Business Cycle Synchronization through the Lens of a DSGE Model - Technical Appendix[1]

Martin SLANICAY - Faculty of Economics and Administration, Masaryk University, Brno (slanicay@mail.muni.cz)

[1]The content of this Appendix is posted in its original, unedited form.

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Appendix - Model

It is a New Keynesian model of two economies, originally presented in Kolasa (2009). The model assumes that there are only two economies in the world: domestic economy (indexed by H and represented by the Czech economy) and the foreign economy (indexed by F and represented by the Euro Area). Each economy is populated by a continuum of infinitely-lived inhabitants, in the domestic economy distributed over the interval $[0, n]$ and in the foreign economy over the interval $[n, 1]$. Both economies produce a continuum of differentiated tradable (non-tradable) goods that is also distributed over the interval $[0, n]$ in the domestic economy, and over the interval $[n, 1]$ in the foreign economy. The parameter n, therefore, represents the relative size of the domestic economy with respect to the foreign economy. Because both economies are modeled in the same way, the assumptions about representative agents as well as the parameters and variables of the model have the same interpretation in both economies. Moreover, derived equations describing the behavior of the economy have the same structural form in both economies. Therefore, I will describe the assumptions about agents and their optimization problems only in the domestic economy, knowing that the same optimization problems hold for the foreign economy. Parameters and variables in the foreign economy are distinguished from those in the domestic economy by an asterisk and for distinguishing tradable goods produced in the domestic economy and foreign economy I employ the subscripts "H" and "F". For example, C_H^* denotes foreign consumption of goods produced in the domestic economy (i.e. Czech export of consumption goods), while C_F denotes domestic consumption of goods produced in the foreign economy (i.e. Czech import of consumption goods).

Households

Households in a given economy are assumed to be homogenous. Households consume tradable and non-tradable goods produced by firms and make their intertemporal decisions about consumption by trading bonds. Households also supply labor and rent capital to firms. A typical household j in a domestic economy seeks to maximize its life-time utility function, which is function of household's consumption $C_t(j)$ and labor effort $L_t(j)$. The utility function is in the form CRRA function (constant relative risk aversion function)

$$U_t(j) = E_t \sum_{k=0}^{\infty} \beta^k \left[\frac{\varepsilon_{d,t+k}}{1-\sigma} (C_{t+k}(j) - H_{t+k})^{1-\sigma} - \frac{\varepsilon_{l,t+k}}{1+\phi} L_{t+k}(j)^{1+\phi} \right], \qquad (1)$$

where E_t denotes expectations in the period t, β is a discount factor, σ is an inverse elasticity of intertemporal substitution in consumption, $H_t = hC_{t-1}$ is an external habit taken by the household as exogenous, h is a parameter of habit formation in consumption, C_t is a composite consumption index (to be defined later), ϕ is an inverse elasticity of labor supply, $\varepsilon_{d,t}$ is a preference shock in the period t, which influences intertemporal decisions about consumption and $\varepsilon_{l,t}$ is a labor supply shock in the period t.

Maximization of the utility function (1) is subject to a set of flow budget constraints given by

$$\begin{aligned} P_{C,t}C_t(j) + P_{I,t}I_t(j) + E_t\{\Upsilon_{t,t+1}B_{t+1}(j)\} = B_t(j) + W_t(j)L_t(j) \\ +R_{K,t}K_t(j) + \Pi_{H,t}(j) + \Pi_{N,t}(j) + T_t(j), \qquad \text{for } t = 0, 1, 2 \ldots, \end{aligned} \qquad (2)$$

where $P_{C,t}$ denotes the price of the consumption C_t, $P_{I,t}$ is the price of investment goods I_t, B_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t,W_t is the nominal wage, $R_{K,t}$ denotes income of households achieved from renting capital K_t, $\Pi_{H,t}$ and $\Pi_{N,t}$ are dividends from tradable and non-tradable goods producers and T_t denotes lump sum government transfers net of lump sum taxes. $\Upsilon_{t,t+1}$ is the stochastic discount factor for nominal payoffs, such that $E_t\Upsilon_{t,t+1} = R_t^{-1}$, where R_t is the gross return on a riskless one-period bond.

Consumption Choice

First order condition of optimality for intertemporal decisions about consumption is in the form of a standard Euler equation

$$\beta R_t E_t \left\{ \frac{\varepsilon_{d,t+1}}{\varepsilon_{d,t}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \frac{P_{C,t}}{P_{C,t+1}} \right\} = 1. \tag{3}$$

Consumption index C_t consists of final tradable goods index $C_{T,t}$ and non-tradable goods index $C_{N,t}$ which are aggregated according to

$$C_t = \frac{C_{T,t}^{\gamma_c} C_{N,t}^{1-\gamma_c}}{\gamma_c^{\gamma_c}(1-\gamma_c)^{1-\gamma_c}},$$

where γ_c denotes share of final tradable goods in consumption of households. Following Burstein et al. (2003) and Corsetti and Dedola (2005), it is assumed that consumption of a final tradable good requires ω units of distribution services $Y_{D,t}$, which implies

$$C_{T,t} = \min\{C_{R,t}; \omega^{-1} Y_{D,t}\}. \tag{4}$$

The consumption index of raw tradable goods is defined as

$$C_{R,t} = \frac{C_{H,t}^{\alpha} C_{F,t}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}},$$

where α denotes share of domestic goods in the domestic basket of raw tradable goods[2] , $C_{H,t}$ is an index of home-made raw tradable goods and $C_{F,t}$ is

[2]Here I depart from the original specification of the model. Following Herber (2010) and Herber and Němec (2012) I am using a modified version of the model. Besides correcting several obvious typos, the modification is based on a different definition of the parameter α^*. In the original specification this parameter would be defined as a share of the Czech tradable goods in the overall index of the tradable goods in the Euro Area, while in the modified specification this parameter is defined as a share of the tradable goods produced in the Euro Area in the overall index of tradable goods in the Euro Area. It implies that the parameter α^* in the original specification is equal to $1-\alpha^*$ in the modified specification, which results in different structural forms of several equations. However, after substituting the actual calibrated values of the parameter α^* into the equations and correcting two obvious typos, we can see that the equations in both specifications are the

an index of foreign-made raw tradable goods, both consumed in the domestic economy and defined as

$$C_{H,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\phi_H}} \int_0^n C_t(z_H)^{\frac{\phi_H - 1}{\phi_H}} dz_H \right]^{\frac{\phi_H}{\phi_H - 1}},$$

$$C_{F,t} = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\phi_F}} \int_n^1 C_t(z_F)^{\frac{\phi_F - 1}{\phi_F}} dz_F \right]^{\frac{\phi_F}{\phi_F - 1}},$$

where ϕ_H (ϕ_F) is an elasticity of substitution between domestic (foreign) raw tradable goods, consumed in the domestic economy. Analogously, the consumption index of non-tradable goods is defined as

$$C_{N,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\phi_N}} \int_0^n C_t(z_N)^{\frac{\phi_N - 1}{\phi_N}} dz_N \right]^{\frac{\phi_N}{\phi_N - 1}},$$

where ϕ_N is an elasticity of substitution between domestic non-tradable goods.

Let us now discuss intratemporal decisions households make about consumption. First of all, households have to choose how many tradable goods and non-tradable goods they want to consume. Formally, households want to maximize consumption[3]

$$C_t = \frac{C_{T,t}^{\gamma_c} C_{N,t}^{1-\gamma_c}}{\gamma_c^{\gamma_c}(1-\gamma_c)^{1-\gamma_c}}, \tag{5}$$

conditionally on their consumption expenditures

$$P_{C,t}C_t = P_{T,t}C_{T,t} + P_{N,t}C_{N,t}.$$

same. The reason why I use the modified specification is as follows: In my opinion, the modified definition of α^* better corresponds to the definition of its counterpart in the domestic economy α.

[3]Equivalently, we can think about households wanting to minimize their consumption expenditures for a given level of their consumption.

The first order conditions for an optimal allocation of consumption expenditures between tradable and non-tradable goods imply that

$$C_{N,t} = (1-\gamma_c)\left(\frac{P_{N,t}}{P_{C,t}}\right)^{-1} C_t, \tag{6}$$

$$C_{T,t} = \gamma_c \left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t. \tag{7}$$

After substituting these allocation functions, i.e. (6) and (7), into the composite consumption index (5), we get a corresponding composite price index in the form

$$P_{C,t} = P_{T,t}^{\gamma_c} P_{N,t}^{1-\gamma_c}. \tag{8}$$

Consequently, household have to make a choice between home-made tradable goods and foreign-made tradable goods. As mentioned above, price of tradable goods $P_{T,t}$ depends on the price of raw tradable goods $P_{R,t}$ and also on the price of non-tradable distribution services $P_{N,t}$. Formally,

$$P_{T,t} = P_{R,t} + \omega P_{N,t}. \tag{9}$$

The price of distribution services is the same for both home-made tradable goods and foreign-made tradable goods, so it does not influence households' choice between them. Therefore, it is correct to assume that households want to maximize consumption of raw tradable goods

$$C_{R,t} = \frac{C_{H,t}^{\alpha} C_{F,t}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}, \tag{10}$$

conditional on their expenditures on raw tradable goods

$$P_{R,t}C_{R,t} = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}. \tag{11}$$

First order conditions of this maximization problem require

$$C_{H,t} = \alpha\gamma_c \left(\frac{P_{H,t}}{P_{R,t}}\right)^{-1} \left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t, \tag{12}$$

$$C_{F,t} = (1-\alpha)\gamma_c \left(\frac{P_{F,t}}{P_{R,t}}\right)^{-1} \left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t, \tag{13}$$

where I use the equilibrium condition $C_{R,t} = C_{T,t}$ and FOC condition (7).[4] After substituting these FOCs, i.e. (12) and (13), into the consumption index of raw tradable goods (10), we get a corresponding price index of raw tradable goods in the form

$$P_{R,t} = P_{H,t}^{\alpha} P_{F,t}^{1-\alpha}. \tag{14}$$

Finally, households have to choose which particular goods they want to consume. I show their optimization problem only for non-tradable goods as these optimization problems are analogous for home-made tradable goods and foreign-made tradable goods. Households want to minimize their expenditures on non-tradable goods

$$\int_0^n P_t(z_N) C_t(z_N) dz_N,$$

conditional on their consumption level of non-tradable goods

$$C_{N,t} = \left[\frac{1}{n}\int_0^n P_t(z_N)^{1-\phi_N} dz_N\right]^{\frac{1}{1-\phi_N}}$$

All these three minimization problems lead to first order conditions in the

[4]Condition $C_{R,t} = C_{T,t}$ is implied by (4), where the other potential condition $C_{T,t} = \omega^{-1} Y_{D,t}$ is not stable, because it would suggest that there are not enough distribution services to satisfy demand of households for tradable goods.

form

$$C_t(z_N) = \frac{1}{n}(1-\gamma_c)\left(\frac{P_t(z_N)}{P_{N,t}}\right)^{-\phi_N}\left(\frac{P_{N,t}}{P_{C,t}}\right)^{-1} C_t$$
$$C_t(z_H) = \frac{1}{n}\gamma_c\alpha\left(\frac{P_t(z_H)}{P_{H,t}}\right)^{-\phi_H}\left(\frac{P_{H,t}}{P_{R,t}}\right)^{-1}\left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t$$
$$C_t(z_F) = \frac{1}{1-n}\gamma_c(1-\alpha)\left(\frac{P_t(z_F)}{P_{F,t}}\right)^{-\phi_F}\left(\frac{P_{F,t}}{P_{R,t}}\right)^{-1}\left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t.$$

Corresponding price indices are in the form

$$P_{N,t} = \left[\frac{1}{n}\int_0^n P_t(z_N)^{1-\phi_N} dz_N\right]^{\frac{1}{1-\phi_N}}$$
$$P_{H,t} = \left[\frac{1}{n}\int_0^n P_t(z_H)^{1-\phi_H} dz_H\right]^{\frac{1}{1-\phi_H}}$$
$$P_{F,t} = \left[\frac{1}{1-n}\int_n^1 P_t(z_F)^{1-\phi_F} dz_F\right]^{\frac{1}{1-\phi_F}}$$

Similar optimization problems and resulting optimality conditions hold also for the foreign economy and are distinguished from those in the domestic economy by superscript "$*$".

