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Construction of Commodity Portfolio and Its Hedge Effectiveness Gauging – Revisiting DCC Models

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Abstract

This paper examines how various types of dynamic conditional correlation (DCC) models performs in the construction of risk-minimizing portfolio. Our portfolios consist of SPY-ETF instrument as a primary asset and four commodities – Brent oil, gold, silver and platinum. In the process of hedge effectiveness measurement, we utilize three different performance metrics – Hedge Effectiveness Index in terms of variance, Value at Risk and Conditional Value at Risk, which target different risk minimizing goals. The additional objective is to test whether minimum-variance hedging portfolio yields a similarly large reduction in portfolio VaR and portfolio CVaR. The hedge effectiveness performances are scrutinized via portfolios that are designed with help of three different types of DCC models – DCC-GARCH, DCC-APARCH and DCC-FIAPARCH. In order to gauge how hedge effectiveness of the portfolios alters across periods of different market turbulences, we split full sample into three subsamples applying modified ICSS algorithm. The research determines that minimum-variance targeting portfolios with accounted long memory in volatility demonstrated considerable robustness when it comes to the best performing models, taking into account three different risk metrics. However, in cases of risk/return targeting goals as well as in the process of out-of-sample forecast, the best solution turns out to be the simplest DCC model.

1. Introduction

In the last two decades, commodity markets have become increasingly available for the international investors, which provides a new ways of investment portfolio diversification for various market participants. A good understanding of how financial markets and commodity markets correlate could render lucrative benefits from these investment opportunities. Gorton and Rouwenhorst (2006) documented that utilizing commodities for portfolio construction can be suitable, since commodities have low or even negative correlation with equities. Hence, numerous researchers coupled various commodities with the stocks or stocks indices (see e.g. Filis et al. (2011), Hillier et al. (2006), Mollick and Assefa (2013), Zhang and Wei (2010), Kumar et al. (2012)).

In the process of portfolio design and hedge ratio estimation, the early studies, such as those conducted by Herbst et al. (1989) and Lindahl (1992), employed constant models. However, constant models are widely criticized since they propose very restrictive assumption that the dynamic weights remain stable over the sample period. Because of that, researchers increasingly resort to time-varying weights that are adjusted continuously, since this approach takes into account information sets available at the time the hedging decision was made. Accordingly, in order to measure portfolio hedging effectiveness, we employ bivariate dynamic conditional correlation (DCC) model that is able to provide time-varying correlation and variances, which are subsequently used as inputs for portfolio construction.

This paper strives to evaluate risk/return performances of four portfolios constructed of SPY-ETF (Exchange Traded Fund) and four commodities – Brent oil futures, gold spot, silver spot and platinum spot. We opt for ETF because contrary to the conventional indices, ETFs can be bought and sold throughout the trading day, which some international investors may find appealing (see Blitz and Huij, 2012). Also, we choose Brent oil instead of WTI oil because Brent oil is currently used to price two thirds of the world's internationally traded crude oil supplies. Brent futures are used instead of spot prices because hedging in oil spot market would be prohibitively costly.[1] Future prices for other commodities do not exist in a time-frame that we analyse, so we consider spot prices for gold, silver and platinum.

In order to take into account different risk-preferences of portfolio investors, we expressed risk in three ways – variance, Value at Risk (VAR) and Conditional Value at Risk (CVaR). Additionally, we address a frequent feature of the financial series that has been often overlooked by researchers, i.e. we refer to the studies such as Goddard and Onali (2012), Charfeddine and Ajmi (2013), Bentes (2014), who found that long memory persistence in the conditional variance might be intrinsic property of financial asset series. Hence, our method of choice is DCC models, whereby the estimation is conducted via three different univariate models, namely: GARCH, APARCH and FIAPARCH with two multivariate density functions – normal and Student t. The aim of the paper is to assess which of the applied models gives the best risk/return portfolio when three different hedging metrics are targeted: minimum variance[2], minimum VaR and minimum CVaR, and whether long memory in GARCH models represent an important feature in the process of portfolio construction. Besides, we seek the answer whether best portfolio risk-minimizing DCC model also gives the best return performances. Since our observation span covers the Global financial crisis (GFC) and the subsequent Sovereign debt crisis (SDC), we divide full sample into three subsamples – before, during and after the crisis, by using modified ICSS algorithm of Sansó et al. (2004). In such way, we can empirically pinpoint the exact dates when these crises erupted and ended. The partition of full sample in three subsamples could provide answers on the question how portfolio risk-effectiveness characteristics differentiate between different market stress-periods, and which DCC model yields the best risk-minimizing performances. Also, we test whether minimum-variance hedging portfolio offers a similarly large reduction in portfolio VaR and portfolio CVaR, since it is not obvious what to expect because minimum-variance hedging reduces portfolio standard deviation but can simultaneously increase skewness and kurtosis (see Harris and Shen, 2006). In addition, we endeavour to stipulate via rolling regression the interconnection between dynamic conditional variances and dynamic correlation. The goal is to assess which conditional variance

[1] We thank anonymous reviewer for this useful comment.
[2] The reason why variance minimization makes sense is high uncertainty in estimation of expected returns. In other words, even though investor cares about expected returns, he might be in the end better off if he focuses on minimizing variance.

has higher effect on the mutual dynamic correlation, which might help investors in portfolio decision making. At the end, we consider out-of-sample risk minimizing portfolio performances.

This paper contributes to the related literature by investigating thoroughly and comprehensively risk/return characteristics of four commodity portfolios, taking into account several accessory goals such as subsample analysis, the examination of nexus between conditional correlation and conditional variances and calculation of out-of-sample portfolio risk-minimizing performances.

Besides introduction, the rest of the paper is structured as follows. Second section examines the literature review and related studies. Third section explains DCC methodology, ICSS technique and the portfolio assembling along with the method for measurement of the portfolios hedge effectiveness. Forth section is reserved for data overview. Fifth section presents DCC and rolling regression results, while sixth section reveals portfolio risk/return results in-sample and out-of-sample. The last section concludes.

2. Literature Review and Related Studies

A number of studies have provided in past times insightful information about the links between commodity markets and stock markets (see e.g. Ewing and Thompson, 2007; Park and Ratti, 2008; Aloui et al., 2012; Arouri et al., 2011). This section presents a short literature review of the studies that combine various assets with well-known commodities in order to minimize portfolio risk. Since it is widely recognized that financial asset returns volatility, covariances and correlations are time-varying with persistent dynamics, the DCC method serves as appropriate approach for the optimal hedge ratio risk management with the minimum variance criterion (see Cha and Jithendranathan, 2009; Vivian and Wohar, 2012). In addition, Choi and Hammoudeh (2010) contended that commodity traders concurrently look at both stock and commodity market fluctuations to infer the trend of each market. Also, Creti et al. (2013) documented that regarding the observed links between commodity and stock markets, volatility plays a key role for hedging possibilities, asset allocation across raw materials and their risk-return trade-off.

In the following, the listed studies describe empirical usage of commodities in portfolios for the minimum-risk hedge purposes. For instance, Gorton and Rouwenhorst (2006) contended that investors potential to diversify can be enhanced with the inclusion of commodity futures in portfolios since commodities show equity-like returns and low correlation with traditional assets. Sadorsky (2014) utilized VARMA AGARCH and DCC-AGARCH to model volatilities and conditional correlations between emerging market stock prices, copper prices, oil prices and wheat prices. The results indicated that, on average, oil provides the cheapest hedge for emerging market stock prices, while copper is the most expensive commodity. The author also asserted that hedge ratios and portfolio weights need frequent updating in order to provide optimal values. The manuscript of Basher and Sadorsky (2016) used DCC, ADCC and GO-GARCH to construct two-asset portfolios between emerging market stock prices, as a primary asset, and oil prices, VIX, gold prices and bond prices. They reported that oil is the best asset to hedge emerging market stock prices in the most situations, and that hedge ratios from the ADCC model are most effective for hedging emerging market stock prices with oil, VIX, or bonds. Chkili (2016) examined the dynamic relationships between gold and stock markets using data for the BRICS counties in the Asymmetric DCC framework. His main objective was to examine the time-varying correlations between the two assets and to check the effectiveness of gold as a hedge for equity markets. As for the portfolio diversification and hedging effectiveness, he found that adding gold to a stock portfolio enhances its risk-adjusted return. Arouri et al. (2011) investigated hedging effectiveness in oil and stock markets in Europe and the United States from a sector perspective. They claimed that optimal portfolios in both Europe and the US should have stocks outweigh oil assets and that the stock investment risk can be hedged with relatively low hedging costs by taking a short position in the oil futures markets. The study of Boako and Alagidede (2016) explore the relative potentials of African equities to provide opportunities for hedging and diversification for global commodity investors. They concluded that including African equities in a diversified portfolio has the effect of lowering risk whiles simultaneously increasing expected returns. Khalfaoui et al. (2015) examines the linkage of crude oil market (WTI) and stock markets of the G-7 countries via five wavelet scales. They showed that hedging ratios and optimal weights vary across scales, whereby portfolio investors should hold less stocks than crude oil. They asserted that this may be due to the fact that stock prices of the G-7 markets are more volatile than WTI oil prices.

