformations of random variables. Moreover, the dependence measures can be expressed in terms of copulas and do not depend on marginal distributions.

A very important issue is the dependence of random variables in the tails. For a given α, the lower tail dependence is defined as $P[Y<G^{-1}(\alpha)|X<F^{-1}(\alpha)]$. The dependence in the upper tail is given as $P[Y>G^{-1}(\alpha)|X>F^{-1}(\alpha)]$. Taking the limit, we obtain the following lower tail dependence τ_L between X and Y:

$$\tau_L=\lim_{\alpha\to 0^+} P[Y<G^{-1}(\alpha)|X<F^{-1}(\alpha)] \tag{6}$$

and upper tail dependence τ_U:

$$\tau_U=\lim_{\alpha\to 1^-} P[Y>G^{-1}(\alpha)|X>F^{-1}(\alpha)]. \tag{7}$$

From the above definitions, it follows that the tail dependence coefficients measure the probability that the margins take extreme (small or large) values. In terms of the copulas, they are equal to the following:

$$\tau_L=\lim_{u\to 0^+}\frac{C(u,u)}{u}, \tag{8}$$

$$\tau_U=\lim_{u\to 1^-}\frac{\bar{C}(u,u)}{1-u}, \tag{9}$$

where $\bar{C}$ denotes the survival copula of C: $\bar{C}(u,v)=u+v-1+C(1-u,1-v)$, for $u,v\in[0,1]$. We say that variables X and Y are called asymptotically dependent in the lower (upper) tail if $\tau_L\in(0,1]$ ($\tau_U\in(0,1]$) and asymptotically independent if $\tau_L=0$ ($\tau_U=0$). From these definitions, it follows that, if copula C has upper-tail dependence, then copula $\bar{C}$ has lower-tail dependence and vice versa.

The list of copulas that can be used is extensive (see Nelson [2006], for example). The most popular are Archimedean copulas, which allow us to model asymmetric tail dependence (Gumbel or Clayton, for example), and elliptical copulas, which are used in modeling symmetric tail dependence (copula t) or tail independence (Gaussian copula). An important role in determining correlation is played by Kendall's τ coefficient. This is one of the most popular rank correlation coefficients. It relies on the notion of concordance. Let (x_1, y_1) and (x_2, y_2) be two observations of random vector (X, Y). We say that the pair is concordant whenever $(y_1 - y_2)(x_1 - x_2) > 0$ and discordant whenever $(y_1 - y_2)(x_1 - x_2) < 0$. Intuitively, a pair of random variables are concordant if large values of one variable are more likely to occur with large values of the other variable. For random variables X and Y, Kendall's τ is the difference between the probabilities of concordance and discordance:

$$\tau = P[(y_1 - y_2)(x_1 - x_2) > 0] - P[(y_1 - y_2)(x_1 - x_2) < 0]. \tag{10}$$

For the pair of random variables X and Y and its copula C, Kendall's τ has a very closed form given by the following:

$$\tau = 4 \int_{[0,1]^2} C(u, v) dC(u, v) - 1. \tag{11}$$

Since a copula is invariant with respect to any monotonic transformation, Kendall's τ also has this property. In the next chapter, we describe the stock exchanges under study.

4. Data from Vienna and Warsaw Stock Exchanges

Our dataset contains the tick data of 11 individual stocks from the ATX (covering the period of January 2, 2006, through April 8, 2016) and 20 stocks from the WSE (January 3, 2011, through March 28, 2017). From the data, we collected the daily prices and volumes along with the high-frequency prices.[1]

The Warsaw Stock Exchange (WSE) began operation on April 16, 1991. This was the first trading day of the listing; intensive development on the exchange occurred after Poland's EU accession in 2004. The most important index on the Warsaw Stock Exchange is the WIG20.

The WIG20 index is a price index that is calculated on the basis of the 20 largest and most-liquid companies. Since March 19, 2007, the WIG20 index has been published every 15 seconds during the session; before this date, the value of this index was published every 30 seconds based on the prices of the 20 largest and most-liquid companies.

The stocks of the companies from the Vienna Stock Exchange (VSE) analyzed in this study are listed on the Austrian Traded Index (ATX). The Vienna Stock Exchange (Wiener Börse AG) was created in 1771, making it one of the oldest stock exchanges in the world. Since November 5, 1999, trading on the VSE has been conducted via the Xetra® (Exchange Electronic Trading) system. This is the same

[1] Computations were conducted in R and Matlab environments.

system that is used on the Frankfurt Stock Exchange.

The ATX index is like the WIG20 – a price index. Its value is calculated based on the prices of the 20 largest and most-liquid companies listed on the Vienna Stock Exchange. During continuous trading, the values of the ATX index are published immediately after each transaction of every company included in this index.

Nowadays, the stock exchanges in Warsaw and Vienna are the largest capital markets in Central and Eastern Europe. They exhibited a similar capitalization in 2004; the number of daily transactions in Warsaw and Vienna amounted to 142,500 and 32,200, respectively. The daily turnover in Warsaw was €56.1 million; in Vienna, this amounted to €76.7 million.