Investment Decisions

Households use part of their income to accumulate capital K_t, assumed to be homogenous, which they rent to firms. Capital is accumulated according to the formula

$$K_{t+1} = (1-\tau)K_t + \varepsilon_{i,t}\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t, \tag{15}$$

where τ is a depreciation rate of capital and I_t denotes investment in the period t. Following Christiano et. al. (2005), capital accumulation is subject to investment specific technological shock $\varepsilon_{i,t}$ and adjustment costs represented by function $S(\cdot)$.[5] This function has to satisfy following properties $S(1) = S'(1) = 0$ and $S''(\cdot) = S'' > 0$.

In order to decide how much capital would a household accumulate, it is again necessary to solve the optimization problem. Household wants to maximize its utility expressed by (1), which is subject to the budget constraint (2) and to the formula for capital accumulation (15). First order conditions corresponding to capital K_t and investment I_t imply the following equations

$$\begin{aligned} \frac{P_{I,t}}{P_{C,t}} = \varepsilon_{i,t} \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S^{'}\left(\frac{I_t}{I_{t-1}}\right)\right) Q_{T,t} + \\ + E_t \left\{ \frac{P_{C,t+1}}{P_{C,t} R_t} \varepsilon_{i,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S^{'}\left(\frac{I_{t+1}}{I_t}\right) Q_{T,t+1} \right\}, \end{aligned} \tag{16}$$

$$Q_{T,t} = E_t \left\{ \frac{R_{K,t+1}}{P_{C,t+1}} \frac{P_{C,t+1}}{P_{C,t} R_t} \right\} + (1 - \tau) E_t \left\{ \frac{P_{C,t+1}}{P_{C,t} R_t} Q_{T,t+1} \right\}. \tag{17}$$

The equation (16) represents the demand for investment and the equation (17) determines a relative price of installed capital (known as Tobin's Q) which is defined as

$$Q_{T,t} = \frac{\lambda_{K,t}}{\lambda_{C,t} P_{C,t}},$$

where $\lambda_{C,t}$ is a marginal utility of nominal income (it is also a Lagrange multiplier on households' budget constraint) and $\lambda_{K,t}$ is a Lagrange multiplier on the law of motion for capital.

Homogenous investment goods are produced in a similar way as the final consumption goods, except that there are no distribution costs associated

[5] It is not important to know the exact form of this function, because a log-linearised form of the model contains only a second derivative of the function S'' (regarded as unknown parameter to be estimated).

with using tradable investment goods,[6] which implies the following definitions

$$I_t = \frac{I_{R,t}^{\gamma_i} I_{N,t}^{1-\gamma_i}}{\gamma_i^{\gamma_i}(1-\gamma_i)^{1-\gamma_i}},$$
$$I_{R,t} = \frac{I_{H,t}^{\alpha} I_{F,t}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}},$$
$$P_{I,t} = P_{R,t}^{\gamma_i} P_{N,t}^{1-\gamma_i}$$

It is assumed that a composition of consumption and investment basket in a given economy can differ, i.e. parameters γ_c and γ_i can be different, and that composition of tradable baskets is identical, i.e. parameter α is the same for both tradable consumption goods and tradable investment goods in the given economy.

Wage Setting

Each household is specialized in a different type of labor $L_t(j)$, which it supplies in a monopolistically competitive labor market. All supplied labor types are aggregated into homogenous labor input L_t according to the formula

$$L_t = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_W}} \int_0^n L_t(j)^{\frac{\phi_W - 1}{\phi_W}} dj \right]^{\frac{\phi_W}{\phi_W - 1}},$$

where ϕ_W is the elasticity of substitution between different labor types. A corresponding aggregate wage index is then defined as

$$W_t = \left[\frac{1}{n} \int_0^n W_t(j)^{1-\phi_W} dj \right]^{\frac{1}{1-\phi_W}},$$

where $W_t(j)$ denotes a wage of the household j. Cost minimization of firms implies the following demand schedules for each labor type $L_t(j)$ in the form

$$L_t(j) = \frac{1}{n} \left(\frac{W_t(j)}{W_t} \right)^{-\theta_W} L_t. \tag{18}$$

[6]Following Burstein et al. (2003).

Following Erceg, Henderson and Levin (2000), a wage setting mechanism a-la Calvo in a modified version with partial wage indexation is assumed. According to this set-up, every period only $1-\theta_W$ portion of households (randomly chosen) can reset their wages optimally, while the remaining portion of households θ_W adjust their wages according to the indexation rule

$$W_t(j) = W_{t-1}(j)\left(\frac{P_{C,t-1}}{P_{C,t-2}}\right)^{\delta_W},$$

where $\delta_W \in (0,1)$ is a parameter of wage indexation. If I set $\delta_W = 0$, I get the original Calvo wage setting mechanism, where all households which can not reoptimize their wages leave their wages unchanged. By setting $\delta_W = 1$, I get the Calvo wage setting mechanism with full wage indexation, where all households which can not reoptimize their wages fully adjust their wages according to the past inflation.

Households, which are allowed to reset their wages optimally, want to maximize their utility represented by the utility function (1), subject to the set of budget constraints (2) and labor demand constraints (18), taking into account the Calvo constraint that they can not always reset their wages. Formally, households want to maximize

$$E_t\sum_{k=0}^{\infty}\theta_W^k\beta^k\left[-\frac{\varepsilon_{l,t+k}}{1+\phi}L_{t+k}(j)^{1+\phi}+\lambda_{C,t+k}W_t(j)\left(\frac{P_{C,t+k-1}}{P_{C,t-1}}\right)^{\delta_W}L_{t+k}(j)\right],$$

which is subject to the following constraint

$$L_{t+k}(j) = \frac{1}{n}\left[\frac{W_t(j)}{W_{t+k}}\left(\frac{P_{C,t+k-1}}{P_{C,t-1}}\right)^{\delta_W}\right]^{-\phi_W}L_{t+k}.$$

First order condition of this optimization problem is in the form

$$E_t\sum_{k=0}^{\infty}\theta_W^k\beta^k\left[\frac{W_t(j)}{P_{C,t+k}}\left(\frac{P_{C,t+k-1}}{P_{C,t-1}}\right)^{\delta_W}-\frac{\phi_W}{\phi_W-1}MRS_{t+k}(j)\right]\cdot$$
$$\cdot\,\varepsilon_{d,t+k}(C_{t+k}(j)-hC_{t+k-1})^{-\sigma}L_{t+k}(j)=0,$$

where $\frac{\phi_W}{\phi_W - 1}$ represents gross wage mark-up resulting from certain monopoly power of the household, $MRS_t(j)$ is the marginal rate of substitution between labor and consumption of household j, defined as

$$MRS_t(j) = \frac{\varepsilon_{l,t} L_t(j)^{\phi}}{\varepsilon_{d,t}(C_t(j) - hC_{t-1})^{-\sigma}}.$$

Since all households face the same optimization problem, they all set the same optimal wage $\tilde{W}_t$. Therefore, the aggregate wage index is then defined as a weighted average of optimally set wages, and wages which are partially adjusted according to the past inflation. Formally,

$$W_t = \left[\theta_W \left(W_{t-1}\left(\frac{P_{C,t-1}}{P_{C,t-2}}\right)^{\delta_W}\right)^{1-\phi_W} + (1-\theta_W)\tilde{W}_t^{1-\phi_W}\right]^{\frac{1}{1-\phi_W}}.$$

Similar conditions and formulas hold also for the foreign economy. It is allowed for parameters governing the wage setting of households to differ between countries.

Firms

Production Technology

There is a continuum of homogenous, monopolistic competitive firms in the tradable and non-tradable sectors of the domestic economy. The production functions of firms are represented by Cobb-Douglas functions, homogenous in labor and capital of degree one (i.e. with constant returns to scale)

$$Y_t(z_N) = \varepsilon_{a^N,t} L_t(z_N)^{1-\eta} K_t(z_N)^{\eta},$$
$$Y_t(z_H) = \varepsilon_{a^H,t} L_t(z_H)^{1-\eta} K_t(z_H)^{\eta},$$

where η is the elasticity of output with respect to capital (common to both sectors, but potentially different in individual countries), and $\varepsilon_{a^H,t}$ ($\varepsilon_{a^N,t}$) is a productivity shock in the tradable (non-tradable) sector. The index of output in each sector is given by Dixit-Stiglitz aggregator

$$Y_{N,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_N}} \int_0^n Y_t(z_N)^{\frac{\phi_N - 1}{\phi_N}} dz_N \right]^{\frac{\phi_N}{\phi_N - 1}}$$

$$Y_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_H}} \int_0^n Y_t(z_H)^{\frac{\phi_H - 1}{\phi_H}} dz_H \right]^{\frac{\phi_H}{\phi_H - 1}}.$$

All firms try to minimize their costs for a given level of production. Formally, firms try to minimize

$$\min_{\substack{L_t(z_i)\\K_t(z_i)}} W_t L_t(z_i) + R_{K,t} K_t(z_i) + \lambda_{i,t}(Y_t(z_i) - \varepsilon_{a^i,t} L_t(z_i)^{1-\eta} K_t(z_i)^{\eta}), \qquad (19)$$

for $i = N, H$. Since all firms have the same technology and face the same prices of inputs, cost minimization (19) requires the same ratio of capital and labor for all domestic firms

$$\frac{W_t L_t}{R_{K,t} K_t} = \frac{1-\eta}{\eta}.$$

Lagrange multiplier $\lambda_{i,t}$ can be interpreted as nominal marginal costs. Therefore, the real marginal costs, identical for all firms in the given sector, are defined by the formula

$$MC_{i,t} = \frac{\lambda_{i,t}}{P_{i,t}} = \frac{1}{P_{i,t}\varepsilon_{a^i,t}} \left(\frac{W_t}{1-\eta}\right)^{1-\eta} \left(\frac{R_{K,t}}{\eta}\right)^{\eta}, \quad \text{for } i = N, H. \qquad (20)$$

Price Setting

In this section I shall describe price setting problem of firms in the domestic non-tradable sector. Price setting of foreign firms as well as firms in the tradable sector is defined analogously.

Firms in the non-tradable sector set their prices in order to maximize their profits. It is assumed that firms face modified Calvo restriction with partial indexation on the frequency of price adjustment. According to this restriction, every period only $1 - \theta_N$ portion of firms in non-tradable sector (randomly chosen) can reset their prices optimally, while θ_N portion of firms in non-tradable sector partially adjust their prices according to the past inflation, following the indexation rule

$$P_t(z_N) = P_{t-1}(z_N)\left(\frac{P_{N,t-1}}{P_{N,t-2}}\right)^{\delta_N},$$

where δ_N is a parameter of price indexation. Setting $\delta_N = 0$, I get the original Calvo constraint, as suggested by Calvo (1983). By setting $\delta_N = 1$, I get the Calvo constraint with full price indexation, where all firms which can not reoptimize their prices fully adjust their prices according to the past inflation.

A firm j, which is allowed to reset its price, chooses the price $P_t(z_N)$ in order to maximize current market value of profits generated until the firm can again reoptimize its price. Formally, firms solve maximization problem

$$E_t\sum_{k=0}^{\infty}\theta_N^k\beta^k\lambda_{C,t+k}Y_{t+k}(z_N)\left[P_t(z_N)\left(\frac{P_{N,t+k-1}}{P_{N,t-1}}\right)^{\delta_N} - P_{N,t+k}MC_{N,t+k}\right],$$

taking into account the sequence of demand constraints

$$Y_{t+k}(z_N) = \frac{1}{n}\left[\frac{P_t(z_N)}{P_{N,t+k}}\left(\frac{P_{N,t+k-1}}{P_{N,t-1}}\right)^{\delta_N}\right]^{-\phi_N} Y_{N,t+k},$$

where $\lambda_{C,t}$ is the marginal utility of households' nominal income in period t, considered by firms as exogenous, and $MC_{N,t}$ is the real marginal costs in the period t, defined in (20). The first order condition of the maximization problem of firms implies

$$\begin{aligned} E_t\sum_{k=0}^{\infty}\theta_N^k\beta^k\lambda_{C,t+k}Y_{t+k}(z_N)\Bigg[P_t(z_N)\left(\frac{P_{N,t+k-1}}{P_{N,t-1}}\right)^{\delta_N} - \\ -\frac{\phi_N}{\phi_N - 1}P_{N,t+k}MC_{N,t+k}\Bigg] = 0. \end{aligned} \tag{21}$$

Firms do not face any firm-specific shocks, so all firms in the given sector choose the same optimal price $\tilde{P}_{N,t}$. Hence, it is possible to express the aggregate price index of non-tradable goods as a weighted average of the optimizing firms' price $\tilde{P}_{N,t}$, and the price of firms which adjust their price to the previous inflation

$$P_{N,t} = \left[\theta_N \left(P_{N,t-1} \left(\frac{P_{N,t-1}}{P_{N,t-2}} \right)^{\delta_N} \right)^{1-\phi_N} + (1-\theta_N) \tilde{P}_{N,t}^{1-\phi_N} \right]^{\frac{1}{1-\phi_N}} . \quad (22)$$

Foreign firms and domestic firms in the tradable sector deal with analogous maximization problems. Therefore, first order conditions and resulting price indices associated with maximization problems of foreign firms and domestic firms in tradable sector are analogous to those expressed in equations (21) and (22). It is assumed that structural parameters of price stickiness θ and price indexation δ as well as stochastic properties of shocks in productivity can differ among countries and sectors.