Recent studies such as Aloui and Hamida (2015), Christensen et al. (2010), Chkili, Hammoudeh and Nguyen (2014) reported that long memory and asymmetry properties are important stylized facts which need to be accounted for when modeling and forecasting the conditional volatility of both stock and commodity markets. However, very few academic papers recognized this issue in relation with hedging and risk minimizing strategies. For instance, Chkili, Aloui and Nguyen (2014) used the DCC–FIAPARCH model to examine the time-varying properties of conditional return and volatility of crude oil and US stock markets, with the purpose to design an optimal portfolio. They compared hedging performances of standard DCC-GARCH and long memory DCC-FIAPARCH models and concluded that the latter model enables investors to hedge the risk of their stock portfolios more effectively, and with lower costs. The study of Mensi, Hammoudeh and Kang (2015) examined the time-varying linkages between Saudi Arabian stock market and major commodity futures markets, and drew implications for portfolio risk management. For the purpose, they considered the bivariate DCC–FIAPARCH model with and without structural breaks. Their empirical results reveal evidence of asymmetry and long memory in the conditional volatility and find strong evidence of diversification benefits, hedging effectiveness and downside risk reductions. Also, they concluded that combining long memory GARCH processes with structural breaks can help investors to construct portfolios with lower comparing to the other portfolios created without structural beaks insertion in FIAPARCH model.

3. Methodology

3.1 Dynamic Conditional Correlation Framework

This section explains the DCC methodology used to calculate dynamic conditional volatility and dynamic conditional correlation in the selected asset series, in order to determine optimal weights and hedge effectiveness indices. In addition to the well-known clustering phenomenon and leverage effect in the volatility of financial time series, Baillie et al. (2007) asserted that common characteristic of daily asset returns is presence of persistent autocorrelation in their volatility. They contended that volatility persistence appears because of the long memory (LM) in variance that occurs due to the very slow decay of squared daily autocorrelation coefficients. Due to the fact that methodologies as ordinary GARCH and integrated GARCH are not consistent with long memory problem, Baillie et al. (1996) extended the standard GARCH model with a fractionally integrated process. In order to take into account all these stylized facts, we utilized independent bivariate DCC model considering several univariate GARCH models in the DCC framework, i.e. GARCH, APARCH and FIAPARCH. Particularly, the mean has the same AR(1) form for all GARCH models as suggested by equation (1), while the variance equations take three specifications depending on whether the leverage effect and LM is considered. The simple GARCH model is specified as follows:

$$y_t = C + \phi y_{t+1} + \varepsilon_t; \qquad \varepsilon_t \sim z_t \sqrt{\sigma_t^2} \tag{1}$$

$$\sigma_t^2 = c + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2, \tag{2}$$

where C and c are constants in mean and variance equations, Φ is autoregressive parameter, $y = \left[y_t^{SPY}, y_t^{commodity}\right]'$ represents 2×1 vector of SPY-ETF returns and commodity returns, while $\varepsilon_t = \left[\varepsilon_t^{SPY}, \varepsilon_t^{commodity}\right]'$ is 2×1 vector of error terms and symbol z_t represents an independently and identically distributed process, i.e. $z_t \sim N(0,1)$. All series are observed as log returns, i.e. $r_{i,t} = 100 \times \log(P_{i,t} / P_{i,t-1})$. Parameter β captures the persistence of volatility and α gauges an ARCH effect.

The presence of asymmetric effect, without taking into account LM phenomenon, is measured via APARCH of Ding et al. (1993) that is specified as follows:

$$\sigma_t^{\delta} = c + \alpha(|\varepsilon_t| - \mu\varepsilon_t)^{\delta} + \beta\sigma_t^{\delta}, \tag{3}$$

where parameter μ is the leverage coefficient and μ > 0 implies that negative shocks affect volatility more than positive shocks and *vice-versa*. Parameter δ is the power term parameter, and it takes finite positive values. Parameters μ and δ have the following limitations: −1 < μ < 1 and δ > 0.

Methodologies as ordinary GARCH and integrated GARCH are not consistent with long memory problem, thus we refer to Tse (1998) and estimate FIAPARCH model that is capable to capture both the long memory and the asymmetric processes.

$$\sigma_t^{\delta} = c + \left(1 - (1 - \beta(L))^{-1}\alpha(L)(1-L)^d\right)(|\varepsilon_t| - \mu\varepsilon_t)^{\delta}, \tag{4}$$

where d parameter lies within 0 < *d* < 1, which means that FIAPARCH(1, d, 1) model allows an intermediate range of persistence. Also, it nests the other GARCH-type models, meaning that it is equivalent to FIGARCH model when δ = 2 and μ = 0, while it reduces to APARCH model when d = 0.

In order to estimate dynamic conditional correlations, we employ multivariate DCC model by Engle (2002), which comprises two-stage estimation procedure of the conditional covariance matrix Σ_t. Firstly, for each pair of selected time-series a univariate GARCH, APARCH and FIAPARCH model is fitted and estimates of $\sqrt{\sigma_{ii,t}^2}$ are acquired. In second step, asset-return residuals are standardized, i.e. $v_{i,t} = \varepsilon_{i,t} / \sqrt{\sigma_{ii,t}^2}$ wherein the $v_{i,t}$ is then used to estimate the parameters of the conditional correlation. Accordingly, the multivariate conditional variance is specified as: $\Sigma_t = D_t C_t D_t$. Where $D_t = diag(\sqrt{\sigma_{11,t}^2} \cdots \sqrt{\sigma_{nn,t}^2})$ and $\sigma_{ii,t}^2$ represents the conditional variance, which is obtained from some form of a univariate GARCH model in the first stage. The evolution of correlation in the DCC model is presented as:

$$Q_t = (1-a-b)\bar{Q} + \alpha v_{t-1} v_{t-1}' + \beta Q_{t-1}, \tag{5}$$

where *a* and *b* are nonnegative scalar parameters under condition *a + b < 1*; $Q_t = (q_{ij,t})$ is n × n time-varying covariance matrix of residuals, while $\bar{Q} = E\left[v_t v_t'\right]$ stands for n × n time-invariant variance matrix of v_t. Since Q_t does not have unit elements on the diagonal, it is scaled to obtain proper correlation matrix (C_t) according to following form: $C_t = (diag(Q_t))^{-1/2} Q_t (diag(Q_t))^{-1/2}$. Accordingly, the element of C_t looks like:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}} = \frac{(1-a-b)\bar{q}_{ij} + a v_{i,t-1} v_{j,t-1} + b q_{ij,t-1}}{\sqrt{\left[(1-a-b)\bar{q}_{ii} + a v_{i,t-1}^2 + b q_{ii,t-1}\right]}\sqrt{\left[(1-a-b)\bar{q}_{jj} + a v_{j,t-1}^2 + b q_{jj,t-1}\right]}}, \tag{6}$$

where i ≠ j, and in our bivariate model, n is equal to 2. All DCC models were estimated by quasi maximum likelihood (QMLE) technique. This procedure allows asymptotically consistent parameter estimates even if the underlying distribution is not normal, as asserted by Bollerslev and Wooldridge (1992). In addition, we estimate all bivariate DCC models with both multivariate normal and multivariate Student-t density functions.

3.2 Modified ICSS Algorithm

Since we consider relatively broad time-span, which comprises the periods of intense market straining, i.e. the Global financial crisis and the following Sovereign debt crisis, we decided to split full sample into three subsamples and gauge portfolio hedge effectiveness across periods of different market turbulence. In order to do that, we refer to Mensi et al. (2016) who utilized modified ICSS algorithm of Sans´o *et al*. (2004) to detect exact date of the GFC outbreak and to divide full sample into two subsamples. This particular methodology resolves the problem of oversized break detection, which is the characteristic of basic ICSS algorithm of Inclan and Tiao (1994), by explicitly taking into account the fourth moment properties of the time series. The Modified Inclan and Tiao (MIT) empirical statistics, using a non-parametric adjustment based on Bartlett and Kernel, is presented by:

$$MIT = \sup_k \left| T^{-0.5} G_k \right| , \tag{7}$$

where $G_k = \hat{\lambda}^{-0.5}\left[C_k - (k/T)C_T\right]; \hat{\lambda} = \hat{\gamma}_0 + 2\sum_{l=1}^{m}\left[1 - l(m+1)^{-1}\right]\hat{\gamma}_l; \hat{\gamma}_l = T^{-1}$ $\sum_{t=l+1}^{T}(\tau_t^2 - \hat{\sigma}^2)(\tau_{t-1}^2 - \hat{\sigma}^2)$; $\hat{\sigma}^2 = T^{-1}C_T$.

Referring to the procedure of Newey and West (1994), we set the lag truncation parameter to be $m = 0.75T^{1/3}$. The asymptotic distribution of the MIT statistics under general conditions is given by $\sup_l \left|W^*(l)\right|$ and the 95th percentile critical value for the asymptotic distribution of MIT statistics is 1.4058.