However, the WSE overtook the VSE in 2008 with respect to capitalization. In addition, we can observe different models of development in both stock exchanges. In the case of the WSE, development is stimulated by new offers; we can observe an increase in the number of listed companies. Development of the Vienna Stock Exchange, however, is focused on the creation of a capital group that includes capital markets from southeastern Europe. The WSE and VSE were local competitors at one time; however, they once considered a merger. There was a discussion about different forms of cooperation; however, these discussions were not successful. Unfortunately, the WSE announced that talks about a potential merger with the VSE were cut off in 2014. In the opinion of many investors from both exchanges, this was not a good decision. In this way, a large and strong stock exchange in Central and Eastern Europe was not created – one that could attract many new investors from neighboring countries like Ukraine and the Balkans.

In spite of the similar capitalization, the WSE and VSE are quite different with respect to the number of listed companies. In the case of the WSE, the number of companies continuously increased, while one can observe a reduction in the number of companies in the case of the VSE.

Different patterns of listing are utilized in both markets. During the years after 2004, the schedule of the WSE sessions changed several times. During this time, the WARSET transaction system (existing on the WSE since 2001) was changed. This system was replaced on April 15, 2013, by the UTP (Universal Trading Platform) system, which is also used by the stock exchanges of other groups (the NYSE and Euronext, among others). As compared to the WARSET system, UTP can handle about 20,000 offers a second, while the WARSET system can handle up to 850 offers per second. Also, time has been shortened due to the UTP; this fact has made *high-frequency trading* possible.

Since November 5, 1999, transactions on the VSE have been handled by the Xetra® system. As compared to the WSE, one can notice fewer changes in the schedule of the session on the VSE. Through the end of 2008, the continuous listing of companies from the ATX started at 9:20; however, since January 1, 2009, continuous listing has commenced at 9:01. The most important difference between the scheduling of sessions in Vienna and Warsaw is the intraday auction (which starts at 12:00). In addition, the duration of this part of the listing is not the same on all days. On most days, the intraday auction occurs between 12:00 and 12:03 (or it takes place at 12:05:30 on expiration days on the future market).

In the next chapter, we present our results of our computations using the methodology outlined in this section. At very beginning of the next section, we will provide descriptive statistics of the data under study.

5. Empirical Findings

In the table below, we present the descriptive statistics of the companies under study (to save space, quantiles of the main descriptive statistics are presented).

Table 1 Descriptive Statistics of ATX Companies

		AMI		
	mean	*s.d.*	*kurtosis*	*skewness*
minimum	7.38	1.61	5.34	-2.60
1st quartile	7.49	1.82	6.34	-1.53
median	8.59	1.86	7.08	-1.19
3rd quartile	9.64	2.00	8.72	-0.97
maximum	10.71	2.25	13.28	-0.80
		ILLIX		
	mean	*s.d.*	*kurtosis*	*skewness*
minimum	-7.52	0.21	2.92	-0.44
1st quartile	-7.20	0.24	3.15	-0.32
median	-6.98	0.28	3.53	-0.21
3rd quartile	-6.65	0.30	3.75	-0.01
maximum	-6.45	0.37	5.47	0.65
		RV		
	mean	*s.d.*	*kurtosis*	*skewness*
minimum	0.61	0.85	3.31	-0.79
1st quartile	1.01	0.87	3.59	0.34
median	1.10	0.96	3.81	0.46
3rd quartile	1.29	1.00	3.96	0.53
maximum	1.57	1.15	5.26	0.66

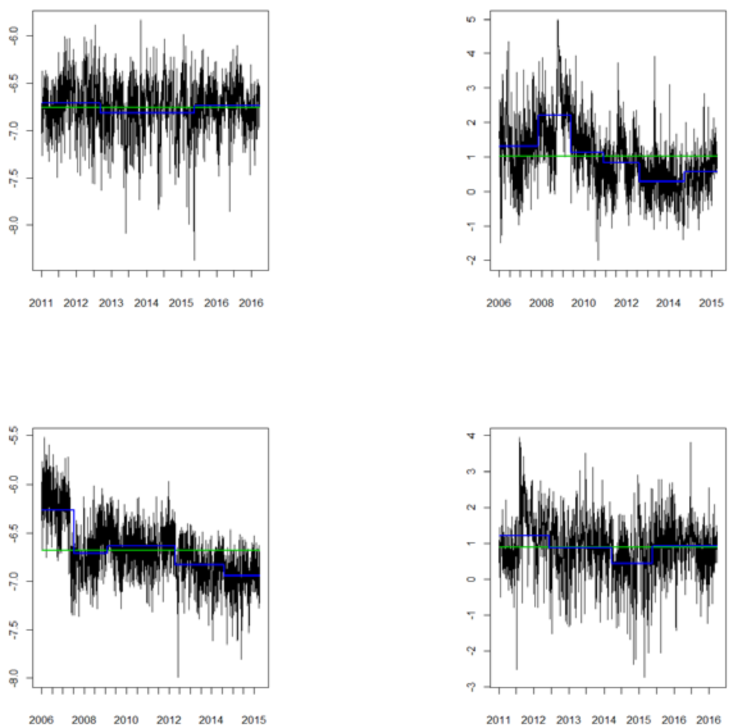
Table 2 Descriptive statistics of WIG companies