It is assumed that prices are set in the producer's currency and that international law of one price holds for intermediate tradable goods. Thus, prices of domestic goods sold in the foreign economy and prices of foreign goods sold in the domestic economy are given by formulas

$$P_t^*(z_H) = ER_t^{-1} P_t(z_H) \qquad P_t(z_F) = ER_t P_t^*(z_F),$$

where ER_t is the nominal exchange rate expressed as units of domestic currency per one unit of foreign currency.

International Risk Sharing

The assumption of complete financial markets implies the perfect risk-sharing condition. Loosely speaking, this condition requires that prices of similar bonds must be the same in the domestic as well as in the foreign economy.

This condition can be expressed using gross returns on these bonds as

$$R_t = R_t^* E_t \left\{ \frac{ER_{t+1}}{ER_t} \right\}.$$

This formula requires that gross returns on domestic bonds must be the same as gross returns on foreign bonds adjusted by expected appreciation (depreciation) of the foreign currency. By substituting for the gross returns on domestic and foreign bonds from the Euler equation (3) and after subsequent mathematical manipulation we get the formula

$$Q_t = \kappa \frac{\varepsilon_{d,t}^*}{\varepsilon_{d,t}} \frac{(C_t^* - h^* C_{t-1}^*)^{-\sigma^*}}{(C_t - hC_{t-1})^{-\sigma}}, \tag{23}$$

where

$$\kappa = E_t \left\{ Q_{t+1} \frac{\varepsilon_{d,t+1}(C_{t+1} - hC_t)^{-\sigma}}{\varepsilon_{d,t+1}^*(C_{t+1}^* - h^* C_t^*)^{-\sigma^*}} \right\}$$

is regarded as a constant, which (using iterations) depends on initial conditions and Q_t is a real exchange rate defined as

$$Q_t = \frac{ER_t P_{C,t}^*}{P_{C,t}}. \tag{24}$$

Formula (23) implies that the real exchange rate is proportional to the ratio of marginal utility of consumption between domestic and foreign households.

The real exchange rate can deviate from purchasing power parity (PPP) because of changes in relative prices of tradable and non-tradable goods, changes in relative distribution costs and changes in terms of trade, as long as there is a difference between household preferences among countries, i.e. $\alpha \neq 1 - \alpha^*$. This can be demonstrated this by substituting for the price indices in the definition of real exchange rate (24) from definitions of these price indices (8), (9) and (14). After some mathematical manipulation we obtain

$$Q_t = S_t^{\alpha+\alpha^*-1} \frac{1 + \omega^* D_t^*}{1 + \omega D_t} \frac{X_t^{*1-\gamma_c^*}}{X_t^{1-\gamma_c}},$$

where S_t are terms of trade defined as domestic import prices relative to domestic export prices[7]

$$S_t = \frac{ER_t P^*_{F,t}}{P_{H,t}},$$

X_t and X^*_t are internal exchange rates defined as prices of non-tradable goods relative to prices of tradable goods

$$X_t = \frac{P_{N,t}}{P_{T,t}} \qquad X^*_t = \frac{P^*_{N,t}}{P^*_{T,t}}$$

and D_t and D^*_t are relative distribution costs, defined as prices of non-tradable goods relative to prices of raw tradable goods

$$D_t = \frac{P_{N,t}}{P_{R,t}} \qquad D^*_t = \frac{P^*_{N,t}}{P^*_{R,t}}.$$

Monetary and Fiscal Authorities

The behavior of central bank is described by a variant of Taylor rule.[8]

$$R_t = R^{\rho}_{t-1}\left[E_t\left\{\left(\frac{Y_{t+1}}{\overline{Y}}\right)^{\phi_y}\left(\frac{P_{C,t+1}}{(1+\overline{\pi})P_{C,t}}\right)^{\phi_\pi}\right\}\right]^{1-\rho}\varepsilon_{m,t},$$

where ρ is a parameter of interest rate smoothing, Y_t is a total output in the economy, $\overline{Y}$ denotes a steady state level of this output, $\overline{\pi}$ is a steady state level of inflation, ϕ_y is an elasticity of the interest rate to the output, ϕ_π is an elasticity of the interest rate to inflation and $\varepsilon_{m,t}$ is a monetary policy shock.

Fiscal policy is modeled in a very simple fashion. Government expenditures and transfers to households are fully financed by lump-sum taxes so

[7] The assumption of law of one price for tradable goods implies $S^*_t = S^{-1}_t$.

[8] Here I depart from the original specification of the model. I changed the specification of the interest rate rules. In the original model, interest rates depend on current inflation and output, while in my specification interest rates depend on expected inflation and expected output. This, in my view, better corresponds with the actual behavior of central banks in both economies.

that state budget is balanced every period. Government expenditures consist only of non-tradable domestic goods and are modeled as a stochastic process $\varepsilon_{g,t}$. Given the assumptions about households, Ricardian equivalence holds in this model.

Market Clearing Conditions

The model is closed by satisfying the market clearing conditions. Goods market clearing requires that output of each firm producing non-tradable goods is either consumed by households in the domestic economy, spent on investment, used for distribution services or purchased by the government. Similarly, output of firms producing tradable goods is either consumed or invested in the domestic or foreign economy. Formally

$$Y_{N,t} = C_{N,t} + I_{N,t} + Y_{D,t} + G_t, \tag{25}$$

$$Y_{H,t} = C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t}. \tag{26}$$

By plugging in allocation functions (6), (7), and (12) together with analogous allocation functions for investment and their foreign counterparts into the goods market clearing conditions (25) and (26), the aggregate output in both domestic sectors can be rewritten as

$$\begin{aligned} Y_{N,t} = (1-\gamma_c)\left(\frac{P_{N,t}}{P_{C,t}}\right)^{-1} C_t + \omega\gamma_c\left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t + \\ +(1-\gamma_i)\left(\frac{P_{N,t}}{P_{I,t}}\right)^{-1} I_t + G_t, \end{aligned} \tag{27}$$

$$\begin{aligned} Y_{H,t} = \alpha\gamma_c\left(\frac{P_{H,t}}{P_{R,t}}\right)^{-1}\left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t + \frac{1-n}{n}(1-\alpha^*)\gamma_c^*\left(\frac{P^*_{H,t}}{P^*_{R,t}}\right)^{-1}\left(\frac{P^*_{T,t}}{P^*_{C,t}}\right)^{-1} C^*_t \\ + \alpha\gamma_i\left(\frac{P_{H,t}}{P_{R,t}}\right)^{-1}\left(\frac{P_{R,t}}{P_{I,t}}\right)^{-1} I_t + \frac{1-n}{n}(1-\alpha^*)\gamma_i^*\left(\frac{P^*_{H,t}}{P^*_{R,t}}\right)^{-1}\left(\frac{P^*_{R,t}}{P^*_{I,t}}\right)^{-1} I^*_t, \end{aligned}$$

where in (27) I use the following condition of optimality

$$Y_{D,t} = \omega C_{T,t},$$

which links distribution services with tradable consumption goods. The total output in the economy is given by the sum of output in tradable and non-tradable sectors

$$Y_t = Y_{N,t} + Y_{H,t}.$$

Finally, market clearing conditions for factor markets requires

$$L_t = \int_0^n L_t(z_N)dz_N + \int_0^n L_t(z_H)dz_H$$
$$K_t = \int_0^n K_t(z_N)dz_N + \int_0^n K_t(z_H)dz_H.$$

Analogous market clearing conditions hold for the foreign economy, too.

Exogenous Shocks

Behavior of the model is driven by seven structural shocks in each economy: productivity shocks in tradable sector ($\varepsilon_{a^H,t}$ and $\varepsilon^*_{a^F,t}$), productivity shocks in non-tradable sector ($\varepsilon_{a^N,t}$ and $\varepsilon^*_{a^N,t}$), labor supply shocks ($\varepsilon_{l,t}$ and $\varepsilon^*_{l,t}$), investment efficiency shocks ($\varepsilon_{i,t}$ and $\varepsilon^*_{i,t}$), consumption preference shocks ($\varepsilon_{d,t}$ and $\varepsilon^*_{d,t}$), government spending shocks ($\varepsilon_{g,t}$ and $\varepsilon^*_{g,t}$) and monetary policy shocks ($\varepsilon_{m,t}$ and $\varepsilon^*_{m,t}$). Except for monetary policy shocks, all other shocks are represented by AR1 processes in the log-linearised version of the model, see (68) - (79). Monetary policy shocks are represented by IID processes in the log-linearised version of the model.[9] I also allow for correlations between innovations in corresponding domestic and foreign shocks.

[9] IID - identically and independently distributed

Log-linearised Model

The model presented above is highly non-linear and does not have any analytical solution. A log-linear approximation around the non-stochastic steady state is employed for the purposes of empirical analysis. For details about methods of log-linear approximation see Uhlig (1995). Nice introduction to methods used for log-linearising around the steady state is provided by Zietz (2006). In this section I present a log-linearised form of the model. All variables of the model are in the form of log-deviations from their respective steady state. Formally, $x_t = \log X_t - \log \overline{X}$, where $\overline{X}$ is a steady state value.

The model is formed by 40 equations describing endogenous variables (from equation (28) to equation (67)) and by 12 equations for exogenous shocks (from equation (68) to equation (79)). An interpretation of the model variables is presented in Table 1. Interpretation of the structural parameters and the parameters related to shocks is presented in Tables 2 and 3.