3.3 Portfolio Construction and Hedging Effectiveness Measurement

This section briefly presents the way in which two-asset portfolios were constructed by using the conditional correlations and the conditional variances, obtained from various DCC models. In order to evaluate risk-reduction performances of the portfolios we used three portfolio optimization indicators – the minimum-variance hedge effectiveness index, VaR and CVaR effectiveness metrics that assess the tail risk of the hedge portfolio. In order to build a portfolio that minimizes risk without lowering expected returns, we constructed portfolios in which SPY index represents the basic asset and the commodity indices serve as auxiliary asset. Referring to Kroner and Ng (1998), the optimal portfolio weight, with the following restrictions, is calculated as in equations (8) and (9).

$$W_t^{SPY,COM} = \frac{\sigma_t^{2\,(SPY)} - \sigma_t^{2\,(SPY,COM)}}{\sigma_t^{2\,(SPY)} - 2\sigma_t^{2\,(SPY,COM)} + \sigma_t^{2\,(COM)}}, \tag{8}$$

$$W_t^{SPY,COM} = \begin{cases} 0, & if \quad W_t^{SPY,COM} < 0 \\ W_t^{SPY,COM}, & if \quad 0 < W_t^{SPY,COM} < 1, \\ 1, & if \quad W_t^{SPY,COM} > 1 \end{cases} \tag{9}$$

where $W_t^{SPY,COM}$ denotes the weight of particular commodity (COM) in a 1$ portfolio of a two-asset holding at time t. The labels $\sigma_t^{2\,(SPY)}$ and $\sigma_t^{2\,(COM)}$ refer to conditional variances of the index and selected commodity (COM), respectively. Symbol $\sigma_t^{2\,(SPY,COM)}$ signifies the conditional covariance between the returns of and commodity, at time t. The weight of the index in the considered 1$ portfolio is $(1 - W_t^{SPY,COM})$.

The hedging effectiveness performances are gauged via variance reduction method, which incorporates both upside and downside risk, assigning an equal weight to positive and negative returns. The variance hedging effectiveness index (HEI) analyses the realized hedging errors, implying that hedging effectiveness of particular portfolio is greater when the HEI is higher (closer to 100), and *vice-versa*. Following Arouri et al. (2011), the HEI_{Var} is expressed as:

$$HEI_{Var} = \frac{Var_{unhedged} - Var_{hedged}}{Var_{unhedged}} \times 100 , \tag{10}$$

where $Var_{unhedged}$ indicates the variance of the unhedged portfolio, which is composed of only SPY, and Var_{hedged} indicates the variance of the portfolio which is composed of the SPY index and some commodity, accordingly to the previously calculated optimal weights.

However, the variance measures only the second moment of the returns distribution and cannot differentiate between positive and negative returns, while some investors rather prefer to know the tail risk of the hedged portfolio. In addition, minimum-variance portfolio can simultaneously increase skewness and kurtosis, and so the overall effect on the quantile of cumulative distribution function (CDF) is uncertain. Therefore, referring to Cao et al. (2009), Cotter and Hanly (2006), Harris and Shen (2006), we address higher moments of portfolio returns and calculate VaR hedge effectiveness index, which measures the effect of hedging on negative tail returns. Due to space parsimony, we tested only the hedging performance of the negative tail of the returns at the 95% confidence level. Thus, the VaR of the portfolio for the long (buy) trading positions at confidence level α looks like:

$$VaR_p = \hat{\mu} + q_p(\alpha)\hat{\sigma}_p , \tag{11}$$

where $q_p(\alpha)$ designates the left quantile at α% of the CDF of portfolio returns, while $\hat{\mu}$ and $\hat{\sigma}_p$ refer to the estimated mean and standard deviation of the particular commodity portfolio. The effectiveness from the point of VaR reduction can be measured as:

$$HEI_{VaR} = \frac{VaR_{unhedged} - VaR_{hedged}}{VaR_{unhedged}} \times 100 , \tag{12}$$

where $VaR_{unhedged}$ and VaR_{hedged} indicate to the VaR of the S&P500 index and the VaR of the portfolio with the selected commodities, respectively.

However, one potentially serious drawback of VaR metric is that it does not consider the expected size of a loss in the event that this loss exceeds the VaR of the portfolio. Particularly, Value-at-Risk is not a coherent measure of risk if portfolio returns are not drawn from a multivariate elliptical distribution, meaning that it could be possible for the VaR of a portfolio to exceed the weighted average VaR of the assets that it involves. Therefore, in order to overcome this VaR shortcoming, we applied an alternative measure of risk – Conditional Value-at-risk (CVaR) that measures the mean loss, conditional upon the fact that the VaR has been exceeded. Alexander and Baptista (2004) showed that, in some cases, the use of CVaR as a measure to control risk is more effective than the use of VaR. Expression of CVaR is given as:

$$CVaR_p = \frac{1}{\alpha}\int_{1-\alpha}^{1} VaR(x)dx = -\frac{\sigma_p}{\alpha}\int_{1-\alpha}^{1} q_p^x dx\,. \tag{13}$$

In the process of CVaR calculation, we used the probability of occurrence, q=5%, to examine the position for different types of hedgers. Accordingly, the performance metric utilized to assess the hedging effectiveness is the percentage reduction in CVaR is as follows:

$$HEI_{CVaR} = \frac{CVaR_{unhedged} - CVaR_{hedged}}{CVaR_{unhedged}} \times 100\,, \tag{14}$$

where $CVaR_{unhedged}$ and $CVaR_{hedged}$ indicate to the CVaR of the SPY index and the CVaR of the portfolio with the selected commodities, respectively.

4. Data and Summary Statistics

The data set used for the portfolio construction comprises daily commodity prices of Brent futures, gold spot, silver spot and platinum spot and daily exchange traded fund (ETF) of S&P500 index, i.e. the S&P500 SPDR, the ticker symbol SPY ('Spider'). The Spider is far the largest passive ETF, in which the share price corresponds to 1/10th of the S&P500 index value. All data series are calculated as continuously compounded daily returns. For calculations of an optimal portfolio we combine SPY-ETF as a primary asset and four commodity series as an auxiliary asset. All series were obtained from Yahoo finance and Quandl.com. The data span ranges from January 1, 2001 to December 31, 2016 for all corresponding asset series. Taking into account the unavailability of some data due to non-working days in observed commodity and financial markets, all pairs of time-series are synchronized according to the existing observations. The concise summary statistics that contains first four moments, JB and LB tests as well as two unit-root tests are presented in Table 1. Table 2 discloses the average levels of Pearson's unconditional correlations between SPY-ETF and four commodities.

The basic statistics reveals that all commodities except platinum have higher average return than SPY. The skewness and kurtosis results, along with the Jarque–Bera test for normality, imply that the daily returns are more left-asymmetric, fat-tailed and high-peaked than the Gaussian distribution. These results justify the usage of the various GARCH type models. The presence of serial correlation and heteroscedasticity is tested by Ljung-Box Q-statistics for level and squared residuals, suggesting that some form of ARMA-GARCH parameterization might be appropriate. In addition, spurious regression is evaded since DF-GLS test suggests that all selected series do not contain unit root, while KPSS test proposes that all series are stationary.

Table 1 Summary Statistics

	Mean	*St. dev*	*Skew.*	*Kurt.*	*JB*	*LB(Q)*	*LB(Q²)*	*DF-GLS*	*KPSS*
SPY	0.014	1.243	-0.054	14.048	19795	0.000	0.000	-2.462	0.175
Brent	0.026	2.192	-0.089	6.105	1566	0.000	0.000	-6.823	0.153
Gold	0.033	1.168	-0.334	7.844	3877	0.069	0.000	-7.361	0.329
Silver	0.018	2.156	-0.566	12.194	13917	0.000	0.000	-11.866	0.263
Platinum	0.009	1.488	-0.553	13.706	18784	0.000	0.000	-16.579	0.197

Notes: JB stands for values of Jarque-Bera coefficients of normality, LB(Q) and LB(Q^2) tests denote p-values of Ljung-Box Q-statistics of level and squared residuals for 20 lags. 1% and 5% critical values for DF-GLS test with 10 lags assuming only constant are -2.566 and -1.941, respectively. 1% and 5% critical values for KPSS test are 0.739 and 0.463, respectively.
Source: Authors' calculation.

The preliminary examination of LM properties of returns and squared returns (as a proxy variable of volatility) is carried out via the Hurst–Mandelbrot R/S test, Lo's modified R/S test and the Gaussian semi-parametric (GSP) test of Robinson and Henry (1999). The results are disclosed in Table 2. For the return series, all tests reject the evidence of the LM property, which is expected since the efficient market hypothesis (EMH) categorically contends that knowledge of past events cannot helps in the prediction of the future outcomes. On the other hand, the LM property is found to be highly significant (at the 1% level of significance) by all tests and for all squared return series. These findings suggest that the squared returns may be modelled by a fractionally integrated model, which justifies the usage of the FIAPARCH specification in the DCC framework.

Table 2 Long Memory Tests for Returns and Squared Returns of SPY and Selected Commodities

	Returns					***Squared Returns***				
	SPY	*Brent*	*Gold*	*Silver*	*Platinum*	*SPY*	*Brent*	*Gold*	*Silver*	*Platinum*
Hurst-Mandelbrot R/S test statistics										
Value	1.209	1.539	1.491	1.404	1.644	4.813[a]	4.879[a]	4.834[a]	4.903[a]	4.041[a]
Lo's R/S test statistics										
q=1	1.252	1.587	1.496	1.472	1.654	4.442[a]	4.524[a]	4.645[a]	4.308[a]	3.617[a]
q=5	1.347	1.584	1.496	1.522	1.669	3.132[a]	3.527[a]	3.927[a]	3.511[a]	2.984[a]
Gaussian semi-parametric (GSP) test										
m=T/4	-0.044	0.030	-0.008	-0.019	0.015	0.333[a]	0.248[a]	0.196[a]	0.212[a]	0.180[a]
m=T/8	-0.049	0.031	0.001	-0.006	0.007	0.415[a]	0.332	0.258[a]	0.231[a]	0.195[a]

Notes: The critical values of the Hurst–Mandelbrot R/S test and Lo's R/S analysis are 2.098 at the 1% significance level. T is the sample size and label "m" denotes the bandwidth for the GSP test. Symbol "a" denotes 1% significance level.