		AMI		
	mean	*s.d.*	*kurtosis*	*skewness*
minimum	3.80	1.79	1.98	-3.04
1st quartile	7.35	2.06	6.77	-2.64
median	9.03	2.30	9.41	-1.92
3rd quartile	10.93	2.91	10.61	-1.62
maximum	12.89	3.68	16.21	0.57
		ILLIX		
	mean	*s.d.*	*kurtosis*	*skewness*
minimum	-8.25	0.20	3.03	-0.93
1st quartile	-7.60	0.26	3.57	-0.52
median	-7.13	0.33	3.88	-0.13
3rd quartile	-6.91	0.46	4.49	0.19
maximum	-5.75	0.81	6.88	0.91
		RV		
	mean	*s.d.*	*kurtosis*	*skewness*
minimum	0.43	0.69	3.60	-1.14
1st quartile	0.92	0.77	3.91	-0.23
median	1.03	0.84	4.70	0.24
3rd quartile	1.25	0.91	7.04	0.58
maximum	1.58	1.22	18.82	2.68

The computed descriptive statistics are similar for both markets. The values of skewness and/or kurtosis indicate a departure from normality. This is confirmed by the Jarque-Bera tests (we reject the null at a 1% significance level in all cases). The results of the Ljung-Box tests along with the slow decay of the autocorrelation function leads us to the conclusion that the series of illiquidity and realized volatility exhibit long memory.

Preliminarily, we computed sample linear and Kendall correlation coefficients for the AMI–RV and $ILLIX$–RV pairs. For the companies from both markets, the values of the linear coefficients are greater than Kendall on average. The dependence is stronger for ATX companies for both coefficients. With one exception, the correlation coefficients between the AMI–RV pair are greater than in the case of the $ILLIX$–RV pair. For the WIG, this is not the case summary for 13 companies.

Figure 1 Structural Breaks of AND (top) and ACP (bottom)



Notes: Figure 1 Illustrates the optimal partition from the structural break estimation for Austrian stock AND (top) and Polish stock ACP (bottom). The green lines represent the mean value of the series, and the blue lines represent the mean values in each segment. The $ILLIX$ illiquidity series are presented on the left side, and the realized volatility is on the right.

At first, we investigate the occurrence of structural breaks as a source of long memory, and we adopt multiple breaks in the mean estimation of Bai and Perron (1998, 2003), which are similar to Yang and Chen (2014). Let $y_t = m_i + \varepsilon_t$ be the series under study, with mean m_i and error term ε_t for $t = T_{i-1} + 1, T_{i-1} + 2, \dots, T_i$ and $i = 1,2, \dots, M + 1$. The unknown break dates (or optimal partition) $T_1, T_2, \dots, T_M$ are estimated using the least squares estimation, and their number is determined via the Bayesian Information Criterion. We observe that the volatility and illiquidity series contain multiple structural changes in the mean, which can explain the long-memory property in the time series (exceptions are the AMI illiquidity series of the ACP, CPS, and MIL companies)[2]. In Figure 1, we present an optimal partition of the $ILLIX$ illiquidity and realized volatility series for Austrian stock AND (four breakpoints in illiquidity and five in realized volatility) and Polish stock ACP (two and three breakpoints in illiquidity and realized volatility, respectively).

After removing the structural breaks in the mean (by subtracting component m_i from series), we estimate the long-memory parameters. During the estimation, we use the ELW estimator described in Section 3.2 with $m = n^{0.6}$ and $m = n^{0.55}$ (where n is the series length). The results of this estimation are presented in Tables 3 and 4.

Table 3 Long-Memory Parameters of ATX Companies after Removing sStructural Breaks