Market Clearing Conditions:

$$
\begin{aligned}
y_{H,t} =& \frac{\overline{C}}{\overline{Y}_H}\frac{\gamma_c\alpha}{1+\omega}\left(c_t + (1-\gamma_c)x_t + (1-\alpha)s_t\right) \\
&+ \frac{\overline{C}^*}{\overline{Y}_H}\frac{1-n}{n}\frac{\gamma_c^*(1-\alpha^*)}{1+\omega^*}\left(c_t^* + (1-\gamma_c^*)x_t^* + \alpha^* s_t\right) \\
&+ \frac{\overline{I}}{\overline{Y}_H}\gamma_i\alpha\left(i_t + (1-\gamma_i)(1+\omega)x_t + (1-\alpha)s_t\right) \\
&+ \frac{\overline{I}^*}{\overline{Y}_H}\frac{1-n}{n}\gamma_i^*(1-\alpha^*)\left(i_t^* + (1-\gamma_i^*)(1+\omega)x_t^* + \alpha^* s_t\right)
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
y^*_{F,t} =& \frac{\overline{C}^*}{\overline{Y}^*_F}\frac{\gamma^*_c\alpha^*}{1+\omega^*}\left(c^*_t + (1-\gamma^*_c)x^*_t - (1-\alpha^*)s_t\right) \\
&+ \frac{\overline{C}}{\overline{Y}^*_F}\frac{n}{1-n}\frac{\gamma_c(1-\alpha)}{1+\omega}\left(c_t + (1-\gamma_c)x_t - \alpha s_t\right) \\
&+ \frac{\overline{I}^*}{\overline{Y}^*_F}\gamma^*_i\alpha^*\left(i^*_t + (1-\gamma^*_i)(1+\omega^*)x^*_t - (1-\alpha^*)s_t\right) \\
&+ \frac{\overline{I}}{\overline{Y}^*_F}\frac{n}{1-n}\gamma_i(1-\alpha)\left(i_t + (1-\gamma_i)(1+\omega)x_t - \alpha s_t\right)
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
y_{N,t} =& \frac{\overline{C}}{\overline{Y}_N}\left((1-\gamma_c)(c_t - \gamma_c x_t) + \frac{\gamma_c\omega}{1+\omega}(c_t + (1-\gamma_c)x_t)\right) \\
&+ \frac{\overline{I}}{\overline{Y}_N}\left(1-\gamma_i)(i_t - \gamma_i(1+\omega)x_t\right) + \frac{\overline{G}}{\overline{Y}_N}\varepsilon_{g,t}
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
y^*_{N,t} =& \frac{\overline{C}^*}{\overline{Y}^*_N}\left((1-\gamma^*_c)(c^*_t - \gamma^*_c x^*_t) + \frac{\gamma^*_c\omega^*}{1+\omega^*}(c^*_t + (1-\gamma^*_c)x^*_t)\right) \\
&+ \frac{\overline{I}^*}{\overline{Y}^*_N}\left(1-\gamma^*_i)(i^*_t - \gamma^*_i(1+\omega^*)x^*_t\right) + \frac{\overline{G}^*}{\overline{Y}^*_N}\varepsilon^*_{g,t}
\end{aligned} \tag{31}
$$

$$
y_t = \frac{\overline{Y}_H}{\overline{Y}}y_{H,t} + \frac{\overline{Y}_N}{\overline{Y}}y_{N,t} \tag{32}
$$

$$
y^*_t = \frac{\overline{Y}^*_F}{\overline{Y}^*}y^*_{F,t} + \frac{\overline{Y}^*_N}{\overline{Y}^*}y^*_{N,t} \tag{33}
$$

Euler Equation:

$$
c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \frac{1-h}{\sigma}E_t\left(r_t - \pi_{t+1} + \varepsilon_{d,t+1} - \varepsilon_{d,t}\right) \tag{34}
$$

$$
c^*_t - h^*c^*_{t-1} = E_t(c^*_{t+1} - h^*c^*_t) - \frac{1-h^*}{\sigma^*}E_t\left(r^*_t - \pi^*_{t+1} + \varepsilon^*_{d,t+1} - \varepsilon^*_{d,t}\right) \tag{35}
$$

International Risk Sharing Condition:

$$
q_t = \varepsilon^*_{d,t} - \varepsilon_{d,t} - \frac{\sigma^*}{1-h^*}(c^*_t - h^*c^*_{t-1}) + \frac{\sigma}{1-h}(c_t - hc_{t-1}) \tag{36}
$$

Capital Accumulation:

$$k_{t+1} = (1-\tau)k_t + \tau(i_t + \varepsilon_{i,t}) \tag{37}$$
$$k^*_{t+1} = (1-\tau^*)k^*_t + \tau^*(i^*_t + \varepsilon^*_{i,t}) \tag{38}$$

Real Costs of Capital:

$$r_{K,t} = w_t + l_t - k_t \tag{39}$$
$$r^*_{K,t} = w^*_t + l^*_t - k^*_t \tag{40}$$

Investment Demand:

$$i_t - i_{t-1} = \beta E_t(i_{t+1} - i_t) + \frac{1}{S''}(q_{T,t} + \varepsilon_{i,t}) - \frac{\gamma_i(1+\omega) - \gamma_c}{S''}x_t \tag{41}$$
$$i^*_t - i^*_{t-1} = \beta^* E_t(i^*_{t+1} - i^*_t) + \frac{1}{S''*}(q^*_{T,t} + \varepsilon^*_{i,t}) - \frac{\gamma^*_i(1+\omega^*) - \gamma^*_c}{S''*}x^*_t \tag{42}$$

Price of Installed Capital:

$$q_{T,t} = \beta(1-\tau)E_t q_{T,t+1} - (r_t - E_t\pi_{t+1}) + (1-\beta(1-\tau))E_t r_{K,t+1} \tag{43}$$
$$q^*_{T,t} = \beta^*(1-\tau^*)E_t q^*_{T,t+1} - (r^*_t - E_t\pi^*_{t+1}) + (1-\beta^*(1-\tau^*))E_t r^*_{K,t+1} \tag{44}$$

Labor Input:

$$l_t = \eta(r_{K,t} - w_t) + \frac{\overline{Y}_H}{\overline{Y}}(y_{H,t} - \varepsilon_{a^H,t}) + \frac{\overline{Y}_N}{\overline{Y}}(y_{N,t} - \varepsilon_{a^N,t}) \tag{45}$$
$$l^*_t = \eta^*(r^*_{K,t} - w^*_t) + \frac{\overline{Y}^*_F}{\overline{Y}^*}(y^*_{F,t} - \varepsilon^*_{a^F,t}) + \frac{\overline{Y}^*_N}{\overline{Y}^*}(y^*_{N,t} - \varepsilon^*_{a^N,t}) \tag{46}$$

Real Wage:

$$\begin{aligned} w_t - w_{t-1} =& \frac{(1-\theta_W)(1-\beta\theta_W)}{\theta_W(1+\phi_W\phi)}(mrs_t - w_t) + \beta E_t(w_{t+1} - w_t) \\ &+ \beta E_t(\pi_{t+1} - \delta_W\pi_t) - (\pi_t - \delta_W\pi_{t-1}) \end{aligned} \tag{47}$$

$$w_t^* - w_{t-1}^* = \frac{(1-\theta_W^*)(1-\beta^*\theta_W^*)}{\theta_W^*(1+\phi_W^*\phi^*)}(mrs_t^* - w_t^*) + \beta^* E_t(w_{t+1}^* - w_t^*) + \beta^* E_t(\pi_{t+1}^* - \delta_W^*\pi_t^*) - (\pi_t^* - \delta_W^*\pi_{t-1}^*) \tag{48}$$

Marginal Rate of Substitution:

$$mrs_t = \varepsilon_{l,t} + \phi l_t - \varepsilon_{d,t} + \frac{\sigma}{1-h}(c_t - hc_{t-1}) \tag{49}$$

$$mrs_t^* = \varepsilon_{l,t}^* + \phi^* l_t^* - \varepsilon_{d,t}^* + \frac{\sigma^*}{1-h^*}(c_t^* - h^* c_{t-1}^*) \tag{50}$$

Phillips Curve for Tradable Sector:

$$\pi_{H,t} - \delta_H\pi_{H,t-1} = \beta E_t(\pi_{H,t+1} - \delta_H\pi_{H,t}) + \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H} mc_{H,t} \tag{51}$$

$$\pi_{F,t}^* - \delta_F^*\pi_{F,t-1}^* = \beta^* E_t(\pi_{F,t+1}^* - \delta_F^*\pi_{F,t}^*) + \frac{(1-\theta_F^*)(1-\beta^*\theta_F^*)}{\theta_F^*} mc_{F,t}^* \tag{52}$$

Phillips Curve for Non-tradable Sector:

$$\pi_{N,t} - \delta_N\pi_{N,t-1} = \beta E_t(\pi_{N,t+1} - \delta_N\pi_{N,t}) + \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N} mc_{N,t} \tag{53}$$

$$\pi_{N,t}^* - \delta_N^*\pi_{N,t-1}^* = \beta^* E_t(\pi_{N,t+1}^* - \delta_N^*\pi_{N,t}^*) + + \frac{(1-\theta_N^*)(1-\beta^*\theta_N^*)}{\theta_N^*} mc_{N,t}^* \tag{54}$$

Real Marginal Costs in Tradable Sector:

$$mc_{H,t} = (1-\eta)w_t + \eta r_{K,t} - \varepsilon_{a^H,t} + (1-\alpha)s_t + (1-\gamma_c+\omega)x_t \tag{55}$$

$$mc_{F,t}^* = (1-\eta^*)w_t^* + \eta^* r_{K,t}^* - \varepsilon_{a^F,t}^* + (1-\alpha^*)s_t + (1-\gamma_c^*+\omega^*)x_t^* \tag{56}$$

Real Marginal Costs in Non-tradable Sector:

$$mc_{N,t} = (1-\eta)w_t + \eta r_{K,t} - \varepsilon_{a^N,t} - \gamma_c x_t \tag{57}$$

$$mc_{N,t}^* = (1-\eta^*)w_t^* + \eta^* r_{K,t}^* - \varepsilon_{a^N,t}^* - \gamma_c^* x_t^* \tag{58}$$

Relative Price of Non-tradable Goods:

$$x_t - x_{t-1} = \pi_{N,t} - \pi_{T,t} \tag{59}$$

$$x_t^* - x_{t-1}^* = \pi_{N,t}^* - \pi_{T,t}^* \tag{60}$$

Inflation of Tradable Goods:

$$\pi_{T,t} = \frac{1}{1+\omega}\left(\pi_{H,t} + (1-\alpha)\Delta s_t + \omega\pi_{N,t}\right) \tag{61}$$

$$\pi_{T,t}^* = \frac{1}{1+\omega^*}\left(\pi_{F,t}^* + (1-\alpha^*)\Delta s_t + \omega^*\pi_{N,t}^*\right) \tag{62}$$

Overall Inflation:

$$\pi_t = \gamma_c\pi_{T,t} + (1-\gamma_c)\pi_{N,t} \tag{63}$$

$$\pi_t^* = \gamma_c^*\pi_{T,t}^* + (1-\gamma_c^*)\pi_{N,t}^* \tag{64}$$

Real Exchange Rate:

$$q_t = (\alpha + \alpha^* - 1)s_t + (1 - \gamma_c^* + \omega^*)x_t^* - (1 - \gamma_c + \omega)x_t \tag{65}$$

Monetary Policy Rule:

$$r_t = \rho r_{t-1} + (1-\rho)(\psi_y E_t\{y_{t+1}\} + \psi_\pi E_t\{\pi_{t+1}\}) + \varepsilon_{m,t} \tag{66}$$

$$r_t^* = \rho^* r_{t-1}^* + (1-\rho^*)(\psi_y^* E_t\{y_{t+1}^*\} + \psi_\pi^* E_t\{\pi_{t+1}^*\}) + \varepsilon_{m,t}^* \tag{67}$$

Productivity Shock in Tradable Sector:

$$\varepsilon_{a^H,t} = \rho_{a^H}\varepsilon_{a^H,t-1} + \mu_{a^H,t} \tag{68}$$

$$\varepsilon_{a^F,t}^* = \rho_{a^F}^*\varepsilon_{a^F,t-1}^* + \mu_{a^F,t}^* \tag{69}$$

Productivity Shock in Non-tradable Sector:

$$\varepsilon_{a^N,t} = \rho_{a^N}\varepsilon_{a^N,t-1} + \mu_{a^N,t} \tag{70}$$

$$\varepsilon_{a^N,t}^* = \rho_{a^N}^*\varepsilon_{a^N,t-1}^* + \mu_{a^N,t}^* \tag{71}$$

Preference Shock:

$$\varepsilon_{d,t} = \rho_d \varepsilon_{d,t-1} + \mu_{d,t} \tag{72}$$

$$\varepsilon^*_{d,t} = \rho^*_d \varepsilon^*_{d,t-1} + \mu^*_{d,t} \tag{73}$$

Labor Supply Shock:

$$\varepsilon_{l,t} = \rho_l \varepsilon_{l,t-1} + \mu_{l,t} \tag{74}$$

$$\varepsilon^*_{l,t} = \rho^*_l \varepsilon^*_{l,t-1} + \mu^*_{l,t} \tag{75}$$

Shock in Government Expenditures:

