Appendix B

Model description

In this appendix, a dynamic stochastic model of general equilibrium (DSGE model) of a small open economy (SOE) with financial frictions is described. The model framework follows Shaari (2008), who enhanced the basic small open economy framework proposed by Galí (1999) and Galí and Monacelli (2005) with the financial accelerator mechanism as described in Bernanke *et al.* (1999). In this paper, we elaborated on the model that was implemented in Tvrz (2012). The model structure was adjusted in order to allow time-varying structural parameters. Several changes were made to the original concept of Shaari (2008). Foreign variables are modelled with the use of vector autoregression model (VAR) of order one. This enables us to impose more structure on the behaviour of foreign variables and to identify the foreign exogenous shocks better than with independent AR(1) processes. Also, exogenous shock in entrepreneurs' net worth was introduced into the model.

B.1 Definitions

B.1.1 Consumption demand and price indices

Consumption index C_t is defined as

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{B.1.1}$$

where γ measures openness of the domestic economy and η is the elasticity of substitution between home and foreign goods. Subscripts H and F denote the country of origin, which means that the consumption index is composed of home and foreign goods.

Home goods consumption index is defined as

$$C_{H,t} = \left(\int_0^1 C_{H,t}(h)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}h\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{B.1.2}$$

with elasticity of substitution between different varieties of goods ε. There is a continuum of home goods varieties. The demand function for particular variety of home goods can be expressed as

$$C_{H,t}(j) = C_{H,t}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}. \tag{B.1.3}$$

The home goods price index is defined as

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\varepsilon}\,\mathrm{d}i\right)^{\frac{1}{1-\varepsilon}}. \tag{B.1.4}$$

Foreign goods consumption index definition is analogous to the definition of home goods index (B.1.2). Therefore, also the demand function for particular variety of foreign goods and foreign goods price index are analogous to (B.1.3) and (B.1.4).

The consumer price index (CPI) P_t is defined as

$$P_t = \left[(1-\gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} . \tag{B.1.5}$$

The demand of households for home and foreign goods is given by following functions:

$$C_{H,t} = C_t(1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}, \tag{B.1.6}$$

$$C_{F,t} = C_t\gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}. \tag{B.1.7}$$

B.1.2 Inflation

A standard definition of CPI inflation is assumed[1]

$$\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right) = p_t - p_{t-1}. \tag{B.1.8}$$

Analogously we define the inflation of home goods prices $\pi_{H,t}$, inflation of foreign goods prices $\pi_{F,t}$ and foreign[2] CPI inflation π_t^*. Note that all the price indices are assumed to be equal to one in steady state. Log-linearization[3] of price index definition (B.1.5) gives this result

$$p_t = (1-\gamma)p_{H,t} + \gamma p_{F,t}, \tag{B.1.9}$$

which implies

$$\pi_t = (1-\gamma)\pi_{H,t} + \gamma\pi_{F,t}. \tag{B.1.10}$$

B.1.3 Terms of trade

The terms of trade, TOT_t, are defined as

$$TOT_t = \frac{P_{F,t}}{P_{H,t}}.$$

This variable measures the relative price of foreign and home goods and it is expressed in domestic currency. Since $tot_t = p_{F,t} - p_{H,t}$, it can be shown that $p_t = p_{H,t} - \gamma tot_t$ and $\pi_t = \pi_{H,t} + \gamma\Delta tot_t$.

B.1.4 Real exchange rate

Denote S_t the nominal interest rate in direct quotation (CZK/EUR). Thus, growth of S_t means a depreciation of domestic currency and vice versa. The real exchange rate, RER_t, is then defined as

$$RER_t = S_t\frac{P_t^*}{P_t},$$

where P_t^* is foreign CPI index.

[1]The logarithmic deviations from steady state are denoted by small letter variables, i.e. $\log\left(\frac{X_t}{X}\right) = x_t$.

[2]Foreign variables are denoted by an asterisk.

[3]The model equations are log-linearized around the deterministic steady state. Log-linear equations are, therefore, only approximations of true behaviour of model economy. Nevertheless, in the vicinity of the steady state the approximation error is negligible.

B.1.5 Law of one price gap

We suppose that law of one price holds for domestic[4] exports, $C^*_{H,t}$, i.e.

$$P^*_{H,t} = \frac{P_{H,t}}{S_t}.$$

However, this is not the case for domestic imports, or foreign goods. We assume that there is a wedge between the price of foreign goods in the domestic economy and the price level of foreign country, this means that the law of one price does not hold for the imported goods. We define a law of one price gap

$$LOP_t = \frac{S_t P^*_t}{P_{F,t}}.$$

In log-linear terms $lop_t = s_t + p^*_t - p_{F,t}$. It can be shown that following relation holds for real exchange rate, terms of trade and law of one price gap

$$rer_t = (1-\gamma) tot_t + lop_t. \tag{B.1.11}$$

The development of the law of one price gap is exogenous in this model and its deviation from steady state is assumed to follow AR(1) process of following form

$$lop_t = \rho_{LOP} lop_{t-1} + \varepsilon^{LOP}_t,$$

where $\rho_{LOP} \in \langle 0, 1\rangle$ is the AR(1) coefficient and the innovation term $\varepsilon^{LOP}_t \sim iid(0, \sigma^2_{LOP})$.

B.2 Households

Households maximize expected discounted sum of utilities by choosing optimal consumption and labour paths and solve following optimization problem

$$\max_{\{C_t, L_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, C_{t-1}, L_{H,t}) \right\}$$

subject to a budget constraint. $\beta \in \langle 0, 1\rangle$ is exogenous discount parameter. Following functional form of utility function is assumed

$$U(C_t, L_t) = \log(C_t - \Upsilon C_{t-1}) - \frac{L_{H,t}^{1+\Psi}}{1+\Psi},$$

where C_t is consumption index, $L_{H,t}$ is household's labour supply, Υ is the parameter of external habit and Ψ is inverse elasticity of labour supply.

The budget constraint of representative household has following form

$$\widetilde{W}_{H,t} L_{H,t} + R_{t-1} D_{t-1} + R^*_{t-1} \Psi^B(Z_{t-1}, A^{UIP}_{t-1}) S_t B_{t-1} + \Pi^r_t + TR_t = P_t C_t + D_t + S_t B_t.$$

This means that the household gets remuneration for provided amount of labour $L_{H,t}$ in a form of nominal[5] wage $\widetilde{W}_{H,t}$. Also, the household receives profits from retail firms Π^r_t

[4] Subscript H and F indicates always the country of origin. Therefore, $C^*_{H,t}$ denotes goods produced in domestic economy but consumed abroad.

[5] Nominal variables are denoted by tilde, i.e. $\widetilde{X}_t = X_t P_t$.

and left-over equity of entrepreneurs that go bankrupt and leave the economy in a form of transfers TR_t.

The representative household spends its income on consumption but they can also buy two kinds of assets: domestic bonds D_t from domestic intermediary and foreign bonds B_t (denominated in foreign currency). Domestic bonds yield nominal interest rate R_t in one period. Foreign bonds yield risk-adjusted[6] return $R_t^* \Psi^B(Z_t, A_t^{UIP})$ in one period. Risk-premium is specified according to Adolfson *et al.* (2008) as

$$\Psi^B(Z_t, A_t^{UIP}) = \exp\left[-\psi^B(Z_t + A_t^{UIP})\right],$$

where R_t^* is foreign nominal interest rate, $Z_t = \frac{S_t B_t}{Y_{H,t} P_{H,t}}$ is the net foreign assets position of the domestic economy and A_t^{UIP} is the debt-elastic risk premium shock.[7] Deviation of this shock from steady state is assumed to follow AR(1) process of standard form

$$a_t^{UIP} = \rho_{UIP} a_{t-1}^{UIP} + \varepsilon_t^{UIP},$$

where $\rho_{UIP} \in \langle 0, 1 \rangle$ and the innovation term $\varepsilon_t^{UIP} \sim iid(0, \sigma_{UIP}^2)$.

The solution of households' optimization problem can be summarized by following optimality conditions:

- Optimal choice between consumption and free time:

$$\frac{\widetilde{W}_{H,t}}{P_t} = W_{H,t} = L_{H,t}^{\psi}(C_t - \Upsilon C_{t-1}). \tag{B.2.1}$$

- Optimal choice between consumption and domestic bonds is given by:

$$R_t = \frac{1}{\beta} \frac{C_{t+1} - \Upsilon C_t}{C_t - \Upsilon C_{t-1}} \frac{P_{t+1}}{P_t}. \tag{B.2.2}$$

- Optimal choice between consumption and foreign bonds:

$$R_t^* \Psi^B(Z_t, A_t^{UIP}) = \frac{1}{\beta} \frac{S_t}{S_{t+1}} \frac{(C_{t+1} - \Upsilon C_t)}{(C_t - \Upsilon C_{t-1})} \frac{P_{t+1}}{P_t} \tag{B.2.3}$$

- Optimal choice between foreign and domestic bonds:

$$\begin{aligned} R_t^* \Psi^B(Z_t, A_t^{UIP}) &= \frac{S_t}{S_{t+1}} R_t, \\ R_t^* \exp\left[-\psi^B(Z_t + A_t^{UIP})\right] &= R_t \frac{RER_t P_t}{P_t^*} \frac{P_{t+1}^*}{P_{t+1} RER_{t+1}}, \end{aligned} \tag{B.2.4}$$

  which is a risk-adjusted uncovered interest parity (UIP) condition.