	$m = n^{0.6}$			$m = n^{0.55}$		
company	AMI	$ILLIX$	RV	AMI	$ILLIX$	RV
VOE	0.489 (0.394;0.584)	0.594 (0.499;0.689)	0.562 (0.467;0.657)	0.542 (0.427;0.657)	0.594 (0.499;0.689)	0.514 (0.401;0.627)
AND	0.371 (0.278;0.464)	0.482 (0.389;0.575)	0.577 (0.484;0.670)	0.348 (0.235;0.461)	0.482 (0.389;0.575)	0.459 (0.346;0.572)
ERST	0.419 (0.326;0.512)	0.358 (0.265;0.451)	0.632 (0.539;0.725)	0.413 (0.300;0.526)	0.358 (0.265;0.451)	0.586 (0.473;0.699)
VERB	0.428 (0.335;0.521)	0.629 (0.536;0.722)	0.339 (0.246;0.432)	0.503 (0.390;0.616)	0.629 (0.536;0.722)	0.437 (0.324;0.550)
OMV	0.316 (0.223;0.409)	0.394 (0.301;0.487)	0.522 (0.429;0.615)	0.336 (0.223;0.449)	0.394 (0.301;0.487)	0.496 (0.383;0.609)
VIGR	0.292 (0.199;0.385)	0.390 (0.297;0.483)	0.558 (0.465;0.651)	0.349 (0.236;0.462)	0.390 (0.297;0.483)	0.540 (0.427;0.653)
RIBH	0.428 (0.335;0.521)	0.373 (0.280;0.466)	0.595 (0.502;0.688)	0.390 (0.277;0.503)	0.373 (0.280;0.466)	0.501 (0.388;0.614)
RHI	0.221 (0.128;0.314)	0.426 (0.333;0.519)	0.428 (0.335;0.521)	0.220 (0.107;0.333)	0.426 (0.333;0.519)	0.437 (0.324;0.550)
WBSV	0.305 (0.212;0.398)	0.347 (0.254;0.440)	0.443 (0.350;0.536)	0.277 (0.164;0.390)	0.347 (0.254;0.440)	0.466 (0.353;0.579)
TELA	0.293 (0.200;0.386)	0.604 (0.511;0.697)	0.596 (0.503;0.689)	0.335 (0.222;0.448)	0.604 (0.511;0.697)	0.602 (0.489;0.715)
POST	0.188 (0.095;0.281)	0.410 (0.317;0.503)	0.323 (0.230;0.416)	0.207 (0.094;0.320)	0.410 (0.317;0.503)	0.276 (0.163;0.389)

Notes: This table reports the results of the estimation of the long-memory parameter using the ELW estimator for two choices of bandwidths ($m = n^{0.6}$ and $m = n^{0.55}$). The asymptotic 95% confidence intervals are constructed by adding and subtracting $1.96 \cdot 1/\sqrt{4m}$ to the estimates. The method is applied to the series after removing the structural breaks in the mean.

From these results, we conclude that the long memory parameters are generally larger for the ATX companies. For the WIG companies, all but one (BZW) are in a stationary region, whereas for ATX companies, there are a few cases (for some of them, the left boundary of the confidence interval is also greater than 0.5). For EUR and TPE, we can observe negative values of the estimates (but insignificantly different from zero for the 5% significance level). We notice that removing the structural breaks leads to a reduction in the variances of the realized volatility and illiquidity series. From now on, we apply a method similar to Rossi and de Magistris (2013) and Gurgul and Syrek (2013). We apply fractional differencing to the series of illiquidity and realized volatility and apply Vector Autoregressive models. The maximum lag length is chosen using the Bayesian Information Criterion and does not exceed four. To explore the dependence structure, we apply the copulas presented in Chapter 3 of our study. Rossi and de Magistris (2013) estimate the parameters of the marginal

[2] Full information about the dates of the breaks are available from the authors upon request.

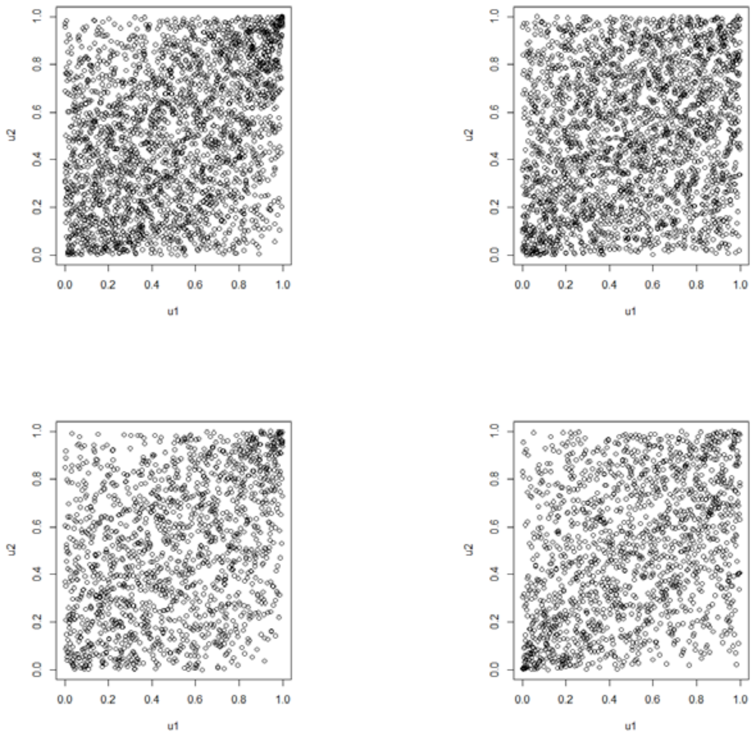
distributions using the maximum likelihood method along with the parameters of the copulas; we instead apply the canonical maximum likelihood method (Cherubini et al., 2004). The residuals from the VAR models are transformed into uniformly distributed $U(0{,}1)$ variables using the empirical cumulative distribution function; then, the parameters of the copulas are estimated with the maximum likelihood method. The selection of the copulas that fit the best are based on the Bayesian Information Criterion. During the estimation procedure, we fit numerous families of copulas. The most often chosen copulas are the Archimedean copulas: Gumbel (G), Clayton (C), Joe (J), BB1 and BB7 (along with their survival versions with prefix "S") as well elliptical copulas t and Gaussian (N). Descriptions of these copulas are given in the appendix. In Figure 2, we present the scatter plots of the (U, V) pairs for the ERST and ACP companies ($AMI - RV$ on the left, $ILLIX - RV$ on the right). When using AMI as the illiquidity measure, we observe a concentration of points in the top-right corner, whereas for the $ILLIX$ a concentration is shown in the bottom-left corner.