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \mu_{g,t} \tag{76}$$

$$\varepsilon^*_{g,t} = \rho^*_g \varepsilon^*_{g,t-1} + \mu^*_{g,t} \tag{77}$$

Shock in Investment Efficiency:

$$\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \mu_{i,t} \tag{78}$$

$$\varepsilon^*_{i,t} = \rho^*_i \varepsilon^*_{i,t-1} + \mu^*_{i,t} \tag{79}$$

Table 1: Interpretation of Variables

variable	interpretation
c_t,c_t^*	consumption
i_t,i_t^*	investment
y_t,y_t^*	total output
$y_{H,t}$,$y_{F,t}^*$	output in tradable sector
$y_{N,t}$,$y_{N,t}^*$	output in non-tradable sector
x_t,x_t^*	internal exchange rates
s_t	terms of trade
r_t,r_t^*	nominal interest rate
q_t	real exchange rate
k_t,k_t^*	capital
$r_{K,t}$,$r_{K,t}^*$	payoff from renting capital
w_t,w_t^*	real wage
$q_{T,t}$,$q_{T,t}^*$	price of installed capital (Tobin's Q)
l_t,l_t^*	labor
mrs_t,mrs_t^*	marginal rate of substitution
π_t,π_t^*	inflation
$\pi_{T,t}$,$\pi_{T,t}^*$	inflation of tradable goods
$\pi_{H,t}$,$\pi_{F,t}^*$	inflation in tradable sector
$\pi_{N,t}$,$\pi_{N,t}^*$	inflation of non-tradable goods
$mc_{H,t}$,$mc_{F,t}^*$	real marginal costs in tradable sector
$mc_{N,t}$,$mc_{N,t}^*$	real marginal costs in non-tradable sector
$\varepsilon_{a^H,t}$,$\varepsilon_{a^F,t}^*$	productivity shock in tradable sector
$\varepsilon_{a^N,t}$,$\varepsilon_{a^N,t}^*$	productivity shock in non-tradable sector
$\varepsilon_{d,t}$,$\varepsilon_{d,t}^*$	preference shock
$\varepsilon_{l,t}$,$\varepsilon_{l,t}^*$	labor supply shock
$\varepsilon_{g,t}$,$\varepsilon_{g,t}^*$	government expenditures shock
$\varepsilon_{i,t}$,$\varepsilon_{i,t}^*$	investment efficiency shock
$\varepsilon_{m,t}$,$\varepsilon_{m,t}^*$	monetary policy shock

Table 2: Interpretation of Structural Parameters

parameter	interpretation	domain
n	relative size of the domestic economy	$\langle 0, 1\rangle$
β, β^*	discount factor	$\langle 0, 1\rangle$
h, h^*	habit formation in consumption	$\langle 0, 1\rangle$
σ, σ^*	inv. elast. of intertemporal substitution	$\langle 0, \infty)$
ϕ, ϕ^*	inv. elast. of labor supply	$\langle 0, \infty)$
ϕ_H, ϕ_F	elast. of subst. among tradable goods	$\langle 1, \infty)$
ϕ_N, ϕ_N^*	elast. of subst. among non-tradable goods	$\langle 1, \infty)$
ϕ_W, ϕ_W^*	elast. of subst. among labor types	$\langle 1, \infty)$
γ_c, γ_c^*	share of tradable goods in consumption	$\langle 0, 1\rangle$
γ_i, γ_i^*	share of tradable goods in investment	$\langle 0, 1\rangle$
α, α^*	share of domestic tradable goods	$\langle 0, 1\rangle$
ω, ω^*	distribution costs	$\langle 0, \infty)$
τ, τ^*	capital depreciation rate	$\langle 0, 1\rangle$
$S^{''}, S^{''*}$	adjustment costs of capital	$\langle 0, \infty)$
η, η^*	elasticity of output with respect to capital	$\langle 0, 1\rangle$
θ_H, θ_F^*	Calvo parameter for tradable sector	$\langle 0, 1\rangle$
θ_N, θ_N^*	Calvo parameter for non-tradable sector	$\langle 0, 1\rangle$
θ_W, θ_W^*	Calvo parameter for households	$\langle 0, 1\rangle$
δ_H, δ_F^*	indexation in tradable sector	$\langle 0, 1\rangle$
δ_N, δ_N^*	indexation in non-tradable sector	$\langle 0, 1\rangle$
δ_W, δ_W^*	indexation of households	$\langle 0, 1\rangle$
ρ_i, ρ_i^*	interest rate smoothing	$\langle 0, 1\rangle$
ψ_π, ψ_π^*	elasticity of interest rate to inflation	$\langle 0, \infty)$
ψ_y, ψ_y^*	elasticity of interest rate to output	$\langle 0, \infty)$

Table 3: Interpretation of Parameters related to Shocks

parameter	interpretation	domain
$\rho_{a^H}, \rho^*_{a^F}$	persistence of productivity shocks - tradables	$\langle 0, 1\rangle$
$\rho_{a^N}, \rho^*_{a^N}$	persistence of productivity shocks - non-tradables	$\langle 0, 1\rangle$
ρ_d, ρ^*_d	persistence of preference shocks	$\langle 0, 1\rangle$
ρ_l, ρ^*_l	persistence of labor supply shocks	$\langle 0, 1\rangle$
ρ_g, ρ^*_g	persistence of shocks in government expenditures	$\langle 0, 1\rangle$
ρ_i, ρ^*_i	persistence of shocks in investment efficiency	$\langle 0, 1\rangle$
$\sigma_{a^H}, \sigma^*_{a^F}$	std. dev. of productivity shocks - tradables	$\langle 0, \infty)$
$\sigma_{a^N}, \sigma^*_{a^N}$	std. dev. of productivity shocks - non-tradables	$\langle 0, \infty)$
σ_d, σ^*_d	std. dev. of preference shocks	$\langle 0, \infty)$
σ_l, σ^*_l	std. dev. of labor supply shocks	$\langle 0, \infty)$
σ_g, σ^*_g	std. dev. of shocks in government expenditures	$\langle 0, \infty)$
σ_i, σ^*_i	std. dev. of shocks in investment efficiency	$\langle 0, \infty)$
σ_m, σ^*_m	std. dev. of monetary shocks	$\langle 0, \infty)$
$cor_{a^H, a^{F*}}$	correlation of productivity shocks - tradables	$\langle -1, 1\rangle$
$cor_{a^N, a^{N*}}$	correlation of productivity shocks - non-tradables	$\langle -1, 1\rangle$
cor_{d,d^*}	correlation of preference shocks	$\langle -1, 1\rangle$
cor_{l,l^*}	correlation of labor supply shocks	$\langle -1, 1\rangle$
cor_{g,g^*}	correlation of shocks in government expenditures	$\langle -1, 1\rangle$
cor_{i,i^*}	correlation of shocks in investment efficiency	$\langle -1, 1\rangle$
$cor_{m,m*}$	correlation of shocks in investment efficiency	$\langle -1, 1\rangle$

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Appendix - Software

Dynare

The model is estimated using the programme Dynare, version 4.2.4. It is a free toolbox for Matlab and it was designed for solving and estimating a wide class of economic models, especially those with rational expectations. It is a very suitable tool for handling DSGE models. Dynare offers two approaches to the estimation of the model: (i) maximal likelihood method and (ii) Random Walk Chain Metropolis-Hastings algorithm. Beside the estimation, it is also able to produce many useful statistics, such as convergence diagnostics of the MH algorithm, checkplots, impulse-response functions, variance decomposition, shock decomposition, conditional and unconditional forecasts, etc. All versions of Dynare toolbox and Dynare manuals are available on the website http://www.dynare.org/.

IRIS

Several exercises which are presented in Appendix, namely (i) comparison of second moments implied by the data and the model; and (ii) comparison of the model predictions with the actual observations; were performed by the programme IRIS developed by Jaromír Beneš.[10] IRIS is a free toolbox for Matlab designed for solving, estimating and evaluating the dynamic economic models. It is a very suitable tool for handling conditional and unconditional forecasts and for comparison of second moments (model vs. data). Unlike

[10] IRIS is an acronym of "I Rest, Iris Solves".

Dynare, it is also able to produce impulse-responses to anticipated shocks. All versions of IRIS toolbox as well as IRIS manuals are available on the website http://code.google.com/p/iris-toolbox-project/

Demetra

Seasonal adjustment of the data was performed in the Demetra programme. Demetra is a free software designed for seasonal adjustment of time series, developed by researchers from Eurostat and the National Bank of Belgium. Demetra offers several specifications of TRAMO/SEATS and X12 methods for seasonal adjustment of time series. It also performs many statistical tests focusing on evaluation of the quality of seasonal adjustment. All versions of the programme Demetra as well as various manuals and guidelines are available on http://www.cros-portal.eu/page/demetra

Appendix - Data

Real GDP, consumption and investment are measured as "Millions of euro, chain-linked volumes, reference year 2005 (at 2005 exchange rates)". Consumption is given by "Household and NPISH final consumption expenditures".[11] Investment is given by "Gross fixed capital formation".

Prices are measured by the "HICP, Index, 2005=100, All-items HICP". Real wage is given by the "Labour Cost Index - Wages and salaries, Nominal value, Business economy, Index, 2008=100, Data adjusted by working days", which is divided by HICP in each period. Short-term interest rate is given by the "Money market interest rates, 3-month rates".

Internal exchange rate defined as prices of non-tradable goods relative to prices of tradable goods is calculated from the components of HICP, where "Services (overall index excluding goods)" and "Energy" are regarded as non-tradable goods, while "Non-energy industrial goods" and "Food including alcohol and tobacco" are regarded as tradable goods.

Except for the nominal interest rates, all observed variables are seasonally adjusted. I used TRAMO/SEATS algorithm for seasonal adjustment of the time series and took advantage of the Demetra software developed for seasonal adjustment of time series.[12] Although many time series are also available as seasonally adjusted on Eurostat, I decided to download all previ-

[11] The data series labeled as "Final consumption expenditures of households" are not available for the Euro Area 12, which is why I use "Household and NPISH final consumption expenditures". However, values of "Final consumption expenditure of NPISH" are so negligible that it does not make make any significant difference.

[12] The best performance from the available variants of TRAMO/SEATS algorithm was made by specification "Tramo-Seats RSA2 log/level, working days, Easter, outliers detection, airline model", which is what I decided to use for seasonal adjustment of employed time series.

ously mentioned time series as not seasonally adjusted and then adjust them myself. There are several reasons for this decision. First, the seasonally adjusted versions of HICP and its components are unavailable. I find it more consistent to use time series all of which are adjusted by the same method than use time series adjusted by possibly different methods. Secondly, some of those time series which are available as seasonally adjusted on Eurostat clearly show some residual seasonality (also detected by Demetra), which raises doubts about quality of the employed seasonal adjustment.

Except for the nominal interest rates, all observed variables are expressed as demeaned 100*log differences. Nominal interest rates are demeaned and expressed as quarterly rates per cent. The following formulas show how are transformed observed variables linked to the model variables.

	CZ:	EA:
consumption:	$C_t^{obs} = c_t - c_{t-1}$	$C_t^{obs*} = c_t^* - c_{t-1}^*$
investment:	$I_t^{obs} = i_t - i_{t-1}$	$I_t^{obs*} = i_t^* - i_{t-1}^*$
GDP:	$Y_t^{obs} = y_t - y_{t-1}$	$Y_t^{obs*} = y_t^* - y_{t-1}^*$
prices:	$HICP_t^{obs} = \pi_t$	$HICP_t^{obs*} = \pi_t^*$
int. exchange rate:	$X_t^{obs} = x_t - x_{t-1}$	$X_t^{obs*} = x_t^* - x_{t-1}^*$
real wage:	$W_t^{obs} = w_t - w_{t-1}$	$W_t^{obs*} = w_t^* - w_{t-1}^*$
interest rate:	$R_t^{obs} = r_t$	$R_t^{obs*} = r_t^*$

Figure 1 displays original and seasonally adjusted data, both transformed as log differences. It shows the performance of the employed seasonal adjustment and also demonstrates how important is correct seasonal adjustment of time series. Figure 2 displays transformed data which enter the estimation.