In log-linear terms we can write:

$$l_{H,t} = \frac{w_{H,t}}{\Psi} - \frac{c_t - \Upsilon c_{t-1}}{\Psi(1 - \Upsilon)} \tag{B.2.5}$$

$$(1 - \Upsilon)(r_t - E_t \pi_{t+1}) = (c_{t+1} - \Upsilon c_t) - (c_t - \Upsilon c_{t-1}) \tag{B.2.6}$$

$$rer_{t+1} - rer_t = (r_t - E_t \pi_{t+1}) - (r_t^* - E_t \pi_{t+1}^*) + \psi^B z_t + \psi^B a_t^{UIP} \tag{B.2.7}$$

[6] Risk-adjustment is introduced into the model in order to stationarize the net foreign assets position in the steady state. See Schmitt-Grohé and Uribe (2003) for detailed explanation.

[7] Since it directly influences the development of real exchange rate via uncovered interest parity condition (UIP condition), it is denoted as UIP shock.

B.3 Entrepreneurs

Following Bernanke *et al.* (1999), we next introduce a production factor of capital into the model and describe the entrepreneur as a representative economic agent. Entrepreneurs play two important roles in the model. Firstly, they own and manage firms that produce intermediate (wholesale) goods and, secondly, they own and produce the capital goods.

In owning and production of capital goods the entrepreneurs face a financing constraint. This means that the entrepreneurs are not fully self-financing, and therefore, they have to borrow resources from commercial banks. The banks always demand higher interest rate than the policy interest rate. The spread between commercial interest rate and policy interest rate is determined by the ratio of the value of capital stock and entrepreneurs' net worth (leverage ratio). What we have just described is the financial accelerator mechanism and it is the source of financial frictions in this model.

To rule out the possibility that the entrepreneurs could accumulate enough net worth to become fully self-financing, the entrepreneurs have to have a finite horizon. For that purpose it is assumed that a fraction $\varsigma \in \langle 0, 1\rangle$ of entrepreneurs goes bakrupt each period in steady state. Remaining share $(1-\varsigma)$ of entrepreneurs survives to the next period.

B.3.1 Intermediate goods production

Firms that produce intermediate goods operate at perfectly competitive market. This means that these firms have no market power and will attain no profits. Intermediate goods $Y_{H,t}$ is produced by combining the production factors of capital K_t and labour L_t. The output is sold at wholesale price $P^W_{H,t}$ to retailers. Standard Cobb-Douglas production technology is assumed,

$$Y_{H,t} = A^Y_t K^\alpha_t L^{1-\alpha}_t,$$

where parameter $\alpha \in \langle 0, 1\rangle$ determines the income share of capital. A^Y_t is a productivity shock; its deviation from steady state is assumed to evolve according to following AR(1) process

$$a^Y_t = \rho_Y a^Y_{t-1} + \varepsilon^Y_t,$$

where $\rho_Y \in \langle 0, 1\rangle$ and $\varepsilon^Y_t \sim iid(0, \sigma^2_Y)$. The total labour input is defined as a composite of the labour provided by households $L_{H,t}$ and by entrepreneurs $L_{E,t}$,

$$L_t = L^\Omega_{H,t} L^{1-\Omega}_{E,t}.$$

In line with Bernanke *et al.* (1999) we normalize the labour input of entrepreneurs to 1. The production function can be then rewritten as

$$Y_{H,t} = A^Y_t K^\alpha_t L^{\Omega(1-\alpha)}_{H,t},$$

or in log-linear terms

$$y_{H,t} = \alpha k_t + (1-\alpha)\Omega l_{H,t} + a^Y_t. \tag{B.3.1}$$

The solution of entrepreneurs' optimization problem can be summarized by following set of optimality conditions:

$$
\begin{aligned}
\frac{\widetilde{R}_{G,t}}{P_t} &= R_{G,t} = \alpha \frac{Y_{H,t}}{K_t} MC_{H,t} \frac{P_{H,t}}{P_t}, \\
\frac{\widetilde{W}_{H,t}}{P_t} &= W_{H,t} = \Omega(1-\alpha) \frac{Y_{H,t}}{L_{H,t}} MC_{H,t} \frac{P_{H,t}}{P_t}, \\
\frac{\widetilde{W}_{E,t}}{P_t} &= W_{E,t} = (1-\Omega)(1-\alpha) Y_{H,t} MC_{H,t} \frac{P_{H,t}}{P_t},
\end{aligned}
$$

where $\widetilde{R}_{G,t}$ is the gross nominal rental rate for capital, $\widetilde{W}_{H,t}$ is the nominal wage paid to households, $\widetilde{W}_{E,t}$ is the nominal wage paid to entrepreneurs themselves and $MC_{H,t}$ are the real marginal costs of home goods production. After log-linearization we receive[8]

$$r_{G,t} = y_{H,t} + mc_{H,t} - k_t - \left[\frac{\gamma}{1-\gamma} (rer_t - lop_t) \right], \tag{B.3.2}$$

$$w_{H,t} = y_{H,t} + mc_{H,t} - l_{H,t} - \left[\frac{\gamma}{1-\gamma} (rer_t - lop_t) \right], \tag{B.3.3}$$

$$w_{E,t} = y_{H,t} + mc_{H,t} - \left[\frac{\gamma}{1-\gamma} (rer_t - lop_t) \right].$$

Plugging for k_t and $l_{H,t}$ from (B.3.2) and (B.3.3) into the production function (B.3.1) we receive the expression for real marginal costs,

$$mc_{H,t} = \frac{(1-\alpha)(1-\Omega) y_{H,t} + \alpha r_{G,t} + (1-\alpha)\Omega w_{H,t} - a_t^Y}{\alpha + (1-\alpha)\Omega} + \frac{\gamma}{1-\gamma}(rer_t - lop_t). \tag{B.3.4}$$

Equation (B.3.4), therefore, suggests that depreciation of the real exchange rate increases the real marginal costs while an increase of law of one price gap decreases them.

B.3.2 Capital goods production

Entrepreneurs produce capital goods and sell it at competitive market at nominal price $\widetilde{Q}_t$. Capital is produced by combining already existing capital with investment INV_t. Investment INV_t is a bundle of home and foreign consumption goods. We assume, that the entrepreneurs choose the optimal mix of goods varieties in the same way as the households. Therefore, the investment is defined analogously to the the consumption index C_t in equation (B.1.1),

$$INV_t = \left[(1-\gamma)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{B.3.5}$$

Thus, the demand of entrepreneurs for home and foreign goods as well as respective price indices are the same as in the case of households.

Stock of capital goods is assumed to evolve according to

$$K_{t+1} = \Phi\left(\frac{INV_t}{K_t}\right) K_t + (1-\delta) K_t, \tag{B.3.6}$$

[8] We used the equation (B.1.11) and relations described in sections B.1.2 and B.1.3.

where $\delta \in \langle 0,1\rangle$ is the rate of capital depreciation and $\Phi(.)$ is an increasing concave production function. Following functional form is assumed for $\Phi(.)$

$$\Phi\left(\frac{INV_t}{K_t}\right) = \frac{INV_t}{K_t} - \frac{\psi^I}{2}\left(\frac{INV_t}{K_t} - \delta\right)^2,$$

where $\psi^I > 0$ is the capital adjustment costs parameter. Following Bernanke *et al.* (1999), the capital adjustment costs are introduced into the model to allow movement in the price of capital, which increases the volatility in entrepreneurs' net worth and contributes to the financial accelerator effect.

In the steady state,[9] the production function has following properties: $\Phi\left(\frac{\overline{INV}}{\overline{K}}\right) = \delta$, $\Phi'\left(\frac{\overline{INV}}{\overline{K}}\right) = 1$. These properties ensure deterministic level of capital stock in the steady state (investment only replaces depreciated capital, $\overline{INV} = \delta\overline{K}$) and also that the price of capital will be equal to one in the steady state ($\overline{Q} = 1$). Therefore, log-linearizing the law of motion of capital (B.3.6) gives

$$k_{t+1} = \delta inv_t + (1-\delta)k_t.$$

The entrepreneur decides about how much new capital to produce. The optimality condition is

$$Q_t = \frac{1}{1 - \psi^I\left(\frac{INV_t}{K_t} - \delta\right)},$$

which in log-linear terms means

$$q_t = \psi^I \delta(i_t - k_t). \tag{B.3.7}$$

Now we define entrepreneur's gross real return on capital investment $R_{K,t}$,

$$R_{K,t} = \frac{[R_{G,t} + (1-\delta)Q_t]K_t}{Q_{t-1}K_t}.$$

In log-linear terms we receive

$$r_{K,t} = \left(1 - \frac{1-\delta}{\overline{R}_K}\right) r_{G,t} + \frac{1-\delta}{\overline{R}_K} q_t - q_{t-1} \tag{B.3.8}$$

Utilization of capital in production of intermediate goods yields gross real rental rate $R_{G,t}$. Since the entrepreneurs own the capital, any change in the price of capital also influences the return on investment and consequently it affects the entrepreneur's net worth.

B.3.3 Financial frictions

Entrepreneurs finance their capital investment projects using their net worth N_{t+1} and borrowed funds F_{t+1}. Thus, the entrepreneurs' budget constraint can be written down as

$$Q_t K_{t+1} = N_{t+1} + F_{t+1},$$

or in log-linear terms

$$\overline{K}(q_t + k_{t+1}) = \overline{N} n_{t+1} + \overline{F} f_{t+1}. \tag{B.3.9}$$

[9] We denote steady state values of all variables by overline.

When borrowing from financial intermediary, entrepreneur has to pay not only the gross real interest rate $R_t\frac{P_t}{P_{t+1}}$, but also the external finance premium, EFP_{t+1}. This premium depends on the leverage ratio of the entrepreneur and it is defined as

$$EFP_{t+1} = \left(\frac{N_{t+1}}{Q_t K_{t+1}}\right)^{-\chi},$$

where $\chi > 0$ is the elasticity of the external finance premium with respect to the leverage ratio $\frac{N_t}{Q_{t-1}K_t}$. Bernanke *et al.* (1999) motivate this set-up by an agent-principal problem at the credit market.