Table 4 Long-Memory Parameters of WIG Companies after Removing Structural Breaks

	$m = n^{0.6}$			$m = n^{0.55}$		
	AMI	$ILLIX$	RV	AMI	$ILLIX$	RV
ACP	0.135 (0.027;0.243)	0.286 (0.178;0.394)	0.255 (0.147;0.363)	0.135 (0.005;0.265)	0.190 (0.06;0.320)	0.322 (0.192;0.452)
BZW	0.301 (0.193;0.409)	0.683 (0.575;0.791)	0.377 (0.269;0.485)	0.274 (0.144;0.404)	0.74 (0.61;0.87)	0.426 (0.296;0.556)
CCC	0.096 (-0.012;0.204)	0.246 (0.138;0.354)	0.105 (-0.003;0.213)	0.076 (-0.054;0.206)	0.359 (0.229;0.489)	0.108 (-0.022;0.238)
CPS	0.098 (-0.01;0.206)	0.156 (0.048;0.264)	0.222 (0.114;0.330)	0.143 (0.013;0.273)	0.160 (0.03;0.29)	0.211 (0.081;0.341)
ENA	0.179 (0.071;0.287)	0.378 (0.27;0.486)	0.355 (0.247;0.463)	0.078 (-0.052;0.208)	0.332 (0.202;0.462)	0.360 (0.23;0.49)
EUR	-0.045 (-0.153;0.063)	0.292 (0.184;0.4)	0.345 (0.237;0.453)	0.075 (-0.055;0.205)	0.299 (0.169;0.429)	0.430 (0.3;0.56)
GTC	0.168 (0.06;0.276)	0.212 (0.104;0.32)	0.387 (0.279;0.495)	0.212 (0.082;0.342)	0.275 (0.145;0.405)	0.418 (0.288;0.548)
ING	0.110 (0.002;0.218)	0.266 (0.158;0.374)	0.163 (0.055;0.271)	0.158 (0.028;0.288)	0.265 (0.135;0.395)	0.128 (-0.002;0.258)
KER	0.140 (0.032;0.248)	0.314 (0.206;0.422)	0.361 (0.253;0.469)	0.090 (-0.04;0.22)	0.219 (0.089;0.349)	0.349 (0.219;0.479)
KGH	0.205 (0.097;0.313)	0.169 (0.061;0.277)	0.360 (0.252;0.468)	0.302 (0.172;0.432)	0.171 (0.041;0.301)	0.41 (0.28;0.54)
LTS	0.224 (0.116;0.332)	0.279 (0.171;0.387)	0.369 (0.261;0.477)	0.262 (0.132;0.392)	0.191 (0.061;0.321)	0.429 (0.299;0.559)
MIL	0.189 (0.081;0.297)	0.201 (0.093;0.309)	0.379 (0.271;0.487)	0.167 (0.037;0.297)	0.066 (-0.064;0.196)	0.476 (0.346;0.606)
PEO	0.248 (0.14;0.356)	0.200 (0.092;0.308)	0.409 (0.301;0.517)	0.251 (0.121;0.381)	0.242 (0.112;0.372)	0.491 (0.361;0.621)
PGE	0.140 (0.032;0.248)	0.264 (0.156;0.372)	0.396 (0.288;0.504)	0.069 (-0.061;0.199)	0.205 (0.075;0.335)	0.38 (0.25;0.51)
PGN	0.133 (0.025;0.241)	0.283 (0.175;0.391)	0.279 (0.171;0.387)	0.205 (0.075;0.335)	0.238 (0.108;0.368)	0.287 (0.157;0.417)
PKN	0.166 (0.058;0.274)	0.186 (0.078;0.294)	0.311 (0.203;0.419)	0.153 (0.023;0.283)	0.279 (0.149;0.409)	0.313 (0.183;0.443)
PKO	0.104 (-0.004;0.212)	0.219 (0.111;0.327)	0.449 (0.341;0.557)	0.133 (0.003;0.263)	0.11 (-0.02;0.24)	0.436 (0.306;0.566)
PZU	0.154 (0.046;0.262)	0.238 (0.13;0.346)	0.309 (0.201;0.417)	0.135 (0.005;0.265)	0.274 (0.144;0.404)	0.315 (0.185;0.445)
SNS	0.050 (-0.058;0.158)	0.342 (0.234;0.45)	0.303 (0.195;0.411)	0.052 (-0.078;0.182)	0.232 (0.102;0.362)	0.247 (0.117;0.377)
TPE	-0.071 (-0.179;0.037)	0.389 (0.281;0.497)	0.384 (0.276;0.492)	-0.119 (-0.249;0.011)	0.177 (0.047;0.307)	0.436 (0.306;0.566)

Notes: This table reports the results of the estimation of the long-memory parameter using the ELW estimator for two choices of bandwidths ($m = n^{0.6}$ and $m = n^{0.55}$). The asymptotic 95% confidence intervals are constructed by adding and subtracting $1.96 \cdot 1/\sqrt{4m}$ to the estimates. The method is applied to the series after removing the structural breaks in the mean.