Figure 1: Original Data (green line) and Seasonally Adjusted Data (blue line) - Log Differences

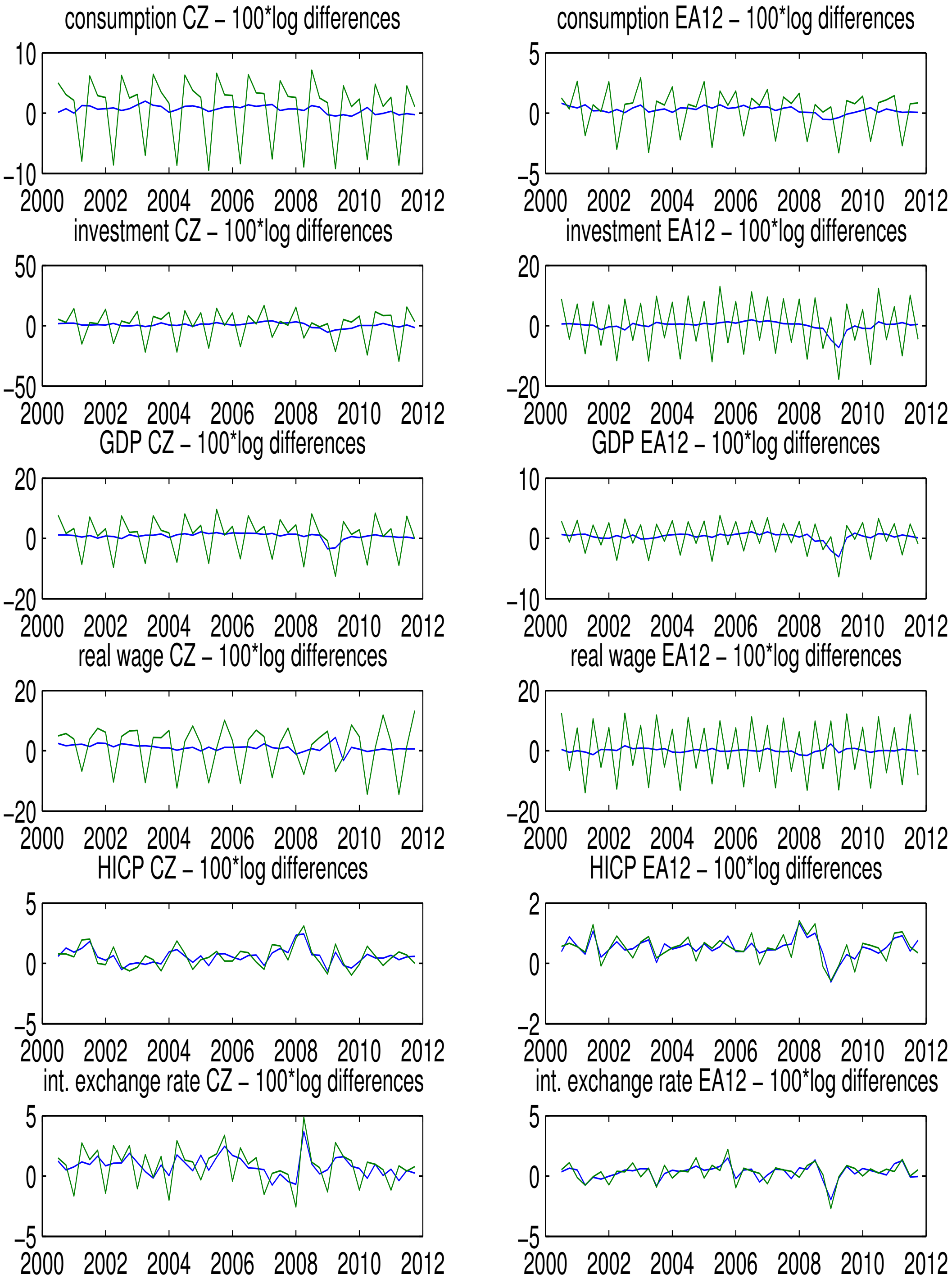
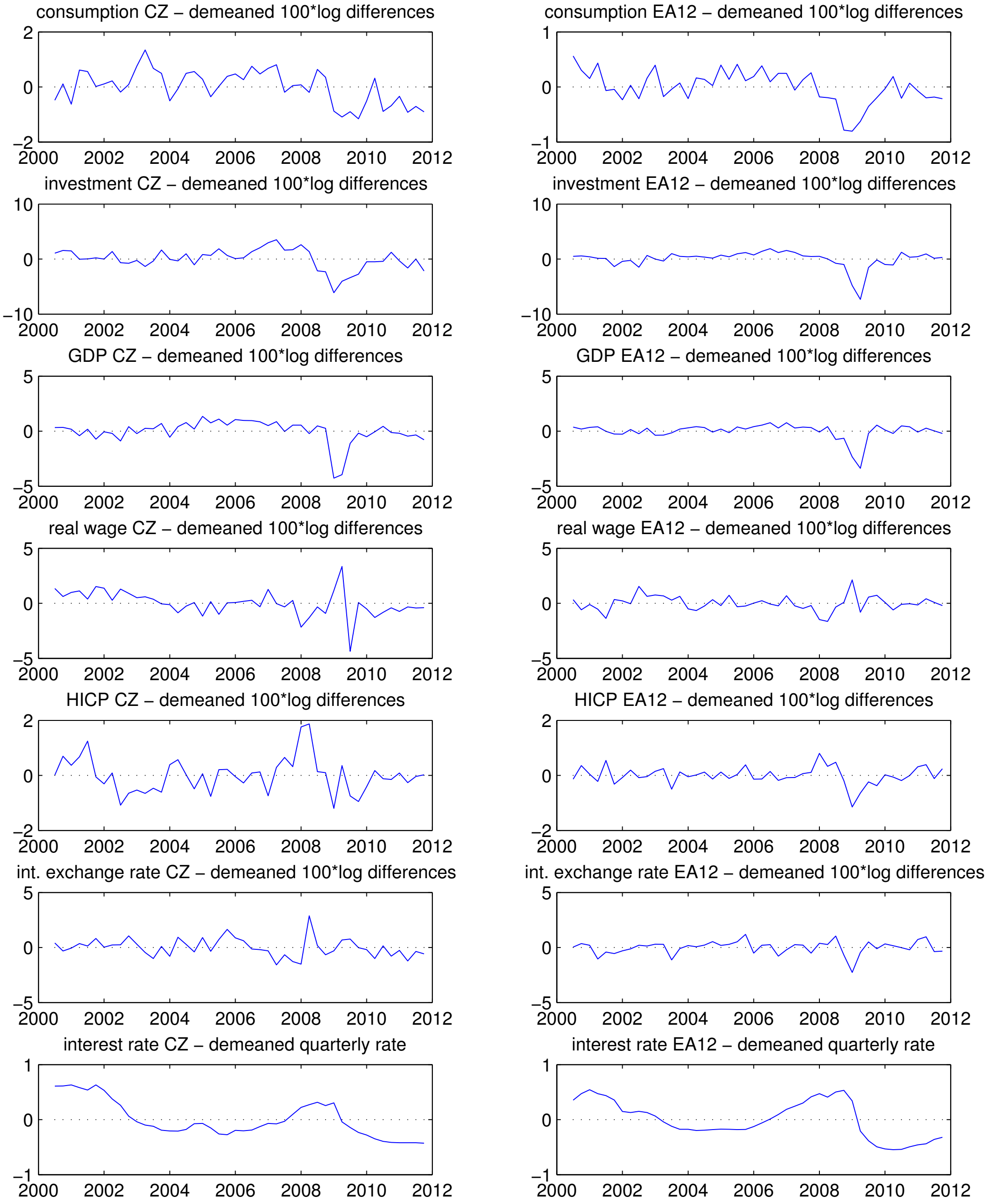


Figure 2: Data for Estimation



Appendix - Estimation

Because of a large number of parameters and a short length of the data sample employed, I decided to calibrate a few parameters. I calibrated those parameters for which I have a good prior information from the data, and those parameters which are known to be weakly identifiable in DSGE models. This mixed approach is quite common in the literature and leads to a better identifiability of non-calibrated parameters, see Canova (2007).

The parameter n governing the relative size of both economies is calibrated to be 0.0135, according to the ratio of nominal GDP levels, averaged over the examined period 2000-2011. The share of tradable goods in consumption in the Czech economy γ_c (in the Euro Area 12 γ_c^*) is calibrated to be 0.5424 (0.5014). These values correspond to the complements of the average shares of services and energy goods in the HICP baskets in the examined period. Parameters γ_i and γ_i^*, which denote share of tradable investment goods, are set equal to 0.4956 and 0.4219, according to the average shares of non-construction works in total investment expenditures in the examined period. The shares of domestic tradable goods α and α^* are set equal to 0.28 and 0.989, following Musil (2009).

The discount factors β and β^* are calibrated to be 0.9975, which implies an annual steady state real interest rate of 1%. This value roughly corresponds to the long term mean of annual real interest rates in both economies. Quarterly depreciation rates τ and τ^* are calibrated to be 0.025, which implies an annual depreciation rate of 10%. Distribution costs ω and ω^* are calibrated to unity following Burstein et al. (2003), which implies that the share of distribution costs in the final price of tradable consumption goods is 50%. Elasticities of output with respect to capital η and η^* are calibrated at 0.3868 and 0.3654, which corresponds to the complement to the average shares of labor on the GDP in the given economy in the period 2000-2010.[13] Elasticities of substitution among labor types ϕ_W and ϕ_W^*, which are known to be badly identifiable, are set equal to 3 following Smets and Wouters (2003). This value implies a wage mark-up of 50%. Following Slanicay and Vašíček (2009, 2011), Čapek (2010) and Matheson (2010), who argue that incorporating price (wage) indexation into the Calvo price (wage) setting mechanism deteriorates the empirical fit of DSGE models, I decided to set indexation parameters δ_H, δ_F^*, δ_N, δ_N^*, δ_W and δ_W^* equal to 0. It implies that the estimated variant of the model employs the original Calvo price (wage) setting mechanism, see Calvo (1983).

Steady state shares of consumption, investment and government spending in the total output correspond to their average shares in the GDP in the examined period. Namely, $\frac{\overline{C}}{\overline{Y}} = 0.4946$, $\frac{\overline{I}}{\overline{Y}} = 0.2620$, $\frac{\overline{C^*}}{\overline{Y^*}} = 0.5691$ and $\frac{\overline{I^*}}{\overline{Y^*}} = 0.2036$. Other steady state shares are calculated consistently with the derivation of the model (analogously for the foreign economy):

$$
\begin{aligned}
\frac{\overline{G}}{\overline{Y}} &= 1 - \frac{\overline{C}}{\overline{Y}} - \frac{\overline{I}}{\overline{Y}}, \\
\frac{\overline{Y_N}}{\overline{Y}} &= \frac{1+\omega-\gamma_c}{1+\omega}\frac{\overline{C}}{\overline{Y}} + (1-\gamma_i)\frac{\overline{I}}{\overline{Y}} + \frac{\overline{G}}{\overline{Y}}, \\
\frac{\overline{Y_H}}{\overline{Y}} &= 1 - \frac{\overline{Y_N}}{\overline{Y}}.
\end{aligned}
$$

Remaining parameters are estimated. Prior setting of the estimated parameters is presented in Table 4. For parameters whose natural domain is the interval between 0 and 1, I chose Beta distribution of priors. For structural parameters whose natural domain is the set of non-negative real numbers, I chose Gamma distribution of priors, except for the parameters of adjustment costs S'', S''^*. For those I chose Normal distribution of priors. For parameters representing standard deviations of shocks, whose natural domain is the set of non-negative real numbers, I chose Inverse Gamma distribution of priors.

[13] See http://stats.oecd.org.

Finally, for parameters representing correlations between shocks, whose natural domain is the interval between -1 and 1, I chose Normal distribution of priors, with the obvious restriction.

Prior means for Calvo parameters of price and wage stickiness θ_H, θ_F^*, θ_N, θ_N^*, θ_W, and θ_W^* are set to be 0.7 which implies average price (wage) duration of 10 months. Priors for parameters in the Taylor rule are set consistently with Taylor (1999). Inverse elasticities of intertemporal substitution σ and σ^* and inverse Frisch elasticities of labor supply ϕ ϕ^* are estimated with relatively loose priors with prior means set to be 1.0, following Galí (2008), and prior std. deviations equal to 0.7, which are values commonly found in the business cycle literature. Parameters of habit formation h and h^* are estimated with prior means set to be 0.7 and prior std. deviations equal to 0.1, as in Smets and Wouters (2003). Priors for capital adjustment costs S'' and S''^* are taken from Kolasa (2009). Priors for shocks are taken from Herber (2010).