5. Empirical results

5.1 Results of DCC models

This section succinctly presents the results of estimated DCC models with three different univariate specifications – GARCH, APARCH and FIAPARCH. Due to space brevity, we reveal only DCC-FIAPARCH point estimates in Table 3[3]. The parameter estimates of other two models are very similar, since GARCH and APARCH models are nested in FIAPARCH model, so there is no need to display their results. Results show that all asymmetric parameters (μ) are positive and highly statistically significant in SPY-ETF marker, which means that negative shocks cause

[3] The results of the other two models are available on request.

stronger change in volatility than positive shocks. This also applies for Brent oil, while in silver market positive shocks have greater impact on volatility than negative shocks. μ parameters in gold and platinum markets are statistically insignificant. The fractionally differencing parameters (d) lay within range $0 < d < 1$, whereby commodity markets have slightly higher values of d parameter comparing to SPY, which indicates higher volatility persistence and lower market efficiency. In order to confirm the models' accuracy, we performed LB test on level and squared residuals, and the obtained results indicate that the hypothesis of no serial correlation and heteroscedasticity should be accepted for all asset return series.

Table 3 Results of AR(1)-FIAPARCH(1, d,1) – DCC Estimates

	SPY – Brent		***SPY – Gold***		***SPY – Silver***		***SPY – Platina***	
Panel A: SPY estimates of variance equation								
ω	0.034*		0.034*		0.034*		0.034*	
α	0.251***		0.251***		0.251***		0.251***	
β	0.560***		0.560***		0.560***		0.560***	
μ	0.922***		0.922***		0.922***		0.922***	
δ	1.423***		1.423***		1.423***		1.423***	
d	0.375***		0.375***		0.375***		0.375***	
Diagnostic tests								
LB(Q)_20	25.22		14.92		15.24		15.88	
LB(Q^2)_20	17.24		17.81		18.25		18.30	
Panel B: Commodity estimates of variance equation								
ω	0.028		0.036**		0.020		0.039**	
α	0.265***		0.233**		0.345***		0.250**	
β	0.674***		0.608***		0.682***		0.527***	
μ	0.198***		-0.102		-0.112***		0.058	
δ	2.099***		2.285***		2.272***		2.139***	
d	0.489***		0.428		0.430***		0.427***	
Diagnostic tests								
LB(Q)_20	21.45		19.04		18.68		21.16	
LB(Q^2)_20	19.97		7.39		11.06		22.28	
Panel C: DCC parameters								
	Normal	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**
a	0.026***	0.023***	0.014***	0.014***	0.009***	0.007***	0.006***	0.007***
b	0.972***	0.974***	0.980***	0.976***	0.985***	0.985***	0.990***	0.989***
St	–	9.203***	–	6.872***	–	7.071***	–	8.429***

Notes: LB(Q) and LB(Q^2) test denote p-values of Ljung-Box Q-statistics for level and squared residuals for 20 lags. Parameter S_t denotes the degrees of freedom, measuring the degree of fat-tails of the residuals density. ***, **, * represent statistical significance at the 1%, 5% and 10% level, respectively.
Source: Authors' calculation.

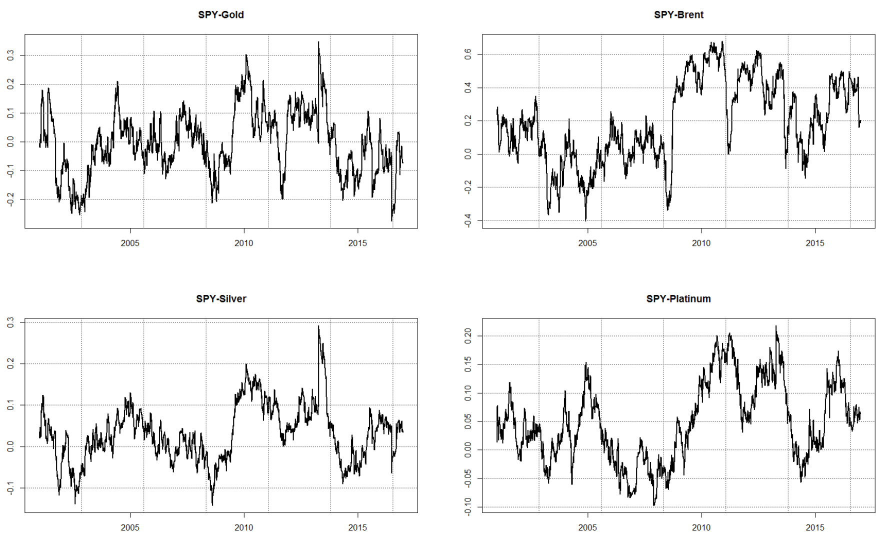
All univariate FIAPARCH models are estimated with univariate normal distribution, while DCC parameters are estimated with multivariate normal and Student t distributions, and these results are presented in Panel C. Estimates of the multivariate DCC models (*a* and *b*) are statistically significant and nonnegative in all cases, also satisfying the condition $a + b < 1$. The highly significant parameter of Student t distribution (S_t) confirms the adequacy of this distribution. In order to show which model and with which density function fits the best, we present log likelihood values and three information criteria – AIC, SIC and HQIC in Table 4. The results disclose that all LL values are the highest for DCC-FIAPARCH model with multivariate Student t distribution, while all information criteria are the lowest for the same model. According to these values, DCC-APARCH is the second-best model. These findings undoubtedly indicate high statistical significance of long memory and asymmetry in commodity and SPY series. Since DCC-FIAPARCH model with the multivariate Student t distribution is the best-fitting for all market-pairs. Figure 1 presents time varying DCC plots estimated with this model, while Table 5 contains average full sample Pearson's correlations.

Table 4 Log-likelihood and Three Information Criteria for all DCC Models

Panel A: Information criteria for DCC-GARCH models								
	SPY – Brent		*SPY – Gold*		*SPY – Silver*		*SPY – Platinum*	
	Normal	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**
LL	-13360	-13192	-11288	-11016	-13486	-13221	-11996	-11803
AIC	6.881	6.796	5.806	5.667	6.936	6.800	6.170	6.071
SIC	6.889	6.815	5.823	5.868	6.953	6.820	6.188	6.090
HQIC	6.888	6.803	5.813	5.674	6.942	6.807	6.177	6.078
Panel B: Information criteria for DCC-APARCH models								
	Normal	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**
LL	-13227	-13116	-11218	-10944	-13414	-13145	-11921	-11729
AIC	6.841	6.759	5.772	5.623	6.901	6.763	6.134	6.036
SIC	6.866	6.785	5.797	5.658	6.924	6.789	6.158	6.061
HQIC	6.850	6.768	5.781	5.641	6.909	6.772	6.142	6.045
Panel C: Information criteria for DCC-FIAPARCH models								
	Normal	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**
LL	-13217	-13095	-11162	-10926	-13332	-13112	-11857	-11629
AIC	6.811	6.749	5.745	5.623	6.860	6.748	6.102	6.017
SIC	6.839	6.778	5.772	5.653	6.888	6.776	6.129	6.046
HQIC	6.821	6.759	5.754	5.634	6.869	6.758	6.112	6.027

Notes: LL stands for log-likelihood, AIK, SIC and HQIC denote Akaike, Schwarz and Hanna-Quinn information criteria, respectively.
Source: Authors' calculation.

Figure 1 DCC-FIAPARCH Plots of SPY and Four Commodities



Source: Authors' calculation.

Table 5 Pearson's unconditional correlation between SPY-ETF and the commodities

	SPY-Brent oil	*SPY-Gold*	*SPY-Silver*	*SPY-Platinum*
Pearson's correlation	0.236*	-0.040*	0.004*	0.045*

Notes: Asterisk denotes 1% statistical significance.

From Table 5 results, we can see that average Pearson's correlations are either very small or even negative (in case of gold), which makes these commodity instruments suitable for the diversification with SPY. More specifically, Figure 1 reveals that Pearson's correlations frequently take negative values, which is a necessary prerequisite to say that these commodity assets are also favourable instruments for hedging in combination with SPY. In particular, we find negative correlations during Iraqi war in cases of all SPY-commodity pairs, while during GFC it is more conspicuous for precious metals and less for Brent oil. In addition, negative correlations of SPY-precious metal pairs appear evident in period 2014-2015. These results justify the usage of selected commodities for the hedging purposes, i.e. our choice of commodities is based on the negative correlation findings.

Numerous researchers reached similar conclusion when it comes to the selection of hedging instruments. For instance, Gorton and Rouwenhorst (2006) and Erb and Harvey (2006) claimed that commodities in general, and particularly precious metals, could have significant role in risk hedging of portfolios with stocks, due to their negative correlation. McCown and Zimmerman (2007) found that silver moves in opposite direction in relation with stock markets on the long-run, which stands as an argument in support of silver's usage as a hedging tool. In the study of Belousova and Doreitner (2012), authors contended that adding silver or platinum to a portfolio with stocks during bull markets reduces volatility and enhances return, thus these commodities can serve as suitable hedging instruments. Hillier et al. (2006) asserted that gold, silver and platinum have good hedging capabilities, particularly in periods of ´abnormal´ stock market volatility, during which portfolios with these precious metals perform significantly better than standard equity portfolios. Besides, Basher and Sadorsky (2016) concluded that oil provides the most effective hedge for emerging market stock prices.