Figure 2 Scatter Plots for ERST (top) and ACP (bottom)



Notes: Figure 2 illustrates the scatter plots of the (U, V) pairs for the ERST (top) and ACP (bottom) companies. The $AMI - RV$ pair is on the left, and the $ILLIX - RV$ pair is on the right.

Despite the similarity between the AMI and $ILLIX$ mentioned in the previous sections, the dependence measures based on the copulas that fit best with respect to the markets are very different. The copulas for the AMI illiquidity-realized volatility pairs are characterized by dependence in the upper tails (high values of illiquidity are associated with high values of volatility), whereas the $ILLIX$-realized volatility pairs are characterized by dependence in the lower tails at most (the low illiquidity is associated with low volatility). For the last case of a few of the WIG companies, we observe that the elliptical copulas Gaussian (with tail independence) and t (with symmetric dependence) best fit the data. On average, the dependence measured by Kendall’s tau is stronger when considering the AMI illiquidity measure and are stronger for the ATX companies. This is also true when comparing the values of the tail dependence coefficient. At the least, we can observe that the computed τ_U from the AMI illiquidity-realized volatility pairs is greater than the τ_L from the $ILLIX$-realized volatility pairs. This is true for both the ATX and WIG companies. It is worth mentioning that the sample Kendall coefficients are generally greater than those computed from the estimated copulas.

Application of the VAR models allows us to analyze the causal dependence. The table below presents the numbers of rejections of the null hypothesis of non-causality (with a 1% significance level; $\nrightarrow$ means "does not Granger cause").

Table 7 Copula Estimation Results for ATX Companies

		AMI				***ILLIX***		
company	*copula*	τ	τ_L	τ_U	*copula*	τ	τ_L	τ_U
VOE	G	0.21	0	0.28	SG	0.17	0.22	0.00
AND	G	0.24	0	0.31	C	0.15	0.14	0.00
ERST	SBB7	0.24	0.09	0.28	SG	0.21	0.27	0.00
VERB	G	0.25	0	0.32	SG	0.22	0.28	0.00
OMV	G	0.31	0	0.38	C	0.16	0.17	0.00
VIGR	G	0.30	0	0.37	C	0.14	0.12	0.00
RIBH	G	0.27	0	0.35	BB1	0.19	0.17	0.06
RHI	G	0.25	0	0.32	SG	0.18	0.24	0.00
WBSV	SC	0.21	0	0.28	SG	0.21	0.27	0.00
TELA	G	0.26	0	0.33	SG	0.21	0.27	0.00
POST	G	0.28	0	0.35	SG	0.19	0.25	0.00

Notes: This table reports the results of the estimation of the copula parameters and dependence measures for the illiquidity-realized volatility pairs. The estimated copulas are Gumbel (G), Clayton (C), BB1 and BB7. The "S" prefix denotes their survival versions.

Table 8 Copula Estimation Results for WIG Companies

		AMI				***ILLIX***		
company	*copula*	τ	τ_L	τ_U	*copula*	τ	τ_L	τ_U
ACP	G	0.23	0	0.29	SG	0.24	0.31	0.00
BZW	SC	0.19	0	0.23	C	0.15	0.13	0.00
CCC	G	0.25	0	0.32	N	0.17	0.00	0.00
CPS	G	0.24	0	0.30	t	0.17	0.05	0.05
ENA	G	0.23	0	0.30	C	0.20	0.25	0.00
EUR	J	0.21	0	0.40	BB7	0.18	0.05	0.21
GTC	SC	0.24	0	0.34	BB1	0.24	0.04	0.22
ING	SC	0.23	0	0.32	SG	0.17	0.22	0.00
KER	G	0.24	0	0.31	SG	0.14	0.18	0.00
KGH	G	0.18	0	0.24	t	0.19	0.02	0.02
LTS	G	0.21	0	0.27	SG	0.14	0.19	0.00
MIL	G	0.24	0	0.31	t	0.15	0.05	0.05
PEO	G	0.19	0	0.24	t	0.22	0.03	0.03
PGE	G	0.23	0	0.30	SG	0.08	0.11	0.00
PGN	G	0.19	0	0.25	SJ	0.12	0.25	0.00
PKN	G	0.26	0	0.33	C	0.17	0.19	0.00
PKO	G	0.27	0	0.34	BB1	0.18	0.07	0.10
PZU	G	0.23	0	0.30	SG	0.20	0.26	0.00
SNS	G	0.25	0	0.32	C	0.14	0.12	0.00
TPE	G	0.22	0	0.28	BB1	0.22	0.08	0.16

Notes: This table reports the results of the estimation of the copula parameters and dependence measures for the illiquidity-realized volatility pairs. The estimated copulas are Gumbel (G), Clayton (C), Joe (J), BB1, BB7, t and Gaussian (N). The "S" prefix denotes their survival versions.