Entrepreneurs are risk-neutral and choose K_{t+1} to maximize profits. The amount of borrowed funds F_{t+1} is implied by chosen K_{t+1}. To maximize the profits, entrepreneurs equate expected marginal return on capital investment with marginal financing costs

$$E_t R_{K,t+1} = E_t\left[\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right)^{-\chi} R_t \frac{P_t}{P_{t+1}}\right], \tag{B.3.10}$$

which again in log-linear terms gives

$$E_t r_{K,t+1} = r_t - E_t\pi_{t+1} - \chi(n_{t+1} - q_t - k_{t+1}).$$

Now we derive the evolution of entrepreneurs' net worth. The new net worth consists of entrepreneurial equity held by the fraction $(1-\varsigma)A_t^{NW}$ of entrepreneurs that will survive current period and entrepreneurs' wage income $W_{E,t}$,

$$N_{t+1} = (1-\varsigma)A_t^{NW} V_t + W_{E,t}, \tag{B.3.11}$$

where A_t^{NW} is shock in entrepreneurial net worth and ς is the steady-state bankruptcy rate. Thus, the net worth shock influences the development of net worth by changing the effective survival rate of entrepreneurs. Its deviation from steady state is assumed to evolve according to AR(1) process,

$$a_t^{NW} = \rho_{NW} a_{t-1}^{NW} + \varepsilon_t^{NW},$$

where $\rho_{NW} \in \langle 0, 1\rangle$ and $\varepsilon_t^{NW} \sim iid(0, \sigma_{NW}^2)$.

The remaining entrepreneurs who leave the economy transfer their equity to households as transfers $TR_t = \varsigma A_t^{NW} V_t$. This mechanism ensures that net worth is pinned down in steady state. We also assume that labour income of entrepreneurs is small $(1-\Omega = 0.01)$. Wage income of entrepreneurs ensures, that they always have positive net worth to do the business with. Entrepreneurs' equity is defined as

$$V_t = R_{K,t} Q_{t-1} K_t - \left(\frac{N_t}{Q_{t-1}K_t}\right)^{-\chi} R_{t-1}\frac{P_{t-1}}{P_t} F_t. \tag{B.3.12}$$

Thus, the entrepreneurs' equity is the realized return on capital investment minus the repayment of loans. Note that an increase in interest rate lowers entrepreneurs' net worth, which increases the premium and further lowers the net worth.

To obtain a log-linear approximation of entrepreneurial net worth dynamics in the neighbourhood of steady state, we log-linearize the entrepreneurial equity definition and rearrange to receive

$$\begin{aligned} n_{t+1} = {} & (1-\varsigma)\overline{R}_K\left[(\Gamma+1)r_{K,t} - \chi\Gamma(q_{t-1} + k_t) - \Gamma(r_{t-1} - \pi_t) + (\chi\Gamma + 1)n_t + a_t^{NW}\right] + \\ & + (\Gamma+1)\frac{\overline{W}_E}{\overline{K}} w_{E,t}, \end{aligned} \tag{B.3.13}$$

where we used this substitution: $\frac{\overline{K}-\overline{N}}{\overline{N}} = \Gamma$, $\frac{\overline{K}}{\overline{N}} = \Gamma + 1$ and $\frac{\overline{W}_E}{\overline{N}} = \frac{\overline{W}_E}{\overline{K}}\frac{\overline{K}}{\overline{N}} = (\Gamma+1)\frac{\overline{W}_E}{\overline{K}}$.

B.4 Retailers

There are two types of retailers in the model economy. Home goods retailers buy intermediate goods from entrepreneurs and sell it as home goods to households or export it abroad. Foreign goods retailers buy final goods abroad and sell it to the households as foreign goods. Both types of retailers operate at monopolistically competitive markets. Thus, the retailers have certain market power and earn non-zero profits. These profits are distributed back to households. The retailers are assumed to practice Calvo-type price setting with inflation indexation, which means that there are nominal price rigidities in the model.

B.4.1 Home goods retailers

Home goods retailers buy the intermediate good from entrepreneurs at the wholesale price $P^W_{H,t}$ and at no additional costs they distribute the home goods to the households. Price of retailer z is denoted by $P_{H,t}(z)$, $z \in \langle 0,1\rangle$. Each period, only a fraction $(1-\theta_H)$ of retailers reset their prices to new optimal price $P^{NEW}_{H,t}$. Since all the home goods retailers face the same optimization problem, the new optimal price is common to all of them. The remaining fraction of the home goods retailers θ_H adjust their price according to last period CPI inflation according to

$$P_{H,t}(z) = P_{H,t-1}(z)(\pi_{t-1})^{\kappa},$$

where $\kappa \in \langle 0,1\rangle$ measures the degree of inflation indexation. Together with (B.1.4) this implies aggregate price level of home goods

$$P_{H,t} = \left[(1-\theta_H)\left(P^{NEW}_{H,t}\right)^{1-\varepsilon} + \theta_H\left(P_{H,t-1}(\pi_{t-1})^{\kappa}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}},$$

or in log-linear terms

$$p_{H,t} = (1-\theta_H)p^{NEW}_{H,t} + \theta_H p_{H,t-1} + \theta_H \kappa \pi_{t-1} \tag{B.4.1}$$

The Calvo parameter θ_H measures the rigidity of home goods prices. It is an exogenous probability of keeping current price of any particular home good for the next period without any change. It can be shown that the expected duration of any particular home good price is given by $\frac{1}{1-\theta_H}$.

The Phillips curve of home goods is given by

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} - \kappa\beta\pi_t + \kappa\pi_{t-1} + \frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H} mc_{H,t}. \tag{B.4.2}$$

B.4.2 Foreign goods retailers

Foreign goods retailers buy the final goods abroad and sell it to the households at price $P_{F,t}$. Law of one price is assumed to hold at the wholesale level. Therefore, the foreign goods retailers buy the goods at a price $P^W_{F,t} = S_t P^*_t$. Since the law of one price does not hold at retail level ($P_{F,t} \neq S_t P^*_t$), the effect of the incomplete exchange rate pass-through is introduced into the model. Similarly to the home goods retailers, the foreign goods retailers set their prices à la Calvo with parameter θ_F and inflation indexation with parameter κ.

To obtain the log-linear approximation of the foreign goods inflation dynamics in the neighbourhood of the steady state, we would have to proceed the same way as in the previous section. However, all the results presented in the section B.4.1 are valid also for foreign goods retailers. Only the subscript H has to be changed to F everywhere it appears. Therefore, we can write

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} - \kappa\beta\pi_t + \kappa\pi_{t-1} + \frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F} mc_{F,t}.$$

Unlike in the section B.4.1, here we can proceed a bit further and use the fact that $MC_{F,t} =$
$= \frac{P^W_{F,t}}{P_{F,t}} = \frac{S_t P^*_t}{P_{F,t}}$, and therefore, $mc_{F,t} = s_t + p^*_t - p_{F,t} = lop_t$ to obtain the final form of foreign goods Phillips curve,

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} - \kappa\beta\pi_t + \kappa\pi_{t-1} + \frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F} lop_t. \tag{B.4.3}$$

B.4.3 CPI inflation

Finally, we will derive the log-linear approximation of CPI inflation dynamics. Plugging the results (B.4.2) and (B.4.3) into the definition of CPI inflation (B.1.10) we obtain

$$\pi_t = \frac{1}{1+\kappa\beta}\left[\beta E_t\pi_{t+1} + \kappa\pi_{t-1} + (1-\gamma)\Lambda_H mc_{H,t} + \gamma\Lambda_F lop_t\right], \tag{B.4.4}$$

where $\Lambda_H = \frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H}$ and $\Lambda_F = \frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F}$.

B.5 Monetary policy

The monetary authority is modelled using standard forward-looking Taylor rule. This interest rate rule specifies how does the central bank react to expected deviations of CPI inflation and aggregate output from steady state when it decides about policy interest rate. In log-linear terms the Taylor rule can be written down as

$$r_t = (1-\rho)\left(\beta_\pi E_t\pi_{t+1} + \Theta_y E_t y_{t+1}\right) + \rho r_{t-1} + \varepsilon^{MP}_t, \tag{B.5.1}$$

where $\rho \in \langle 0,1\rangle$ is smoothing parameter, $\beta_\pi > 1$ represents the elasticity of policy interest rate with respect to the expected CPI inflation[10] , $\Theta_y \geq 0$ stands for the elasticity of policy interest rate with respect to the expected output gap and $\varepsilon^{MP}_t \sim iid(0,\sigma^2_{MP})$ is the monetary policy shock.