Table 4: Priors for Estimated Prameters

parameter	prior mean	prior std. dev.	distribution
h, h^*	0.7	0.1	Beta
σ, σ^*	1.0	0.7	Gamma
ϕ, ϕ^*	1.0	0.7	Gamma
S'', S''^*	4.0	1.5	Normal
θ_H, θ_F^*	0.7	0.05	Beta
θ_N, θ_N^*	0.7	0.05	Beta
θ_W, θ_W^*	0.7	0.05	Beta
ρ, ρ^*	0.7	0.15	Beta
ψ_π, ψ_π^*	1.3	0.15	Gamma
ψ_y, ψ_y^*	0.25	0.1	Gamma
$\rho_{a^H}, \rho_{a^F}^*$	0.7	0.1	Beta
$\rho_{a^N}, \rho_{a^N}^*$	0.7	0.1	Beta
ρ_d, ρ_d^*	0.7	0.1	Beta
ρ_l, ρ_l^*	0.7	0.1	Beta
ρ_g, ρ_g^*	0.7	0.1	Beta
ρ_i, ρ_i^*	0.7	0.1	Beta
$\sigma_{a^H}, \sigma_{a^F}^*$	2	∞	Inv. Gamma
$\sigma_{a^N}, \sigma_{a^N}^*$	2	∞	Inv. Gamma
σ_d, σ_d^*	6	∞	Inv. Gamma
σ_l, σ_l^*	10	∞	Inv. Gamma
σ_g, σ_g^*	3	∞	Inv. Gamma
σ_i, σ_i^*	6	∞	Inv. Gamma
σ_m, σ_m^*	0.3	∞	Inv. Gamma
$cor_{a^H,a^{F*}}$	0	0.4	Normal
$cor_{a^N,a^{N*}}$	0	0.4	Normal
cor_{d,d^*}	0	0.4	Normal
cor_{l,l^*}	0	0.4	Normal
cor_{g,g^*}	0	0.4	Normal
cor_{i,i^*}	0	0.4	Normal
$cor_{m,m*}$	0	0.4	Normal

Table 5: Estimated Values

parameter	posterior mean CZ	90% credible CZ		posterior mean EA	90% credible EA	
h, h^*	0.78	0.68	0.89	0.81	0.72	0.90
σ, σ^*	2.03	0.92	3.09	3.39	1.69	4.99
ϕ, ϕ^*	0.52	0.04	1.00	0.91	0.16	1.64
S'', S''^*	6.01	4.21	7.83	5.26	3.38	7.06
θ_H, θ_F^*	0.72	0.64	0.79	0.7	0.64	0.76
θ_N, θ_N^*	0.76	0.70	0.81	0.68	0.62	0.74
θ_W, θ_W^*	0.75	0.69	0.82	0.78	0.72	0.84
ρ, ρ^*	0.9	0.88	0.93	0.88	0.85	0.91
ψ_π, ψ_π^*	1.22	1.02	1.4	1.41	1.17	1.65
ψ_y, ψ_y^*	0.12	0.07	0.17	0.27	0.16	0.38
$\rho_{a^H}, \rho_{a^F}^*$	0.95	0.92	0.98	0.68	0.55	0.81
$\rho_{a^N}, \rho_{a^N}^*$	0.46	0.33	0.58	0.66	0.56	0.76
ρ_d, ρ_d^*	0.82	0.72	0.91	0.69	0.58	0.81
ρ_l, ρ_l^*	0.49	0.35	0.62	0.49	0.35	0.63
ρ_g, ρ_g^*	0.85	0.79	0.92	0.73	0.64	0.83
ρ_i, ρ_i^*	0.79	0.72	0.87	0.76	0.67	0.84
$\sigma_{a^H}, \sigma_{a^F}^*$	6.01	3.45	8.51	4.75	2.87	6.54
$\sigma_{a^N}, \sigma_{a^N}^*$	6.84	3.55	10.02	2.59	1.60	3.54
σ_d, σ_d^*	5.24	2.74	7.63	5.05	2.85	7.15
σ_l, σ_l^*	32.8	8.57	57.88	25.53	7.07	43.29
σ_g, σ_g^*	3.29	2.61	3.95	1.52	1.26	1.77
σ_i, σ_i^*	4.39	3.09	5.65	4.24	2.93	5.49
σ_m, σ_m^*	0.08	0.07	0.10	0.09	0.08	0.11
$cor_{a^H,a^{F*}}$	-0.26	-0.47	-0.04			
$cor_{a^N,a^{N*}}$	0.27	0.05	0.48			
cor_{d,d^*}	0.45	0.25	0.66			
cor_{l,l^*}	-0.17	-0.40	0.06			
cor_{g,g^*}	0.35	0.15	0.56			
cor_{i,i^*}	0.33	0.13	0.54			
$cor_{m,m*}$	0.57	0.39	0.76			

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Appendix - MCMC Convergence Diagnostics

Figures 3, 4 and 5 depict convergence diagnostics of the Metropolis-Hastings algorithm developed by Brooks and Gelman (1998). Each subplot that corresponds to a particular parameter contains a red and a blue line. Let's now explain how are these lines constructed, what they imply, and how they ideally should look like. Let's denote

- Ψ_{ij} - the i^{th} draw (out of I, in our case $I = 2000000$) in the j^{th} sequence (out of J, in our case $J = 4$)
- $\overline{\Psi}_{.j}$ - the mean of j^{th} sequence
- $\overline{\Psi}_{..}$ - the mean across all available data.

$\widehat{B}$ defined as

$$\widehat{B} = \frac{1}{J-1}\sum_{j=1}^{J}(\overline{\Psi}_{.j} - \overline{\Psi}_{..})^2$$

is an an estimate of the variance of the mean (σ^2/I), and $B = \widehat{B}I$ is therefore an estimate of the variance. Other estimates of the variance are

$$\widehat{W} = \frac{1}{J}\sum_{j=1}^{J}\frac{1}{I}\sum_{t=1}^{I}(\Psi_{tj} - \overline{\Psi}_{.j})^2$$

and

$$W = \frac{1}{J}\sum_{j=1}^{J}\frac{1}{I-1}\sum_{t=1}^{I}(\Psi_{tj} - \overline{\Psi}_{.j})^2.$$

Ideally, we would like to achieve such a result that the variance between streams should go to zero, i.e. $\lim_{I\to\infty}\widehat{B} \to 0$, and the variance within stream should settle down, i.e. $\lim_{I\to\infty}\widehat{W} \to \mathit{constant}$. If we plot W (red line) and $\widehat{W}+\widehat{B}$ (blue line), then the previous proposition about ideal result for the variance between and within streams can be reformulated so that the red and blue lines should get close to each other, and that both of them should remain constant after a certain amount of draws. We can see that in general the reported plots have the required form.