5.2 Complementary Analysis via Rolling Regression

As an additional overview, this section deals with the exploration of the dynamic nexus between conditional correlation and conditional volatilities, which some international investors may find interesting. Since previous section revealed that DCC-FIAPARCH with Student t distribution is the best model for all examined asset-pairs, we employ conditional correlations and conditional volatilities from this model as inputs in the estimation of a linear rolling stepwise regression. In this way, it is possible to discover which variance factor has dominant influence on the common correlation throughout the time. The idea for such approach was borrowed from the papers of Syllignakis and Kouretas (2011), Moore and Wang (2014) and Živkov et al. (2016). The size of the rolling window is set to be one year, which is approximately 250 daily observations. Linear interconnection between conditional correlation and conditional volatilities is described by the following regression:

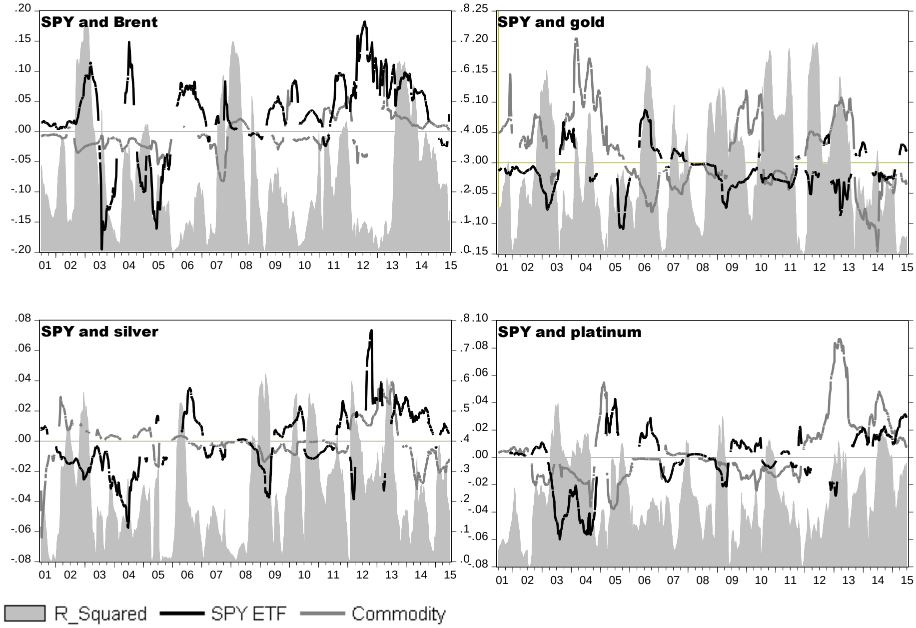
$$\rho_{ij,t} = C_{ij} + \omega_i \sigma^2_{i,t} + \varpi_j \sigma^2_{j,t} + \varepsilon_{ij,t} \,, \tag{15}$$

where ρ_{ij} depicts the estimated pair-wise conditional correlation coefficients between SPY and the selected commodities, where *i* stands for SPY and *j* labels the particular commodity. $\sigma^2_{i,t}$ and $\sigma^2_{j,t}$ are conditional volatilities of corresponding assets, while ω and ϖ are sequentially calculated parameters in each observing window. ε is the common white noise error term. In order to overcome possible spurious regression, we used a GLS approach to correct standard errors for autocorrelation and heteroscedasticity (see MacKinnon and White, 1985). The equation 15 can be criticized as overly restricted, but it was intentionally set up in that way because we solely tried to measure the isolated effects of the conditional volatilities on the conditional correlation. The Figure 2 presents statistically significant plotted rolling point estimates of the conditional volatilities, along with rolling R^2 results for each of the four pairs examined.

The evidence suggests that majority of slope coefficients are statistically significant, taking both positive and negative values. The general rule is that when conditional correlations are above zero, the positive values of ω and ϖ imply that DCCs rise when volatilities increase. In the case of negative DCCs, the negative values of ω and ϖ mean that DCCs rise when volatilities increase. It can be seen that SPY parameters have a higher impact on dynamic correlation throughout the observed sample in Brent and silver cases, while in some instances gold and platinum had much higher influence on DCCs. For example, in case of SPY-Brent, it is obvious that increased SPY volatility in 2003 and 2005 caused higher negative dynamic correlation, while in period 2012-2014, increased SPY volatility resulted in higher positive DCCs. In case of SPY-gold, it can be noticed that higher gold volatility influenced higher positive DCCs around 2004, 2010 and 2012, while in 2015 higher gold volatility was responsible for rise in negative DCCs. In case of SPY-silver, increased SPY volatility caused rise in negative correlation during 2004, while in 2012 it induced rise in positive correlation. On the other hand, in case of SPY-platinum, DCCs are determined in great measure by platinum volatility in 2013.

The explanatory power (R^2) of the specified model fluctuates considerably across time with an average value of 24.5%, 28.5%, 23% and 25.1% for Brent oil, gold, silver and platinum, respectively. Taking into account that the specification of rolling regression involves only two regressors per country, determination coefficients are relatively high. In addition, one can recognize similar patterns across selected pairs of assets in the sense that on average R^2 coefficient rises in the crisis period, which means that assets' conditional volatilities play an important role in the determination of conditional correlation in the periods of increased market stress. This is particularly noticeable in the SPY-Brent case, when R^2 coefficients went even beyond 70% in periods of Iraqi crisis and GFC.

Figure 2 Stepwise Regression Analysis



Notes: Left Y axis displays the values of conditional volatility parameters and right Y axis displays the values of R^2. The estimated rolling parameters are considered significant only if their p-values are lower than 10%.

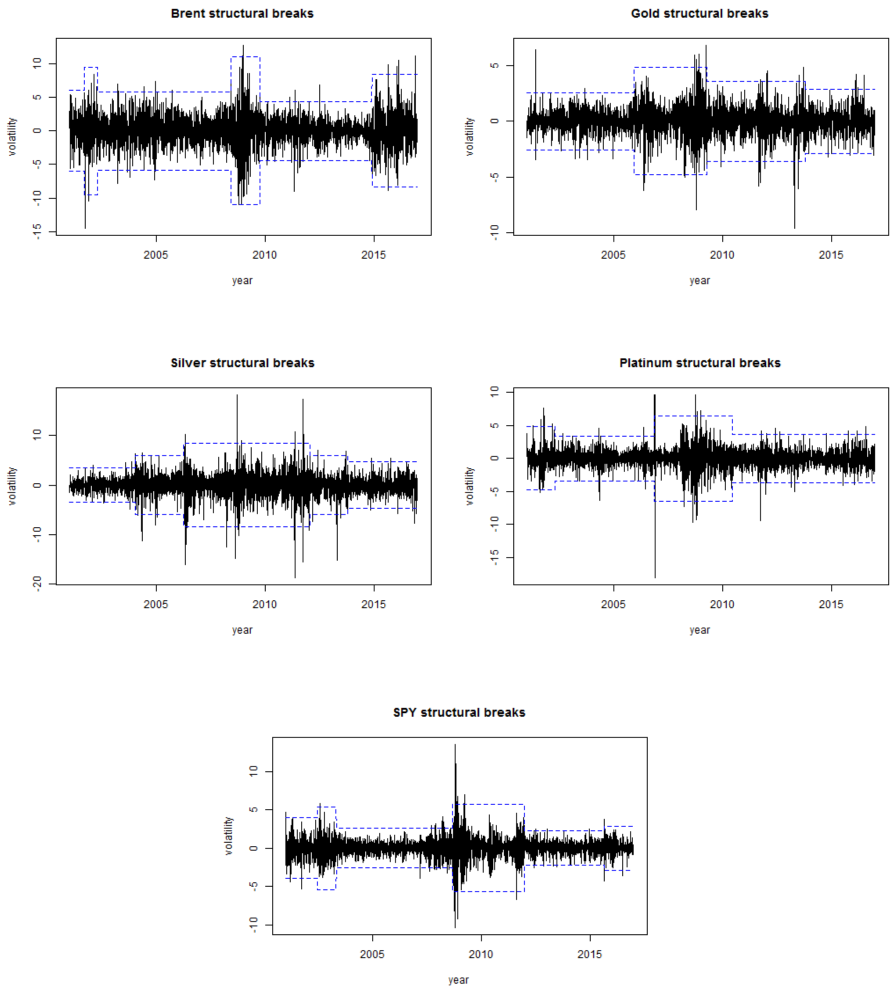
6. Empirical findings

6.1 Results of three hedge effectiveness measures

This Section evaluates overall in-sample portfolio performance in terms of various risk measures, while Sections 6.2 and 6.3 additionally consider portfolios' return-performances and out-of-sample HEI results, respectively. In particular, we gauge hedge effectiveness of the risk-reducing portfolios that are composed via different DCC models – simple DCC-GARCH, DCC-APARCH that recognizes asymmetry in the variances, and DCC-FIAPARCH that takes into account variance asymmetry as well as long memory in variance. The objective is to determine whether the long memory in variance plays an important role in the composition of risk-minimizing portfolio, and how minimum-variance hedging portfolio influences a reduction in portfolio VaR and CVaR. Three indicators are used for the purpose – HEI_{Var} that gives an equal weight to positive and negative returns, HEI_{VaR} that takes into account higher moments of the distribution and so improves upon the variance and HEI_{CVaR} that gauge the mean loss, conditional upon the fact that the VaR has been exceeded. Besides, referring to GFC and SDC as one comprehensive period of increased market turmoil, since they happened consequently one after another, we endeavour to see how risk-reducing characteristics of assembled portfolios alter if we observe different subsamples. The idea came from the study of Arouri et al. (2015), who investigated separately the level of HEI values during the GFC period. In that manner, besides full sample, we calculate HEI_{Var}, HEI_{VaR} and HEI_{CVaR} for three sub-periods – before, during and after GFC and SDC.