B.6 Foreign sector

Following Christiano *et al.* (2011), the foreign economy variables are modelled using a VAR(1) model of this form,

$$\begin{pmatrix} y^*_t \\ \pi^*_t \\ r^*_t \end{pmatrix} = \begin{pmatrix} \rho_{y^*y^*} & \rho_{y^*\pi^*} & \rho_{y^*r^*} \\ \rho_{\pi^*y^*} & \rho_{\pi^*\pi^*} & \rho_{\pi^*r^*} \\ \rho_{r^*y^*} & \rho_{r^*\pi^*} & \rho_{r^*r^*} \end{pmatrix} \begin{pmatrix} y^*_{t-1} \\ \pi^*_{t-1} \\ r^*_{t-1} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \sigma_{\pi^*y^*} & 1 & 0 \\ \sigma_{r^*y^*} & \sigma_{r^*\pi^*} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon^{y^*}_t \\ \varepsilon^{\pi^*}_t \\ \varepsilon^{r^*}_t \end{pmatrix}, \tag{B.6.1}$$

[10] We assume that the Taylor principle holds and the central bank adjusts the policy interest rate more than one-for-one with inflation. Violation of this condition can lead even to indeterminacy of equilibrium, see Davig and Leeper (2007) for more details.

where $\varepsilon_t^{y^*} \sim iid(0, \sigma_{y^*}^2)$, $\varepsilon_t^{\pi^*} \sim iid(0, \sigma_{\pi^*}^2)$ and $\varepsilon_t^{r^*} \sim iid(0, \sigma_{r^*}^2)$. Autocorrelation coefficients satisfy $\rho_{y^*y^*}, \rho_{\pi^*\pi^*}, \rho_{r^*r^*} \in \langle 0, 1\rangle$. Remaining coefficients are not constrained.

For the purposes of estimation, the VAR structure allows us to identify the exogenous shocks in foreign variables better. Since we assume interdependency of foreign exogenous shocks a shock in foreign output will at the same time influence also foreign CPI inflation and foreign interest rate, and similarly, a shock in foreign CPI inflation will have instant impact on foreign interest rate. Compared to foreign variables modelled as independent AR(1) processes this approach should capture the relations between foreign variables more closely.

B.7 Market clearing and equilibrium

B.7.1 Domestic bond market

Financial intermediaries sell domestic bonds to households and lend obtained funds to entrepreneurs. All the financial intermediaries are assumed to operate at perfectly competitive market, generating no profits. The intermediary borrows the funds from households at cost R_t and receives returns from entrepreneurs of $R_t \left(\frac{N_{t+1}}{Q_t K_{t+1}}\right)^{-\chi} > R_t$. The risk premium $EFP_{t+1} = \left(\frac{N_{t+1}}{Q_t K_{t+1}}\right)^{-\chi}$ is assumed to exactly cover the monitoring costs to fulfill the zero-profit condition. Thus, in equilibrium the intermediaries lend all the funds obtained from households to entrepreneurs,

$$F_t = D_t.$$

B.7.2 Foreign bond market

Households can also buy foreign bonds that yield debt-elastic interest rate $R_t^* \Psi^B(Z_t, A_t^{UIP})$. The higher the amount of foreign bonds kept by the households, the lower the risk-premium $\Psi^B(Z_t, A_t^{UIP})$ and the lower the returns. Since the households can hold negative amounts of foreign bonds as well, this relation works also vice versa. The higher the debt of households, the higher the risk-premium and the higher the costs. Since the risk-premium of foreign bonds is given by net foreign assets position (and exogenous UIP shocks), we need to describe the behaviour of this variable. Net foreign assets position Z_t is defined as a ratio of foreign bonds value and nominal GDP,

$$Z_t = \frac{S_t B_t}{P_{H,t} Y_{H,t}}.$$

The evolution of net foreign assets position can be, therefore, approximated by following log-linear equation

$$z_t = \frac{1}{\beta} z_{t-1} + \gamma y_t^* + \gamma \frac{2\eta - \gamma\eta - 1}{1-\gamma} rer_t + \frac{\gamma - \eta}{1-\gamma} lop_t - \gamma \frac{\overline{C}}{\overline{Y}_H} c_t - \gamma \frac{\overline{INV}}{\overline{Y}_H} inv_t. \tag{B.7.1}$$

B.7.3 Aggregate budget constraint

The production of domestic firms, the gross domestic product, is either consumed and invested in the domestic economy or it is exported to the foreign economy,

$$Y_{H,t} = C_{H,t} + INV_{H,t} + C_{H,t}^*.$$

Using the domestic demand for home goods (B.1.6) and its analogue for investment demand and also the foreign demand for home goods we get the aggregate resource constraint,

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\gamma)(C_t + INV_t) + \gamma\left(\frac{1}{RER_t}\right)^{-\eta} Y_t^*\right],$$

which in log-linear terms means

$$y_{H,t} = \frac{\overline{C}}{\overline{Y}_H}(1-\gamma)c_t + \frac{\overline{INV}}{\overline{Y}_H}(1-\gamma)inv_t + \gamma y_t^* + \frac{\eta\gamma(2-\gamma)}{1-\gamma}rer_t - \frac{\eta\gamma}{1-\gamma}lop_t. \tag{B.7.2}$$

B.8 Log-linearized equations

The model can be summarized by following set of log-linear equations:

- intratemporal optimality condition of households,

$$l_{H,t} = \frac{w_{H,t}}{\Psi} - \frac{c_t - \Upsilon c_{t-1}}{\Psi(1-\Upsilon)}, \tag{B.8.1}$$

- intertemporal optimality condition of households,

$$r_t = \frac{c_{t+1} - \Upsilon c_t}{1-\Upsilon} - \frac{c_t - \Upsilon c_{t-1}}{1-\Upsilon} + E_t\pi_{t+1}, \tag{B.8.2}$$

- uncovered interest parity condition,

$$rer_{t+1} - rer_t = (r_t - E_t\pi_{t+1}) - (r_t^* - E_t\pi_{t+1}^*) + \psi^B z_t + \psi^B a_t^{UIP}, \tag{B.8.3}$$

- equilibrium gross rental rate of capital,

$$r_{G,t} = y_{H,t} + mc_{H,t} - k_t - \left[\frac{\gamma}{1-\gamma}(rer_t - lop_t)\right], \tag{B.8.4}$$

- equilibrium wage of households,

$$w_{H,t} = y_{H,t} + mc_{H,t} - l_{H,t} - \left[\frac{\gamma}{1-\gamma}(rer_t - lop_t)\right], \tag{B.8.5}$$

- equilibrium wage of entrepreneurs,

$$w_{E,t} = y_{H,t} + mc_{H,t} - \left[\frac{\gamma}{1-\gamma}(rer_t - lop_t)\right], \tag{B.8.6}$$

- intermediate goods production function,

$$y_{H,t} = \alpha k_t + (1-\alpha)\Omega l_{H,t} + a_t^Y, \tag{B.8.7}$$

- law of motion for capital,

$$k_{t+1} = \delta inv_t + (1-\delta)k_t, \tag{B.8.8}$$

- equilibrium real price of capital,

$$q_t = \psi^I \delta(inv_t - k_t), \tag{B.8.9}$$

- gross return on capital investment definition,

$$r_{K,t} = \left(1 - \frac{1-\delta}{\overline{R}_K}\right) r_{G,t} + \frac{1-\delta}{\overline{R}_K} q_t - q_{t-1}, \tag{B.8.10}$$

- capital investment optimality condition of entrepreneurs,

$$E_t r_{K,t+1} = r_t - E_t \pi_{t+1} - \chi(n_{t+1} - q_t - k_{t+1}), \tag{B.8.11}$$

- law of motion for entrepreneurial net worth,

$$\begin{aligned} n_{t+1} = (1-\varsigma)\overline{R}_K \Big[&(\Gamma+1) r_{K,t} - \chi\Gamma(q_{t-1} + k_t) - \Gamma(r_{t-1} - \pi_t) + (\chi\Gamma + 1) n_t + \\ &+ a_t^{NW} \Big] + (\Gamma+1) \frac{\overline{W}_E}{\overline{K}} w_{E,t}, \end{aligned} \tag{B.8.12}$$

- domestic CPI inflation,

$$\pi_t = \frac{1}{1+\kappa\beta} \left[\beta E_t \pi_{t+1} + \kappa \pi_{t-1} + (1-\gamma)\Lambda_H mc_{H,t} + \gamma \Lambda_F lop_t\right], \tag{B.8.13}$$

- domestic monetary policy rule,

$$r_t = (1-\rho)\left(\beta_\pi E_t \pi_{t+1} + \Theta_y E_t y_{t+1}\right) + \rho r_{t-1} + \varepsilon_t^{MP}, \tag{B.8.14}$$

- law of motion for net foreign assets position,

$$z_t = \frac{1}{\beta} z_{t-1} + \gamma y_t^* + \gamma \frac{2\eta - \gamma\eta - 1}{1-\gamma} rer_t + \frac{\gamma - \eta}{1-\gamma} lop_t - \gamma \frac{\overline{C}}{\overline{Y}_H} c_t - \gamma \frac{\overline{INV}}{\overline{Y}_H} inv_t, \tag{B.8.15}$$

- gross domestic product definition, aggregate budget constraint,

$$y_{H,t} = \frac{\overline{C}}{\overline{Y}_H}(1-\gamma)c_t + \frac{\overline{INV}}{\overline{Y}_H}(1-\gamma)inv_t + \gamma y_t^* + \frac{\eta\gamma(2-\gamma)}{1-\gamma} rer_t - \frac{\eta\gamma}{1-\gamma} lop_t, \tag{B.8.16}$$