Table 9 Granger Causality Testing Results

	ATX	
	$H_0: volatility \nrightarrow illiqudity$	$H_0: illiqudity \nrightarrow volatility$
AMI	8	4
ILLIX	2	1
	WIG	
	$H_0: volatility \nrightarrow illiqudity$	$H_0: illiqudity \nrightarrow volatility$
AMI	4	7
ILLIX	1	1

Notes: This table reports the numbers of rejection of null hypothesis with significance level 1%. Symbol $\nrightarrow$ means "does not Granger cause."

The results of the tests are similar when using $ILLIX$ as an illiquidity measure. We could conclude that non-causality occurs in both directions. For the ATX companies, volatility Granger causes AMI illiquidity in 8 out of the 11 cases; however, the opposite statement is true in 4 cases. For the WIG companies, the situation is different; we more often observe a rejection of the null: illiquidity does not Granger cause volatility. For both markets and measures, the percentage of rejected null hypotheses is greater for the ATX companies.

Based on the estimates of the VAR models, we found some similarities in the impulse response functions[3] The strongest are the responses of AMI illiquidity to the realized volatility (RV->AMI) shocks in the first step. For the ATX companies, the minimum value of response is equal to 0.04, and the maximum value is 0.31 (with a median of 0.16). For the WIG companies, these values are 0.03, 0.41, and 0.19, respectively. In the next two steps, the responses ranges from negative values (minimum) to positive (maximum), with the median at around zero. The responses of the realized volatility to ILLIX shocks have a similar pattern. In the first step for the ATX companies, the ranges are found within a minimum of -0.07 to a maximum of 0.28 (with a median of 0.04), and for the WIG companies, the ranges are found within a minimum of -0.18 and a maximum of 0.18 (with a median of 0.07). For either RV->ILLIX or AMI->RV relationship, the responses are weak (even for the first steps).

6. Summary and Conclusions

Liquidity is the ability to quickly sell or purchase a number of securities at a low cost without significantly changing the assets' prices. There are many known alternative variables that stand for liquidity; transaction value, size, volume, turnover, and bid–offer spread. Instead of liquidity, we used its opposite notion (called illiquidity) in our study. The second most important characteristic of an asset is volatility (which stands for risk). The study uses high-frequency data to estimate the realized volatility, whereas the illiquidity is based on the daily data. The main goal of our empirical part was to check the relationships between illiquidity and realized volatility (but also to discover the properties of these series).

Both the realized volatility and illiquidity exhibit structural changes in the mean. We observe that, on average, the estimated long memory parameters are higher for the ATX companies. For a few analyzed companies, the estimated values of the long memory parameters indicate nonstationarity.

The copulas are proper tools that determine the dependence structure between realized volatility and illiquidity. By mean of copulas, the study provides important insights towards understanding these relationships (especially in the tails).

The structure of the dependence of illiquidity-realized volatility (i.e., $AMI - RV$ and $ILLIX - RV$) calculated by the copulas give quite different pictures. The copulas for the AMI illiquidity-realized volatility pairs show a dependence in the upper tails. This means that the high values of illiquidity are related to high volatility. Quite the opposite is true for the $ILLIX$-realized volatility pairs; they mostly show dependence in the lower tails. This means that low illiquidity is accompanied by low volatility. In the case of some WIG companies, we observe that the elliptical copulas

[3] All results are available from the authors upon request.

(i.e., Gaussian or t-copulas) are most adequate with respect to the data. In most cases, the dependence measured by Kendall's tau is stronger for the AMI illiquidity measure. It is also stronger for the companies included in the ATX as compared to those companies included in the WIG. This is also true when comparing the values of the tail dependence coefficient. An interesting observation for all of the companies is that the τ_U for the AMI illiquidity-realized volatility pairs outperform the τ_L for the $ILLIX$-realized volatility pairs.

At last, we can make our conclusions about the Granger causality between the realized volatility and illiquidity and reactions to shocks (measured with the impulse response functions).

When using $ILLIX$ as an illiquidity measure, the results are similar for both markets; non-causality exists in both directions. For most of the ATX companies, volatility Granger causes AMI illiquidity. The opposite statement is true in four cases. For most of the WIG companies, the rejection of the null hypothesis that illiquidity does not Granger cause volatility is observed. Regarding the impulse response function, we can observe some similarities; the strongest are the responses of AMI illiquidity to the realized volatility and responses of the realized volatility to ILLIX shocks.