Since we analyse five asset-markets and every market reported its own distinctive break dates that was caused due to various political, social and economic events, our focus is on the structural breaks that occurred in the dynamics of the SPY returns because this index appropriately reflects all major global happenings.

Figure 3 Daily Returns and Detected Breaks for SPY and Selected Commodities



Notes: Doted lines denote bands of ±3 standard deviations, where change points are estimated by modified ICSS algorithm.
Source: Authors' calculation.

In order to avoid arbitrariness in the determination of the sub-periods, we followed Mensi et al. (2016) who separated their full sample with the use of the modified ISCC algorithm of Sans´o *et al*. (2004). Utilizing the same technique, we define the exact break points around GFC and SDC in the SPY returns, and accordingly we divide full sample of every portfolio into three subsamples. The detected break dates around GFC and SDC in the SPY returns are September 3, 2008 and December 20, 2011. This period comprises the outset of the GFC (Lehman brothers declared bankruptcy at September 15, 2008), the European sovereign debt crisis in 2010 and the US sovereign debt crisis in 2011. Figure 3 displays the return behaviour of SPY and four selected commodities, and depicts the points of sudden changes detected by the modified ICSS algorithm. It can be seen that above mentioned events are clearly visible in SPY returns as distinct patches of volatility clusters. Also, the number of breaks vary between three and five in all observed series, and the probable reasons for an erratic market behaviour lays in the various local, regional and especially global events such as: the 2001 terrorist attack, the Iraqi war, 2008–2009 GFC, the Greek and US sovereign debt crisis, the Arab Spring, the Ukrainian crisis, dramatic oil price drop etc.

Table 6 Calculated HEI$_{Var}$ Values for the Full Sample and Three Subsamples

	DCC-GARCH		***DCC-APARCH***		***DCC-FIAPARCH***	
	Normal	*Student-t*	*Normal*	*Student-t*	*Normal*	*Student-t*
Panel A: Full sample: January 1 2001 – December 31 2016						
SPY – Brent oil	0.1609	0.1608	0.1744	0.1745	0.1712	0.1711
SPY – Gold	0.6005	0.6007	0.6058	0.6059	0.6050	0.6051
SPY – Silver	0.3867	0.3866	0.3954	0.3954	0.3935	0.3934
SPY – Platinum	0.4452	0.4456	0.4550	0.4555	0.4629	0.4633
Panel B: First subsample: January 1 2001 – September 3 2008						
SPY – Brent oil	0.3100	0.3099	0.3118	0.3119	0.3053	0.3044
SPY – Gold	0.6303	0.6304	0.6361	0.6363	0.6342	0.6344
SPY – Silver	0.4812	0.4813	0.4893	0.4895	0.4836	0.4839
SPY – Platinum	0.4929	0.4932	0.4955	0.4958	0.4973	0.4978
Panel C: Second subsample: September 3 2008 – December 20 2011						
SPY – Brent oil	0.0869	0.0870	0.1098	0.1098	0.1088	0.1090
SPY – Gold	0.6377	0.6378	0.6385	0.6387	0.6349	0.6350
SPY – Silver	0.3784	0.3781	0.3868	0.3867	0.3830	0.3825
SPY – Platinum	*0.4637*	*0.4641*	*0.4746*	0.4751	0.4869	0.4873
Panel D: Third subsample: December 20 2011 – December 31 2016						
SPY – Brent oil	-0.0001	0.0092	0.0232	0.0232	0.0215	0.0217
SPY – Gold	0.3742	0.3746	0.3944	0.3946	0.4071	0.4072
SPY – Silver	0.1418	0.1416	0.1538	0.1537	0.1700	0.1698
SPY – Platinum	0.2397	0.2402	0.2675	0.2679	0.2762	0.2763

Notes: Greyed values indicate the highest HEI$_{VaR}$.
Source: Authors' calculation.

Table 6 reports in-sample HEI$_{Var}$ results for portfolios assembled with inputs from different bivariate DCC models. The models were estimated with two different multivariate distribution functions, observing full sample as well as three subsamples. Regarding the full sample results, it can be noticed that portfolios with gold have the best variance-reducing performances, with substantially higher indices comparing to the second-best platinum portfolios. These findings are consistent with some previous studies like those of Arouri et al. (2015), Chkili (2016) and Basher and Sadorsky (2016) who documented that the inclusion of gold in a portfolio of stocks increases hedge effectiveness index considerably. Also, it should be mentioned that gold has the lowest risk of all examined assets, and as Khalfaoui et al. (2015) contended, the instrument with the lowest risk makes its presence in a two-asset portfolio very desirable. Although DCC-FIAPARCH model has the best fitting characteristics for all pairs of assets, as we have seen in Section 5.1, it does not imply automatically that this model will yield best variance-minimizing features for portfolios. Observing full sample, it is evident that in three out of four cases DCC-APARCH portfolios are slightly better in terms of variance reduction than DCC-FIAPARCH portfolios, while ordinary DCC-GARCH portfolios give worst performances in all the cases. It is an indication that variance asymmetry always plays an important role when it comes to the construction of variance-reducing portfolio, while variance LM shows its significance occasionally.

As for the findings in different subsamples, the HEI_{Var} values are pretty heterogeneous in terms of the size, which justifies the full sample partition. However, in terms of best variance minimizing portfolio, all subsample portfolios resemble those of full sample, i.e. gold is the best auxiliary asset, platinum is the second-best and silver follows. In third subsample, all HEI_{Var} values diminished significantly.

Table 7 Calculated HEI_{VaR} Values for the Full Sample and the Three Subsamples

	DCC-GARCH		***DCC-APARCH***		***DCC-FIAPARCH***	
	Normal	*Student-t*	*Normal*	*Student-t*	*Normal*	*Student-t*
Panel A: Full sample: January 1 2001 – May 31 2016						
SPY – Brent oil	0.0797	0.0796	0.0864	0.0864	0.0853	0.0851
SPY – Gold	0.3763	0.3764	0.3787	0.3788	0.3774	0.3775
SPY – Silver	0.2190	0.2189	0.2220	0.2219	0.2205	0.2204
SPY – Platinum	0.2574	0.2576	0.2605	0.2608	0.2639	0.2642
Panel B: First subsample: January 1 2001 – September 3 2008						
SPY – Brent oil	0.1748	0.1747	0.1739	0.1740	0.1707	0.1699
SPY – Gold	0.4132	0.4132	0.4115	0.4116	0.4058	0.4060
SPY – Silver	0.2856	0.2857	0.2892	0.2894	0.2847	0.2849
SPY – Platinum	0.3009	0.3011	0.3000	0.3003	0.3012	0.3015
Panel C: Second subsample: September 3 2008 – December 20 2011						
SPY – Brent oil	0.0323	0.0324	0.0429	0.0430	0.0427	0.0428
SPY – Gold	0.4132	0.4132	0.4115	0.4116	0.4086	0.4086
SPY – Silver	0.2172	0.2171	0.2182	0.2182	0.2152	0.2150
SPY – Platinum	0.2657	0.2660	0.2683	0.2687	0.2736	0.2740
Panel D: Third subsample: December 20 2011 – May 31 2016						
SPY – Brent oil	-0.0120	-0.0123	-0.0025	-0.0023	-0.0020	-0.0019
SPY – Gold	0.1881	0.1883	0.2010	0.2010	0.2100	0.2099
SPY – Silver	0.0573	0.0570	0.0625	0.0622	0.0726	0.0722
SPY – Platinum	0.1088	0.1089	0.1208	0.1211	0.1220	0.1221

Notes: Greyed values indicate the highest HEI_{VaR}.
Source: Authors' calculation.

These findings are in line with the assertion that precious metals often serve as a safe haven for global investors during the crisis periods, which is especially true for the gold. In addition, HEI_{Var} results are consistent with the rolling regression findings in the Section 5.2, which indicates that gold prices move in reverse directions in regard to SPY instrument, particularly during the crisis periods, and thus it serves as suitable asset for portfolio diversification. However, the results in subsamples are indecisive when it comes to the question which DCC model is better in design of variance-minimizing portfolio. In first and second subsamples DCC-APARCH proved to be better in three out of four cases, but in third subsample DCC with long memory gained the upper hand also in three out of four cases. Besides, indicative finding is that multivariate Student-t distribution fits slightly better in terms of risk reduction for all asset pairs. However, these improvements do not have economic significance because they appear at third or fourth decimal.

As Harris and Shen (2006) contended, while minimum-variance hedging unambiguously reduces the standard deviation of portfolio returns, the effect on skewness and kurtosis could be ambiguous. Moreover, they showed that minimum-variance hedging can potentially increase left-skewness and kurtosis (which determine VaR), and the expectation of the returns that are less than or equal to this quantile (which determines CVaR), and thus the outcome for investors who seek minimum VaR or CVaR is uncertain. Therefore, in the following text we disclose whether minimum variance portfolio could offer similar VaR and CVaR hedging performances, and whether long memory DCC model plays an important part in the portfolio VaR and CVaR reduction. Tables 7 and 8 give an overview of the HEI_{VaR} and HEI_{CVaR} results for the full sample and the three subsamples.