- exogenous stochastic process for law of one price gap shock,

$$lop_t = \rho_{LOP} lop_{t-1} + \varepsilon_t^{LOP}, \tag{B.8.17}$$

- exogenous stochastic process for domestic productivity shock,

$$a_t^Y = \rho_Y a_{t-1}^Y + \varepsilon_t^Y, \tag{B.8.18}$$

- exogenous stochastic process for debt-elastic interest rate shock,

$$a_t^{UIP} = \rho_{UIP} a_{t-1}^{UIP} + \varepsilon_t^{UIP}, \tag{B.8.19}$$

- exogenous stochastic process for shock in entrepreneurial net worth,

$$a_t^{NW} = \rho_{NW} a_{t-1}^{NW} + \varepsilon_t^{NW}, \tag{B.8.20}$$

- exogenous VAR(1) block for foreign variables,

$$\begin{pmatrix} y_t^* \\ \pi_t^* \\ r_t^* \end{pmatrix} = \begin{pmatrix} \rho_{y^*y^*} & \rho_{y^*\pi^*} & \rho_{y^*r^*} \\ \rho_{\pi^*y^*} & \rho_{\pi^*\pi^*} & \rho_{\pi^*r^*} \\ \rho_{r^*y^*} & \rho_{r^*\pi^*} & \rho_{r^*r^*} \end{pmatrix} \begin{pmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ r_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \sigma_{\pi^*y^*} & 1 & 0 \\ \sigma_{r^*y^*} & \sigma_{r^*\pi^*} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^{y^*} \\ \varepsilon_t^{\pi^*} \\ \varepsilon_t^{r^*} \end{pmatrix}. \tag{B.8.21}$$

B.9 Steady state conditions

Following conditions are assumed to hold in steady state:

- $\overline{P} = \overline{P}_H = \overline{P}_F = \overline{P}^* = 1,$
- $\overline{lop} = 0 \Rightarrow \overline{P}_F = \overline{SP}^*,$
- $\overline{A}^Y = \overline{A}^{UIP} = \overline{A}^{NW} = 1,$
- $\overline{D} = \overline{B} = 0 \Rightarrow \overline{Z} = \frac{\overline{SB}}{\overline{Y}_H \overline{P}_H} = 0,$
- $\overline{R}_G = \alpha \frac{\overline{Y}_H \overline{P}_H^W \overline{P}_H}{\overline{K} \overline{P}_H \overline{P}} \Rightarrow \frac{\overline{Y}_H}{\overline{K}} = \overline{R}_G \frac{\mu}{\alpha},$
- $\frac{\overline{C}}{\overline{Y}_H} = 1 - \frac{\overline{INV}}{\overline{Y}_H},$
- $\overline{R}_K = \left(\frac{\overline{N}}{\overline{QK}}\right)^{-\chi} \overline{R} = \left(\frac{\overline{N}}{\overline{K}}\right)^{-\chi} \frac{1}{\beta},$
- $\overline{R}_K = \frac{\overline{R}_G + (1-\delta)\overline{Q}}{\overline{Q}} \Rightarrow \overline{R}_G = \left(\frac{\overline{N}}{\overline{K}}\right)^{-\chi} \frac{1}{\beta} - (1-\delta),$
- $\frac{\overline{W}_E}{\overline{K}} = (1-\alpha)(1-\Omega)\frac{\overline{Y}_H}{\overline{K}},$
- $\overline{\pi} = \overline{\pi}_H = \overline{\pi}_F = \overline{\pi}^* = 0,$
- $\overline{S} = \overline{RER} = 1,$
- $\frac{\overline{P}_H^W}{\overline{P}_H} = \overline{MC}_H = \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon},$
- $\overline{R} = \frac{1}{\beta},$
- $\overline{INV} = \delta \overline{K},$
- $\frac{\overline{INV}}{\overline{Y}_H} = \delta \frac{\overline{K}}{\overline{Y}_H},$

B.10 State-space representation

B.10.1 Model with time-invariant structural parameters

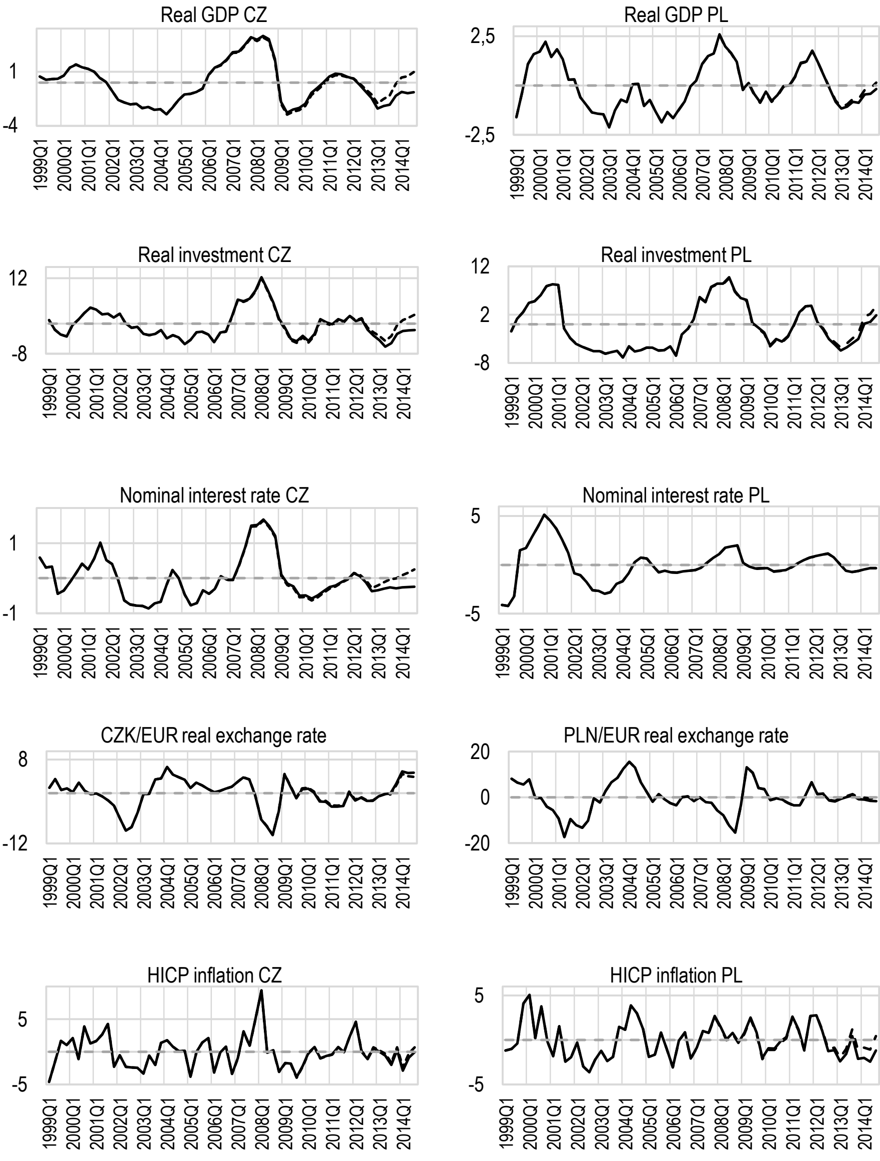
Observables: $y_t, inv_t, i_t, \pi_t, rer_t, y_t^*, i_t^*, \pi_t^*$.
Endogenous variables: $c_t, l_t, w_{H,t}, w_{E,t}, z_t, lop_t, mc_t, k_t, efp_t, r_{G,t}, r_{K,t}, q_t, , n_t, a_t^{NW}, a_t^{UIP}, a_t^Y$.
Exogenous shock innovations: $\varepsilon_t^{UIP}, \varepsilon_t^{LOP}, \varepsilon_t^{NW}, \varepsilon_t^{Y}, \varepsilon_t^{MP}, \varepsilon_t^{i^*}, \varepsilon_t^{\pi^*}, \varepsilon_t^{y^*}$.

B.10.2 Model with time-varying structural parameters

Observables: $y_t, inv_t, i_t, \pi_t, rer_t, y_t^*, i_t^*, \pi_t^*$.
Endogenous variables: $c_t, l_t, w_{H,t}, w_{E,t}, z_t, lop_t, mc_t, k_t, efp_t, r_{G,t}, r_{K,t}, q_t, , n_t, a_t^{NW}, a_t^{UIP}, a_t^Y$, $\Upsilon_t, \Psi_t, \psi_t^B, \psi_t^I, \eta_t, \kappa_t, \gamma_t, \chi_t, \Gamma_t, \varsigma_t, \theta_{H,t}, \theta_{F,t}, \beta_{\pi,t}, \Theta_{y,t}, \alpha_t^\theta$.
Exogenous shock innovations: $\varepsilon_t^{UIP}, \varepsilon_t^{LOP}, \varepsilon_t^{NW}, \varepsilon_t^{Y}, \varepsilon_t^{MP}, \varepsilon_t^{i^*}, \varepsilon_t^{\pi^*}, \varepsilon_t^{y^*}, \nu_t^\Upsilon, \nu_t^\Psi, \nu_t^{\psi^B}, \nu_t^{\psi^I}, \nu_t^\eta, \nu_t^\kappa$, $\nu_t^\gamma, \nu_t^\chi, \nu_t^\Gamma, \nu_t^\varsigma, \nu_t^{\theta_H}, \nu_t^{\theta_F}, \nu_t^{\beta_\pi}, \nu_t^{\Theta_y}, \nu_t^\alpha$.