APPENDIX

Gaussian Copula (N)

The distribution function of the bivariate Gaussian copula is given by

$$C_{\rho}^{Ga}(u_1, u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi(1-\rho^2)^{1/2}} exp\left(\frac{-\left(s_1^2 - 2\rho s_1\ s_2 + s_2^2\right)}{2(1-\rho^2)}\right) ds_1 ds_2, \quad (12)$$

where Φ^{-1} denotes the inverse cumulative distribution of the standard normal, and ρ is the linear correlation coefficient of the corresponding bivariate normal distribution. The Gaussian copulas has zero upper and lower tail dependence (one exception is the case of perfect dependence $\rho = 1$). Kendall's tau is simply given by $\frac{2}{\pi}\arcsin(\rho)$.

t copula (t)

The distribution function of bivariate *t* copula is defined as

$$C_{\nu,\rho}^{t}(u_1, u_2) = \int_{-\infty}^{t_\nu^{-1}(u)} \int_{-\infty}^{t_\nu^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \left(1 + \frac{s_1^2 - 2\rho s_1\ s_2 + s_2^2}{\nu(1-\rho^2)}\right)^{-\frac{\nu+2}{2}} ds_1 ds_2, \quad (13)$$

where t_ν^{-1} is the inverse cumulative distribution of the univariate *t* with $\nu > 2$ degrees of freedom, whereas $\rho \in (-1,1)$ is the linear correlation coefficient of the corresponding bivariate *t* distribution. Because of the radial symmetry, the tail dependence coefficient of the *t* copula equals

$$\tau_L = \tau_U = 2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right). \quad (14)$$

The relationship between Kendall's tau and the linear correlation coefficient is the same as in the case of the Gaussian copula.

Clayton Copula

The Clayton copula is given by

$$C(u_1, u_2; \theta) = \max([u_1{}^{-\theta} + u_2{}^{-\theta} - 1]^{-\frac{1}{\theta}}, 0), \quad (15)$$

with $\theta \in [-1, \infty)\backslash\{0\}$. For positive values of θ, we have:

$$C(u_1, u_2; \theta) = (u_1{}^{-\theta} + u_2{}^{-\theta} - 1)^{-\frac{1}{\theta}}. \quad (16)$$

The Clayton copula is an asymmetric copula and exhibits only lower tail dependence: $\tau_L = 2^{-1/\theta}$. The coefficient of Kendall's tau is given by $\tau = \frac{\theta}{\theta+2}$.

Gumbel Copula

For parameter $\theta \in [1, \infty)$, the distribution function of the Gumbel copula is given by

$$C(u_1, u_2; \theta) = \exp(-[(-lnu_1)^{\theta} + (-lnu_2)^{\theta}]^{\frac{1}{\theta}}). \quad (17)$$

The Gumbel copula exhibits only upper tail dependence: $\tau_U = 2 - 2^{1/\theta}$. The Kendall's tau measure is calculated as $\tau = 1 - \theta^{-1}$.

Joe Copula

Defined for parameter $\theta \in [1, \infty)$, the distribution function of the Joe copula is given by

$$C(u_1, u_2; \theta) = 1 - [(1-u_1)^{\theta} + (1-u_2)^{\theta} - (1-u_1)^{\theta}(1-u_2)^{\theta}]^{\frac{1}{\theta}}. \quad (18)$$

The Joe copula exhibits only upper tail dependence: $\tau_U = 2 - 2^{1/\theta}$. The Kendall's tau measure is calculated as $\tau = 1 + \frac{4}{\theta^2}\int_0^1 x\log(x)(1-x)^{2(1-\theta)/\theta}dx$.

BB1 Copula

The two parameter BB1 copula is given by

$$C(u_1, u_2; \theta, \delta) = \left\{1 + [({u_1}^{-\theta} - 1)^{\delta} + ({u_2}^{-\theta} - 1)^{\delta}]^{1/\delta}\right\}^{-1/\theta}, \quad (19)$$

where $\theta > 0, \delta \geq 1$. The tail dependence coefficient are given by $\tau_U = 2 - 2^{1/\delta}$ and $\tau_L = 2^{-1/(\delta\theta)}$. The coefficient of Kendall's tau is calculated by τ =1- $2/(\delta(\theta+2))$.

BB7 Copula

This family is given by

$$C(u, v; \theta, \delta) = 1 - \left(1 - \left[\left(1 - \bar{u}^{\theta}\right)^{-\delta} + \left(1 - \bar{v}^{\theta}\right)^{-\delta} - 1\right]^{-\frac{1}{\delta}}\right)^{\frac{1}{\theta}}, \quad (20)$$

where $\bar{u} = 1 - u$, $\bar{v} = 1 - v$ and $\theta \geq 1, \delta > 0$. The tail dependence coefficients are simply given by $\tau_U = 2 - 2^{1/\theta}$ and $\tau_L = 2^{-1/\delta}$. The coefficients of Kendall's tau are computed by the following formula:

$$\tau = \begin{cases} 1 + \frac{2}{\delta(\theta-2)} + \frac{4(\delta+1)}{\theta\delta(\theta-2)} B\left(\frac{2}{\theta}, \delta+1\right), & \theta \neq 2 \\ 1 + \frac{1}{\delta} - \frac{1}{\delta}\big(\psi(\delta+2) - \psi(1)\big), & \theta = 2 \end{cases}, \tag{21}$$

where $B(x, y) = \int_0^1 t^{x-1}(t-1)^{y-1}dt$ for $x, y > 0$ is the Beta function, $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ is the Gamma function, and $\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$ for $x > 0$.

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