It can be seen that HEI_{VaR} results in all cases are consistent with the HEI_{Var} findings when it comes to the selection of the most favourable VaR reducing portfolios, but also significantly lower than the corresponding minimum-variance hedge ratios, which happened probably due to increased portfolio kurtosis and skewness (see Harris and Shen, 2006). For instance, observing the full sample, the reduction in VaR, for SPY-gold and SPY-silver portfolios, on average is about 40% of the average reduction in variance. SPY-Brent combination in third subsample is the most extreme, since the HEI_{VaR} results are negative, which means that all portfolios constructed with different DCC models have inferior HEI_{VaR} performances comparing to the investment in the unhedged SPY asset. Third subsample was characterized by extreme oil price oscillations, which suggests that when third and fourth moment are increased, investors who target minimum VaR should avoid portfolios which combine SPY and Brent oil. On the other hand, this portfolio gave decent minimum VaR results to some extent only in the first subsample, since the average reduction in VaR is about 40% of the average reduction in variance. Of all commodity assets that were combined with SPY, gold and platinum yielded the best VaR minimizing output, which is especially true for the gold in the first and second subsamples. These findings indicate that minimum-variance hedging portfolio, in general, offers a lower reduction in a VaR portfolio, which is consistent with Harris and Shen (2006), and that HEI_{VaR} metrics particularly depends on the observing period.

Looking at the Table 8, it can be seen that HEI_{CVaR} findings are very similar with the HEI_{VaR} results in full sample as well as in all subsamples. Once again, it is obvious that HEI_{CVaR} performances significantly differentiate across determined subsamples, which justifies full sample partition. Relatively equable CVaR results with VaR measures indicate that expected size of a loss does not exceeds the VaR of the portfolio. As in the case of VaR results, the CVaR performances are less than the variance counterparts in all cases. In the first subsample, HEI_{CVaR} values are slightly lower than HEI_{VaR} measures in cases of Brent oil and gold, while for silver and platinum the reverse is true. In the second subsample, HEI_{CVaR} values of SPY-Brent and SPY-platinum portfolios slightly exceed HEI_{VaR} values, but these differences are so small that economic significance for international investors does not exist. In the third subsample, those portfolio investors who seek minimum CVaR recorded bad results if they combined SPY with Brent oil and made a portfolio construction with DCC-FIAPARCH model. Gold emerges as best commodity asset in combination with

SPY for CVaR seekers, although in third subsample these results are significantly mitigated. These results are much in line with Reboredo (2013) who investigated the hedge and safe-haven properties of portfolios, where gold is one constituent part in portfolio. He confirmed the usefulness of gold in the portfolio risk-management, and also demonstrated that portfolio composed of gold experience VaR and CVaR reductions. Our results indicate that, portfolios designed via Kroner and Ng (1998) equation, in general, produce a lower reduction in a CVaR portfolio in comparison with minimum variance portfolios, but it should be added that HEI_{CVaR} metrics notably depends on the observing sub-period as well as on the assets series that form particular portfolio.

Table 8 Calculated HEI_{CVaR} Values for the Full Sample and Three Subsamples

	DCC-GARCH		*DCC-APARCH*		*DCC-FIAPARCH*	
	Normal	*Student-t*	*Normal*	*Student-t*	*Normal*	*Student-t*
Panel A: Full sample: January 1 2001 – May 31 2016						
SPY – Brent oil	0.0806	0.0805	0.0874	0.0874	0.0861	0.0859
SPY – Gold	0.3746	0.3747	0.3773	0.3775	0.3762	0.3763
SPY – Silver	0.2186	0.2184	0.2221	0.2221	0.2207	0.2206
SPY – Platinum	0.2569	0.2572	0.2607	0.2610	0.2646	0.2649
Panel B: First subsample: January 1 2001 – September 3 2008						
SPY – Brent oil	0.1699	0.1691	0.1732	0.1733	0.1737	0.1736
SPY – Gold	0.4036	0.4038	0.4066	0.4068	0.4036	0.4039
SPY – Silver	0.2844	0.2845	0.2884	0.2886	0.2840	0.2843
SPY – Platinum	0.2982	0.2984	0.2979	0.2982	0.2991	0.2994
Panel C: Second subsample: September 3 2008 – December 20 2011						
SPY – Brent oil	0.0349	0.0349	0.0457	0.0458	0.0455	0.0456
SPY – Gold	0.4102	0.4102	0.4090	0.4091	0.4061	0.4061
SPY – Silver	0.2162	0.2160	0.2180	0.2180	0.2151	0.2150
SPY – Platinum	0.2661	0.2663	0.2696	0.2700	0.2756	0.2760
Panel D: Third subsample: December 20 2011 – May 31 2016						
SPY – Brent oil	0.0007	0.0008	0.0004	0.0006	-0.0085	-0.0088
SPY – Gold	0.1924	0.1927	0.2054	0.2054	0.2142	0.2142
SPY – Silver	0.0607	0.0604	0.0662	0.0659	0.0760	0.0757
SPY – Platinum	0.1128	0.1130	0.1257	0.1259	0.1277	0.1278

Notes: Greyed values indicate the highest HEI_{CVaR}.
Source: Authors' calculation.

As for the best performing DCC models, the situation regarding VaR and CVaR little bit change in respect to the previous minimum Var analysis. Once again, nuances rule which DCC model is better. Unlike the results from the Table 6, when DCC-APARCH model was the best performing model in 10 cases regarding both full sample and all subsamples, the Table 7 shows that this model is the best in six cases. By contrast, DCC-FIAPARCH retain the consistency and provided the portfolios with the best minimum VaR and CVaR results in seven instances, and with the minimum variance in six instances. The presented findings could indicate that long memory in variance should be considered by investors when it comes to the construction of minimum variance, minimum VaR and minimum CVaR portfolios.

Generally, observing the results from the HEI_{Var}, HEI_{VaR} and HEI_{CVaR}, it can be concluded that none of DCC models predominantly gives the best risk minimizing results. Most likely is that the choice of the most appropriate model depends on the particular characteristics of empirical series as well as on a particular time period that is scrutinized. However, since it was shown in this paper as well as in many other empirical studies that financial and commodity series frequently exhibit long memory property in variance, DCC-FIAPARCH should be considered as one of the options in the process of minimum-variance, minimum-VaR and minimum-CVaR portfolio designing.

6.2 Return-performances and Sharpe Ratio of Constructed Portfolios

Previous section has assessed the portfolios' performances when investors have a single goal – minimization of risk. However, investor never cares about minimizing the risk solely, so this chapter analyses how well constructed portfolios execute when goal is the maximization of expected returns. In addition, we evaluate whether more elaborate DCC models, i.e. with variance asymmetry and long memory, which proved to be the best risk-minimizing models, also have the best expected-return results. Table 9 shows calculated in-sample return values of constructed portfolios for four SPY-commodity pairs.

Table 9 Calculated Expected-return Values for the Full Sample and Three Subsamples

	DCC-GARCH		*DCC-APARCH*		*DCC-FIAPARCH*	
	Normal	*Student-t*	*Normal*	*Student-t*	*Normal*	*Student-t*
Panel A: Full sample: January 1 2001 – December 31 2016						
SPY – Brent oil	0.0069	0.0069	0.0054	0.0054	0.0067	0.0065
SPY – Gold	0.0263	0.0262	0.0226	0.0225	0.0213	0.0213
SPY – Silver	0.0158	0.0158	0.0106	0.0105	0.0100	0.0100
SPY – Platinum	0.0155	0.0154	0.0082	0.0082	0.0043	0.0044
Panel B: First subsample: January 1 2001 – September 3 2008						
SPY – Brent oil	0.0090	0.0089	0.0056	0.0055	0.0068	0.0064
SPY – Gold	0.0290	0.0291	0.0248	0.0249	0.0218	0.0219
SPY – Silver	0.0133	0.0133	0.0096	0.0096	0.0086	0.0087
SPY – Platinum	0.0264	0.0263	0.0215	0.0215	0.0213	0.0213
Panel C: Second subsample: September 3 2008 – December 20 2011						
SPY – Brent oil	-0.0278	-0.0280	-0.0330	-0.0329	-0.0320	-0.0321
SPY – Gold	0.0376	0.0374	0.0304	0.0303	0.0306	0.0303
SPY – Silver	0.0046	0.0047	-0.0091	-0.0089	-0.0109	-0.0105
SPY – Platinum	-0.0175	-0.0174	-0.0325	-0.0322	-0.0429	-0.0425
Panel D: Third subsample: December 20 2011 – December 31 2016						
SPY – Brent oil	0.0270	0.0269	0.0301	0.0303	0.0318	0.0320
SPY – Gold	0.0161	0.0160	0.0155	0.0153	0.0160	0.0159
SPY – Silver	0.0292	0.0289	0.0272	0.0268	0.0283	0.0280
SPY – Platinum	0.0223	0.0222	0.0164	0.0163	0.0112	0.0112

Notes: Greyed values indicate the highest return values.
Source: Authors' calculation

Observing the full sample results, it can be noticed that three out of four portfolios (SPY-gold, SPY-silver and SPY-platinum) constructed with DCC-GARCH model have higher returns than unhedged investment (SPY = 0.014). In addition, we found that the simplest DCC model, i.e. DCC-GARCH produces the best expected-return results in almost all cases. More specifically, DCC-GARCH model with multivariate normal distribution is slightly better in nine out of sixteen portfolios. The same model with Student-t distribution has tiny upper hand in three cases, while in three instances the estimated performances are equable regarding both normal and Student-t distribution. Only in case of SPY-Brent at third subsample, the most elaborate DCC proved to have the best expected-return results. As for the best return-performing portfolio, Table 9 suggests that gold in combination with SPY produced the highest returns in full sample as well as in first and second subsamples. Conversely, portfolio with gold had the worst result in the third subsample, while silver reported the best outcomes.