Appendix C

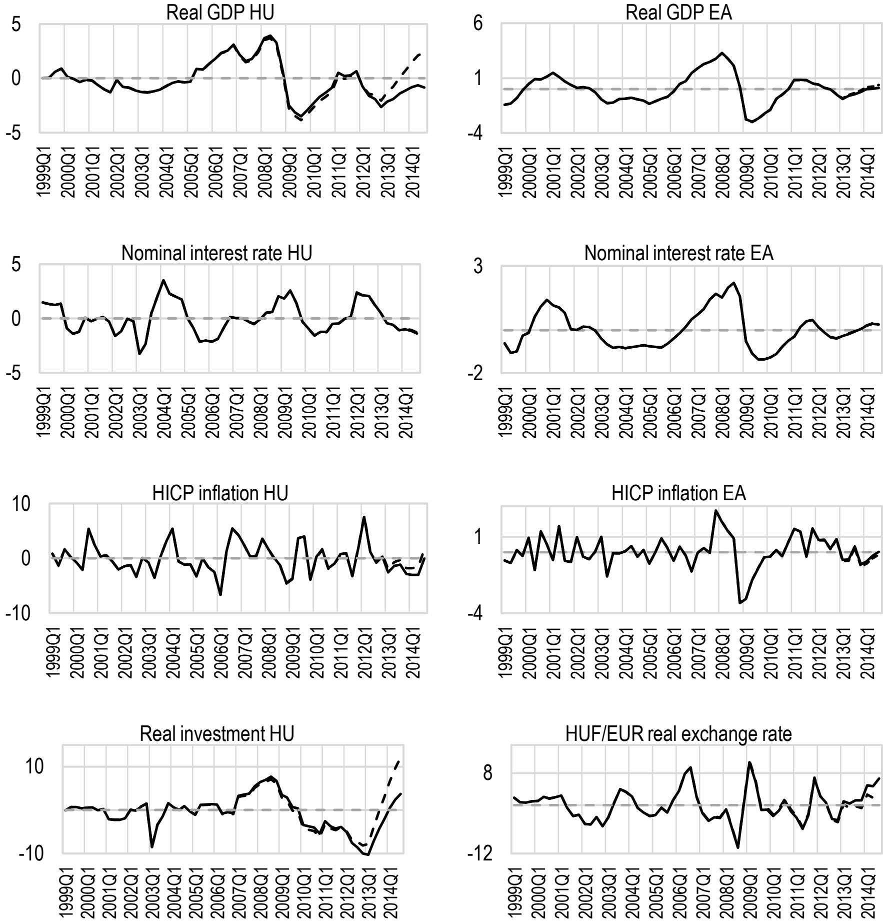
Figure C1 Source Data (CZ, SK)



Notes: Percentage deviations from steady state, data detrended with VAR forecast—black solid line, data detrended without VAR forecast—black dashed line.

Source: Authors' calculations.

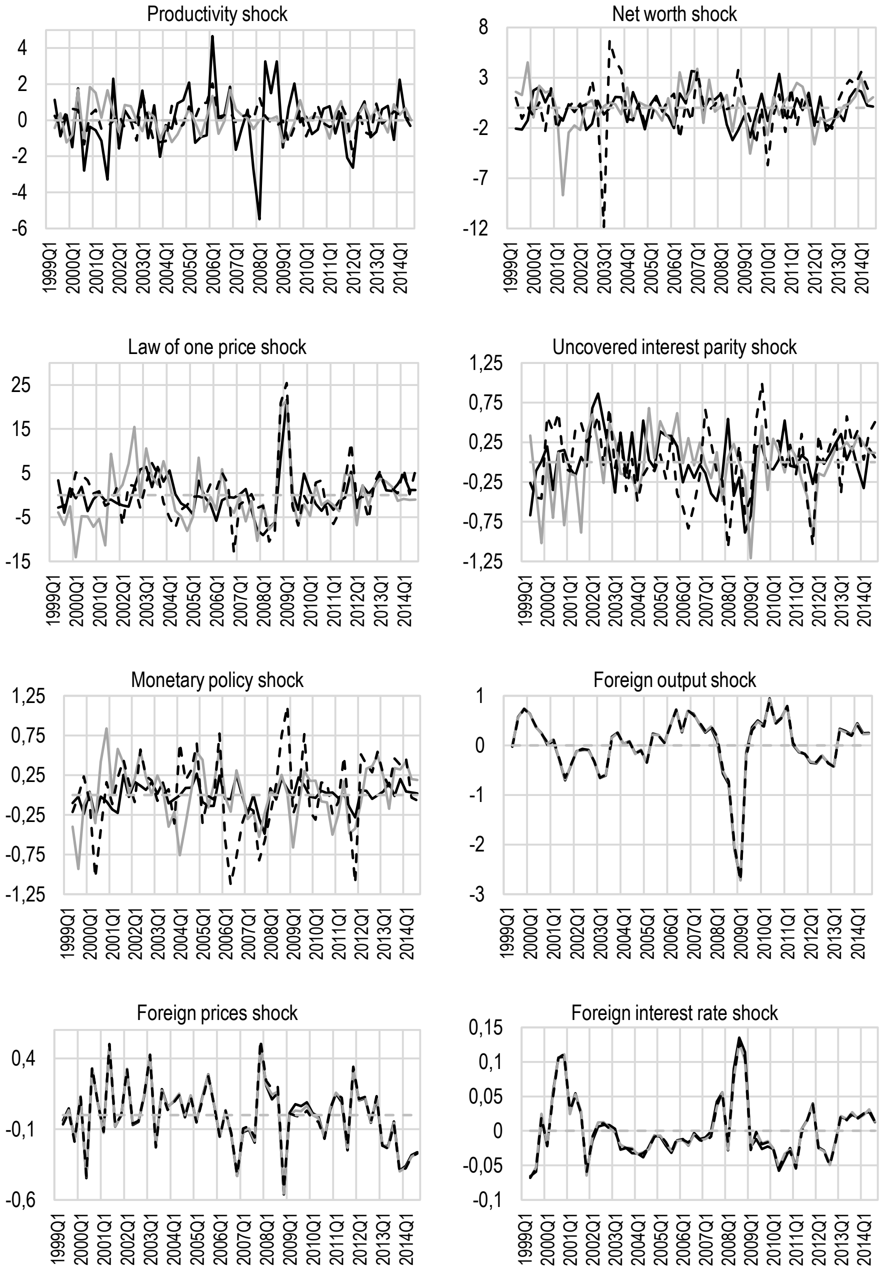
Figure C1 Source data (HU, EA)



Notes: Percentage deviations from steady state, data detrended with VAR forecast—black solid line, data detrended without VAR forecast—black dashed line.

Source: Authors' calculations.

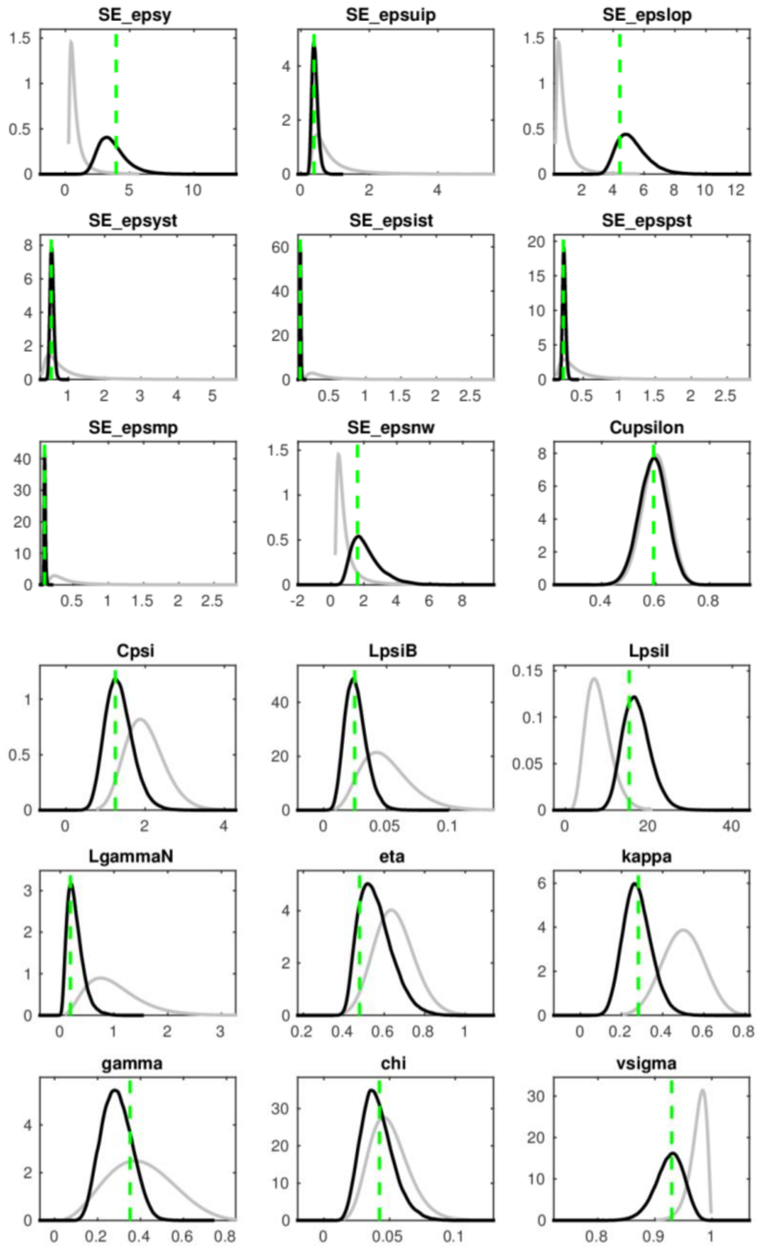
Figure C2 Filtered shock innovations



Notes: Percentage deviations from initial values, CZ—black solid line, PL—grey solid line, HU—black dashed line.