In addition, we calculate Sharpe ratio that includes into consideration both risk and returns of portfolios, and Table 10 contains the results. As can be viewed, in terms of best performing DCC model, results in Table 10 slightly differentiate from the results in Table 9. In other words, when risk is taken into account besides expected returns, DCC-FIAPARCH model has the best Sharpe ratio in two out of sixteen cases, while in all other instances simple DCC-GARCH dominates. Therefore, our findings undoubtedly suggest that the simplest DCC model produces the best risk-return results of portfolios, while for investors who pursue only risk-reducing portfolios better option would be more complex DCC models, i.e. those with asymmetry and long memory in variance.

Table 10 Calculated Sharpe Ratio for the Full Sample and Three Subsamples

	DCC-GARCH		***DCC-APARCH***		***DCC-FIAPARCH***	
	Normal	*Student-t*	*Normal*	*Student-t*	*Normal*	*Student-t*
Panel A: Full sample: January 1 2001 – December 31 2016						
SPY – Brent oil	0.0060	0.0061	0.0048	0.0048	0.0059	0.0058
SPY – Gold	0.0335	0.0334	0.0289	0.0288	0.0273	0.0273
SPY – Silver	0.0162	0.0162	0.0110	0.0109	0.0103	0.0103
SPY – Platinum	0.0167	0.0166	0.0089	0.0089	0.0047	0.0048
Panel B: First subsample: January 1 2001 – September 3 2008						
SPY – Brent oil	0.0099	0.0098	0.0062	0.0061	0.0075	0.0070
SPY – Gold	0.0433	0.0435	0.0374	0.0375	0.0328	0.0329
SPY – Silver	0.0168	0.0168	0.0122	0.0122	0.0109	0.0110
SPY – Platinum	0.0337	0.0336	0.0275	0.0275	0.0273	0.0273
Panel C: Second subsample: September 3 2008 – December 20 2011						
SPY – Brent oil	-0.0151	-0.0152	-0.0182	-0.0181	-0.0176	-0.0177
SPY – Gold	0.0327	0.0326	0.0265	0.0264	0.0265	0.0263
SPY – Silver	0.0031	0.0031	-0.0061	-0.0060	-0.0073	-0.0070
SPY – Platinum	-0.0125	-0.0125	-0.0235	-0.0233	-0.0314	-0.0311
Panel D: Third subsample: December 20 2011 – December 31 2016						
SPY – Brent oil	0.0333	0.0332	0.0374	0.0377	0.0395	0.0398
SPY – Gold	0.0251	0.0250	0.0246	0.0243	0.0256	0.0255
SPY – Silver	0.0389	0.0385	0.0365	0.0359	0.0383	0.0379
SPY – Platinum	0.0316	0.0314	0.0236	0.0235	0.0162	0.0162

Notes: Greyed values indicate the highest Sharpe ratio.
Source: Authors' calculation.

6.3 Out-of-sample Analysis

The previously calculated in-sample findings provide the history performance of our portfolios, constructed by the selected DCC models. Nonetheless, the market participants are very keen to learn what are the out-of-sample performances, since they are concerned with how well these portfolios can do in future. In order to avoid extreme market occurrences that happened during GFC and SDC, we focus on third subsample only. Thus, the in-sample data cover the period from January 1, 2012 to December 31, 2014, while the out-of-sample data for evaluation of the portfolio forecasting performance comprises the period from January 1, 2015 to December 31, 2016. The estimation period is rolled forward by adding a new daily observation and dropping the most distant one. The intention was to estimate out-of-sample variances and covariances and subsequently design portfolios according to Kroner and Ng (1998) equation. However, in the process of DCC model rolling estimation, the majority of rolling estimations of more elaborate models, i.e. DCC-APARCH and DCC-

FIAPARCH, did not converge. On the other hand, the simplest DCC model, DCC-GARCH, showed considerable robustness when it comes to the rolling model convergence. All rolling estimations of this model for all selected pairs, with both multivariate normal and Student-t distributions, were successful. As a consequence, we could not collect a continuous set of forecasting values for DCC-APARCH and DCC-FIAPARCH models, and thus the out-of-sample construction of these portfolios remained unattainable. More specifically, as in a case of risk-return calculations, it turns out that the simplest model is also the best solution. DCC-GARCH model overpowers more complex models in this segment of analysis. Therefore, Table 11 presents calculated out-of-sample HEI values of portfolios constructed only with DCC-GARCH model.

Table 11 Calculated Out-of-sample HEI Values

	SPY – Brent oil		*SPY – Gold*		*SPY – Silver*		*SPY – Platinum*	
	GARCH-n	*GARCH-t*	*GARCH-n*	*GARCH-t*	*GARCH-n*	*GARCH-t*	*GARCH-n*	*GARCH-t*
HEI_{Var}	-0.0911	-0.0912	0.5831	0.5788	0.2460	0.2445	0.3456	0.3443
HEI_{VaR}	-0.0631	-0.0633	0.3487	0.3428	0.1348	0.1342	0.1840	0.1829
HEI_{CVaR}	-0.0592	-0.0594	0.3499	0.3445	0.1341	0.1335	0.1855	0.1844

Notes: Greyed values indicate the best out-of-sample option.
Source: Authors’ calculation.

Comparing the in-sample and out-of-sample HEI values, one can notice that results are somewhat surprising, in a sense that out-of-sample portfolios, designed of SPY-ETF and precious metals, have better results than in-sample portfolios, regarding all three risk-metrics. On the other hand, in case of SPY-Brent pair, in sample portfolios have better hedging performances than out-of-sample counterparts. Once again, SPY-gold portfolio has the best risk minimizing performances and platinum is the second-best. As we have stated earlier, this probably happens due to the low or negative correlation between SPY-ETF and gold, and the fact that gold is the least risky asset (see Khalfaoui et al., 2015). According to out-of-sample results, the difference between portfolios designed with normal and Student-t distribution is almost negligible, visible only at third decimal. Thus, international portfolio investors do not need to care about which multivariate distribution they will apply in the process of out-of-sample portfolio construction.

7. Conclusion

This paper strives to answer whether long memory in variance plays an important part in the process of portfolio hedging assessment. For the purpose, we combined SPY-ETF index as a primary instrument in the two-asset portfolio with each of four well known commodities – Brent oil, gold, silver and platinum. Therefore, we estimate three different bivariate DCC models with both multivariate normal and Student t density functions, and subsequently we construct minimum-variance portfolios with the aim to test whether this portfolio provides a similarly large reduction in portfolio VaR and portfolio CVaR. The validity of hedging strategies is examined by using three different performance metrics (HEI_{Var}, HEI_{VaR} and HEI_{CVaR}). These hedging approaches are adopted in order to see what the outcome is for investors with different risk minimizing goals. As it turned out, long memory DCC models has the best fitting performances in all cases, and complementary rolling regression revealed that SPY-ETF conditional variance has predominant influence on the conditional correlation, while the only exception is gold. Rolling regression indicate that conditional volatility of gold affected frequently the conditional correlation with an opposite sign in regard to SPY-ETF asset, which makes gold a suitable instrument for diversification. Also, we stipulate that minimum-variance hedging portfolio, in general, offers a lower reduction in VaR and CVaR portfolios. In addition, we split full sample into three distinctive subsamples utilizing modified ICSS algorithm in order to gauge how portfolio hedge effectiveness alter across periods of different market turbulence.

The findings of three different hedge effectiveness metrics are pretty heterogeneous across subsamples in terms of the size, which justifies the full sample partition. The results indicate that gold provide the best hedging benefits for international investors in terms of HEI_{Var}, HEI_{VaR} and HEI_{CVaR}, although in the third sub-period (i.e. after GFC and SDC) the hedging performances of gold portfolios are significantly diminished. Besides, the best DCC model in portfolio construction alters between DCC-APARCH and DCC-FIAPARCH across three subsamples. However, when we investigate return-performances of constructed portfolios, the simplest DCC-GARCH model outperform the other two more elaborate models, while investors who pursue only risk-reducing portfolios, the better option would be more complex DCC models. Also, in case of out-of-sample analysis, the rolling DCC-GARCH model estimation demonstrate significant robustness in terms of convergence, unlike the other two DCC models, showing that in this case the simplest model is also the best solution.

Regarding the future research, this study could be extended in several ways. For instance, we showed that long memory in DCC models is an important issue which should be accounted for in the process of risk-minimizing portfolio designing. However, we did not investigate whether inserted structural breaks in the conditional variances of DCC model contribute to the improvement of portfolio hedging performances. Also, for some researchers it could be interesting to evaluate total performance of hedging strategies, both including and excluding transaction costs. In addition, our research was limited on four commodity assets that served as auxiliary instrument in our portfolios, and future studies could address other instruments, e.g. agricultural products, emerging market indices, exchange rates, etc.

We believe that various portfolio managers, market analysts and market participants could find the results useful, since they could benefit from knowing that portfolio selection and risk reduction is better achieved if long memory is recognized in the DCC models, while for the risk/return objectives and for out-of-sample calculations they should use the simplest DCC model.

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