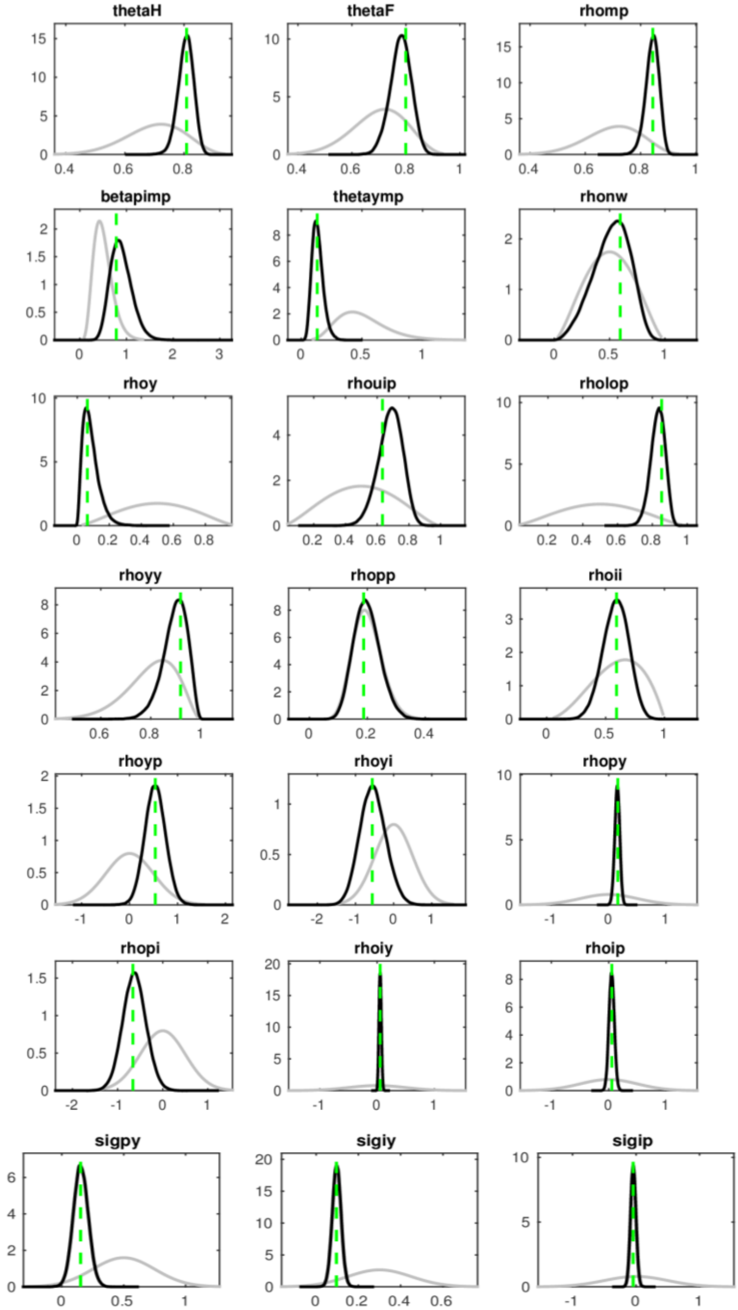
Source: Authors' calculations.

Figure C3 Prior and posterior densities, CZ (1 of 2)



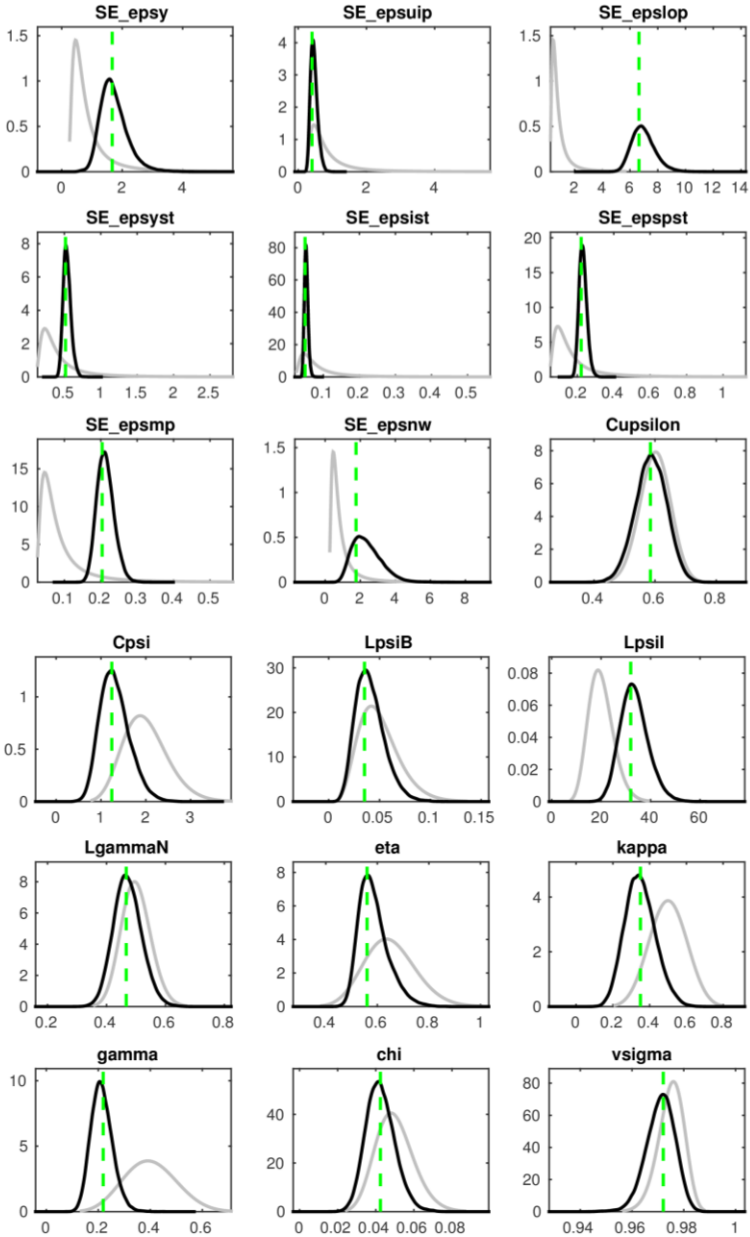
Notes: prior distribution—grey, posterior distribution—black, posterior mode—green dashed line.
Source: Authors' calculations.

Figure C4 Prior and posterior densities, CZ (2 of 2)



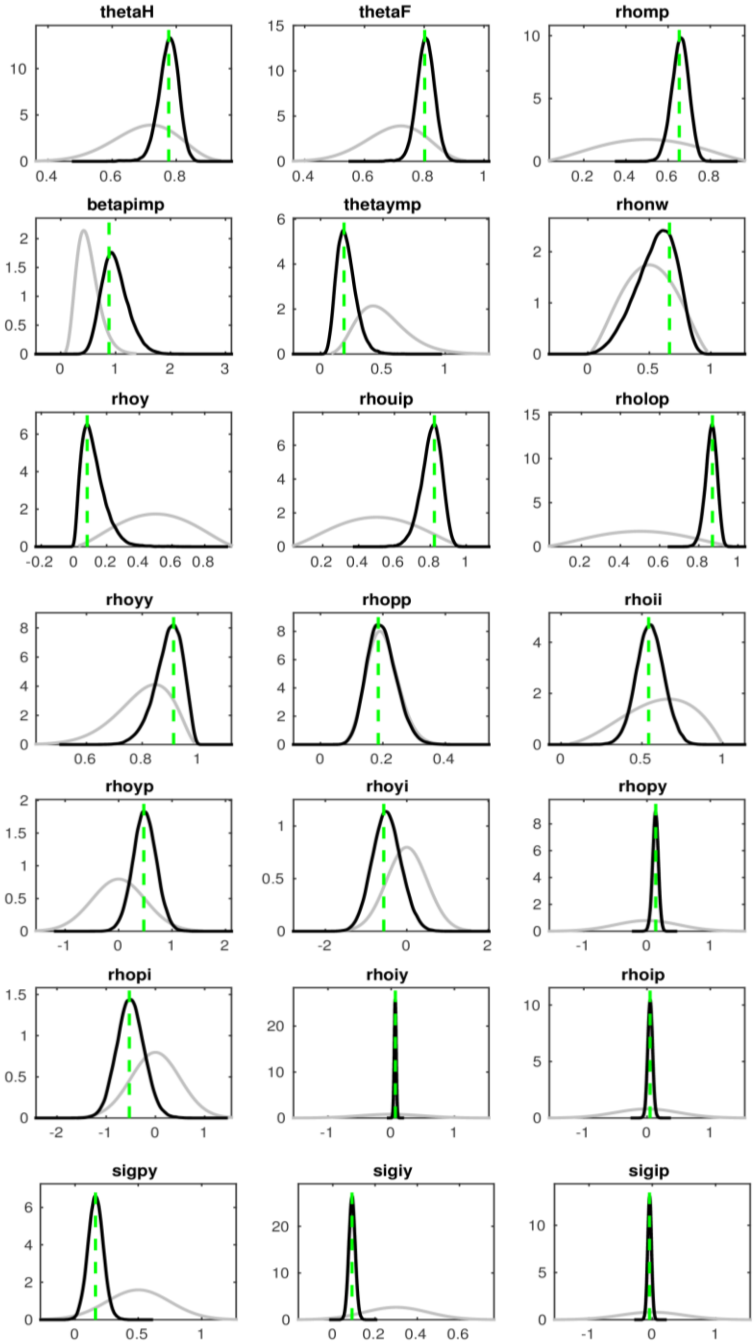
Notes: Prior distribution—grey, posterior distribution—black, posterior mode—green dashed line.
Source: Authors' calculations.

Figure C5 Prior and posterior densities, PL (1 of 2)



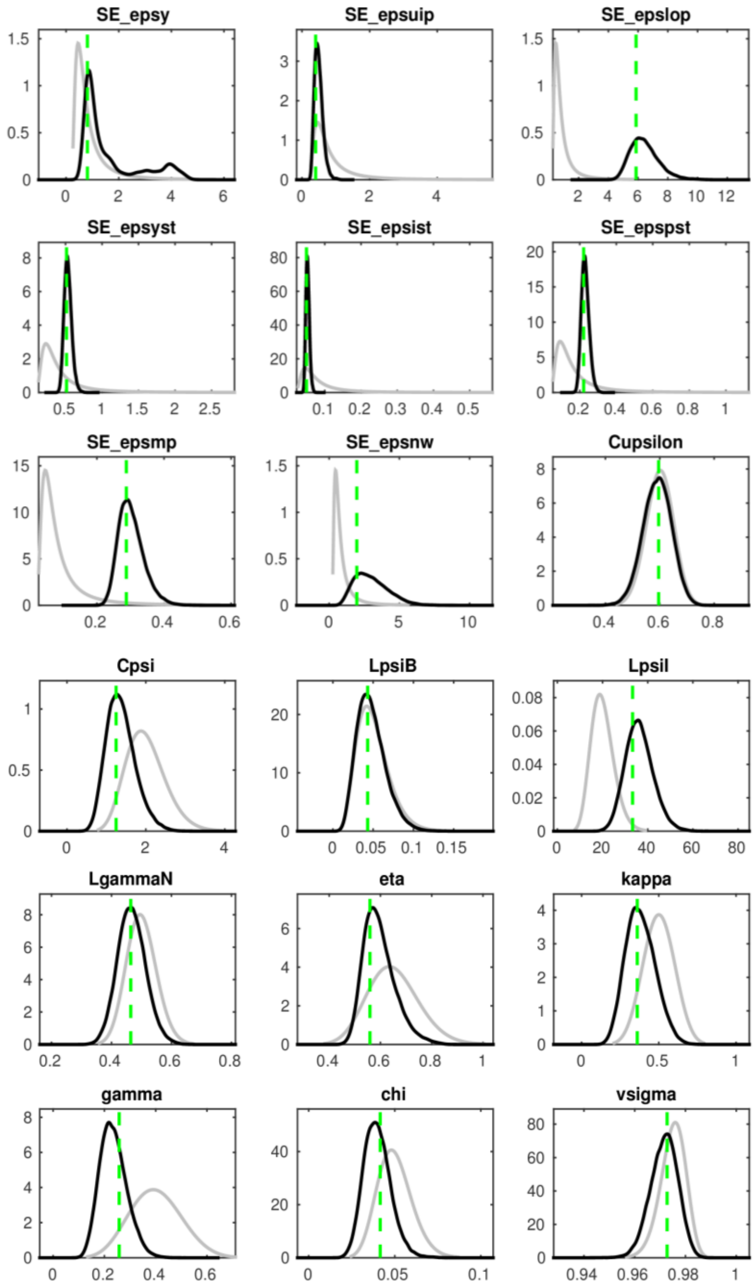
Notes: Prior distribution—grey, posterior distribution—black, posterior mode—green dashed line.
Source: Authors' calculations.

Figure C6 Prior and posterior densities, PL (2 of 2)



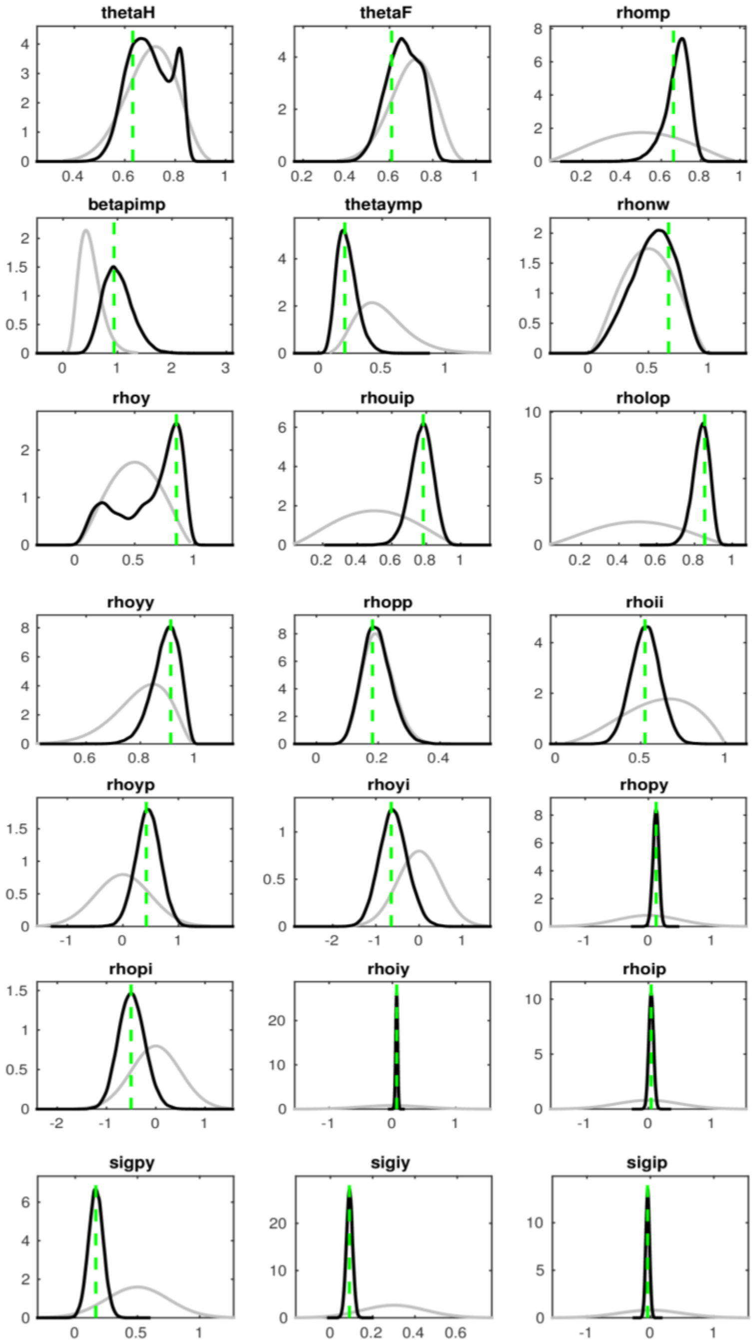
Notes: Prior distribution—grey, posterior distribution—black, posterior mode—green dashed line.
Source: Authors' calculations.

Figure C7 Prior and posterior densities, HU (1 of 2)



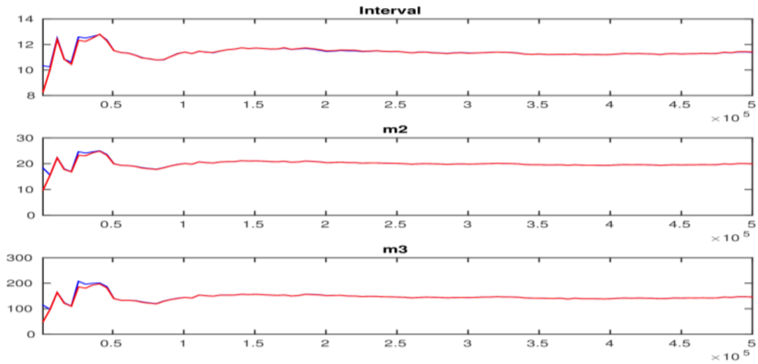
Notes: Prior distribution—grey, posterior distribution—black, posterior mode—green dashed line.
Source: Authors' calculations.

Figure C8 Prior and posterior densities, HU (2 of 2)



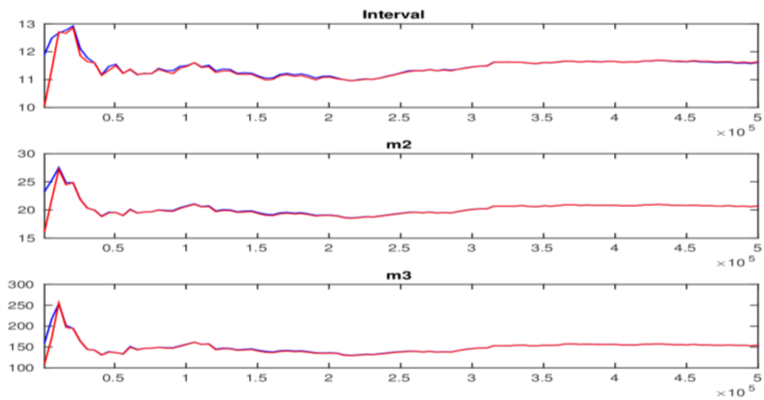
Notes: Prior distribution—grey, posterior distribution—black, posterior mode—green dashed line.
Source: Authors' calculations.

Figure C9 Convergence diagnostics, multivariate, CZ



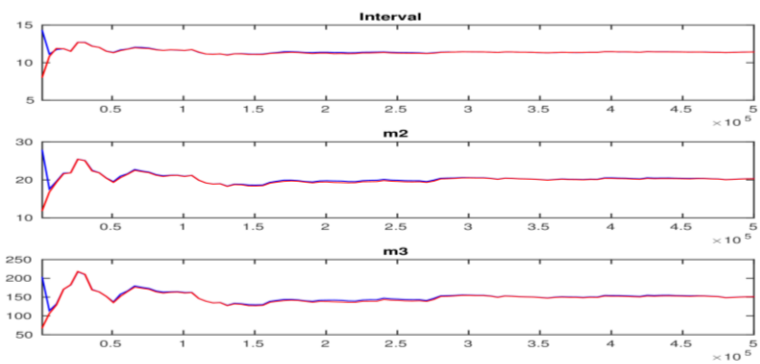
Source: Authors' calculations.

Figure C10 Convergence diagnostics, multivariate, PL



Source: Authors' calculations.

Figure C11 Convergence diagnostics, multivariate, HU



Source: Authors' calculations.