Figure 3: MCMC Convergence Diagnostics 1

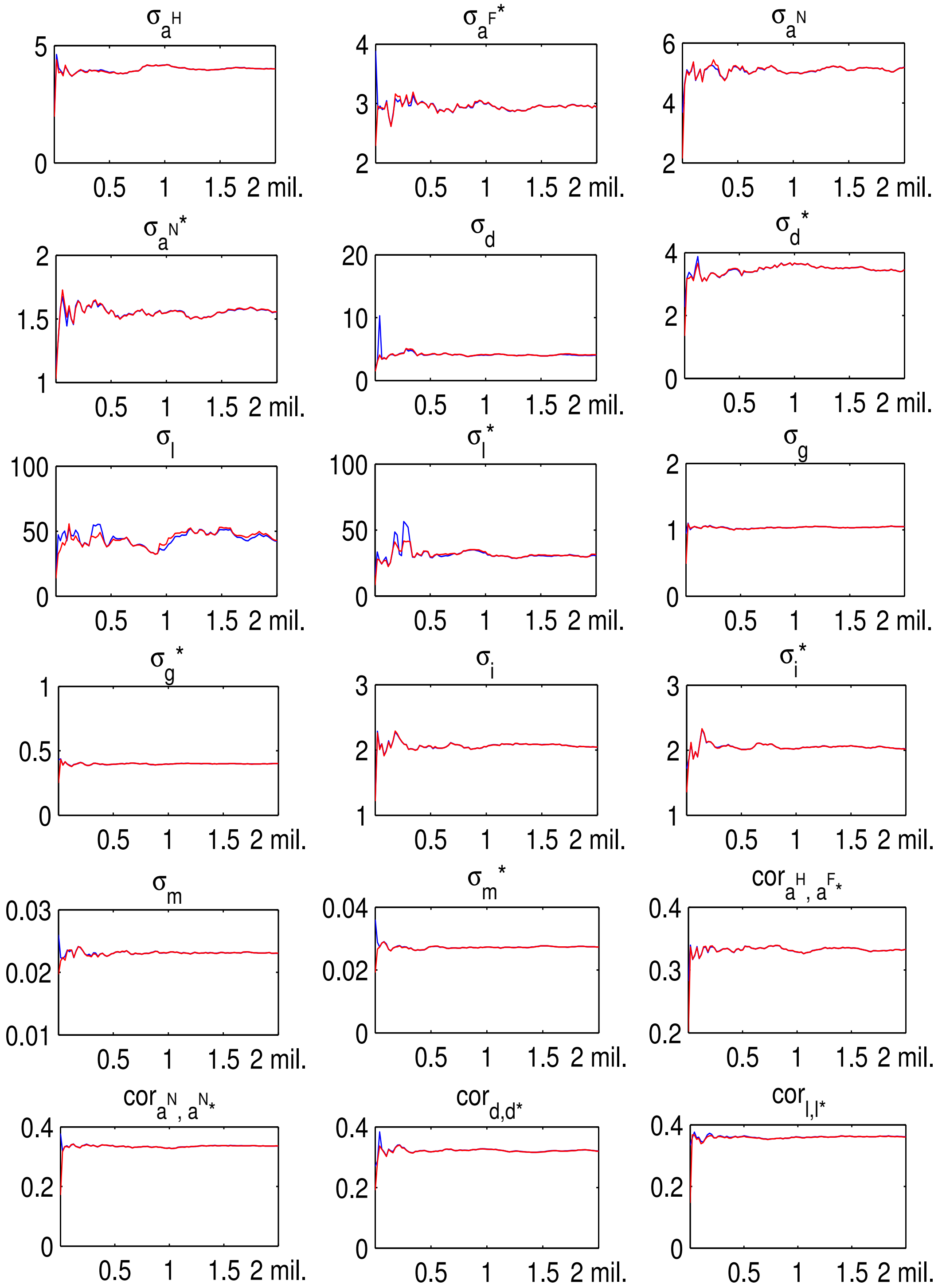


Figure 4: MCMC Convergence Diagnostics 2

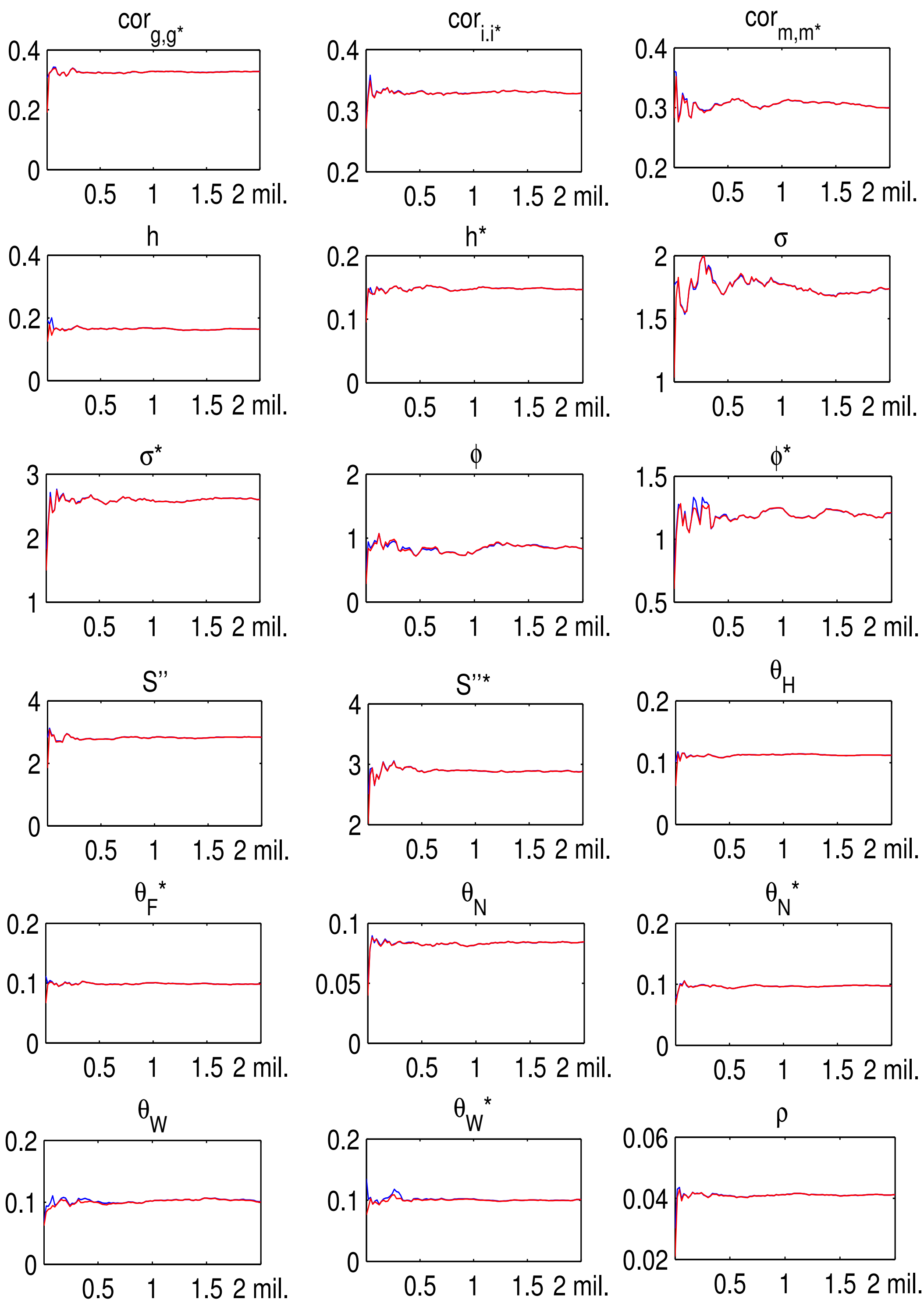
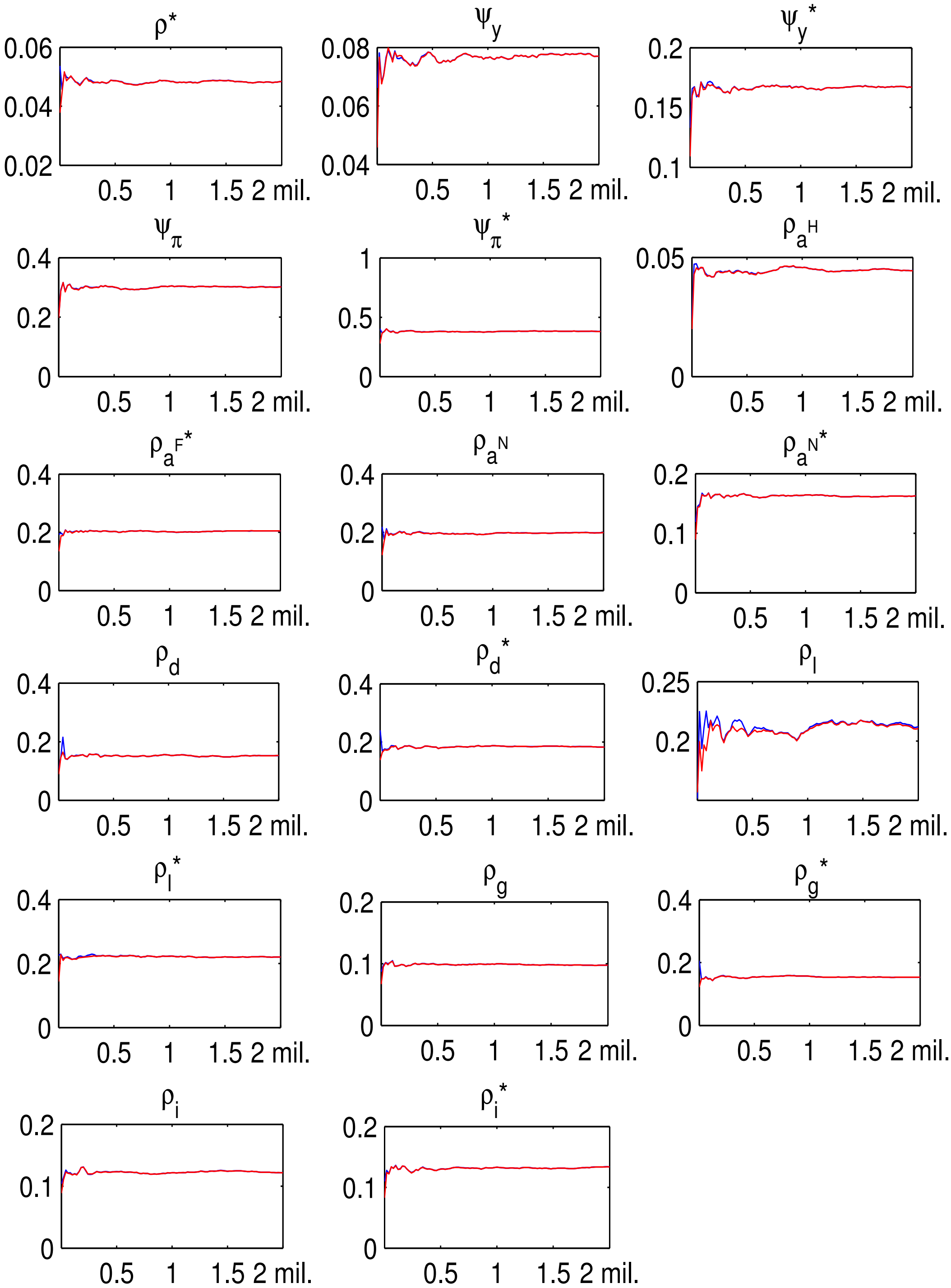


Figure 5: MCMC Convergence Diagnostics 3



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Appendix - Smoothed Shocks

Another type of evaluation can be whether the smoothed shocks (innovations) look like IID processes.[14] Figure 6 depicts the smoothed shocks. From the eyeball test we can say that the smoothed shocks (innovations) roughly resemble the IID processes.

Table 6: Autocorrelations of the Smoothed Shocks

	order of autocorrelations			
	1st	2nd	3rd	4th
domestic shocks				
productivity in tradables *	-0.21*	-0.04	0.22*	-0.20
productivity in non-tradables	-0.08	0.01	0.07	-0.03
investment efficiency **	-0.05	0.33**	0.17	0.02
consumption preferences	0.10	0.09	0.09	-0.08
labor supply	-0.21*	0.19	0.30**	0.11
government expenditures *	0.49***	0.20*	0.04	0.06
monetary policy	-0.16	0.02	-0.07	0.06
foreign shocks				
productivity in tradables	-0.8	-0.23*	0.06	0.12
productivity in non-tradables	-0.09	-0.20*	0.26**	-0.04
investment efficiency	0.21*	-0.1	0.21*	0.10
consumption preferences **	0.11	0.30**	0.42***	-0.08
labor supply	-0.38***	0.09	-0.02	0.01
government expenditures	0.33**	-0.00	-0.1	-0.19
monetary policy	0.42***	0.06	0.04	0.09

Note:*/**/*** - significance level 0.1/0.05/0.01

It is also possible to formally evaluate the statistical properties of the smoothed shocks. Table 6 shows the computed autocorrelations of the smoothed

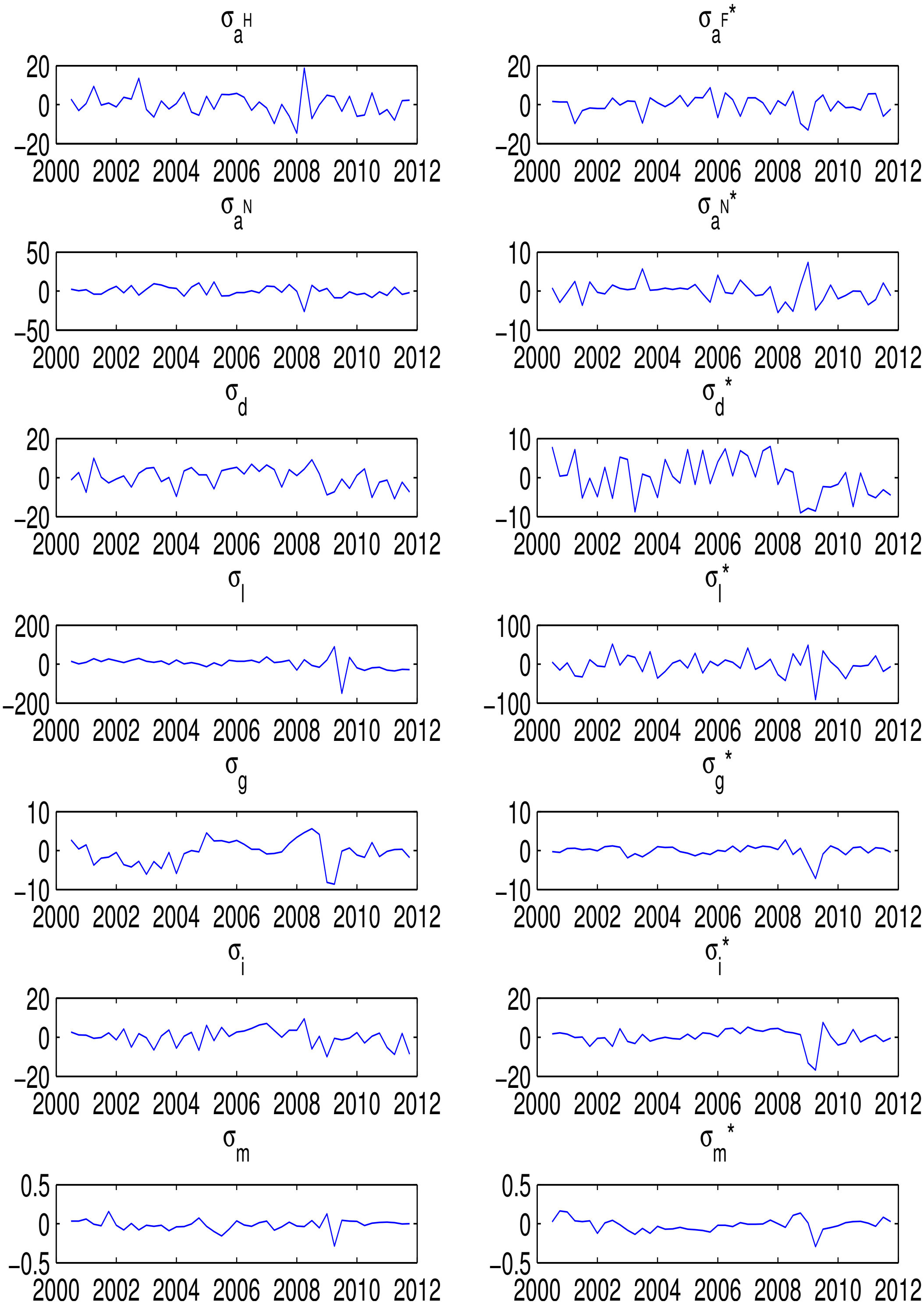
[14]IID - identically and independently distributed

shocks and their statistical significance (highlighted by the asterisks). Besides testing statistical significance of each computed autocorrelation, it is also possible to test the joint hypothesis that the shocks are independently distributed, against the alternative hypothesis that the shocks are not independently distributed. This can be tested via the Ljung-Box Q test, see Ljung and Box (1978). Let's define the test statistic

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k},$$

where n is the sample size, $\hat{\rho_k}$ is the sample autocorrelation at time lag k, and h is the number of lags being tested (in this case $h = 20$). If the shocks are independently distributed, then the test statistic Q is distributed according to the chi-squared distribution with h degrees of freedom (i.e. $Q \sim \chi^2_h$). If $Q > \chi^2_{1-\alpha,h}$ ($\chi^2_{1-\alpha,h}$ is the α-quantile of the chi-squared distribution with h degrees of freedom), then we can reject the null hypothesis of randomness on the significance level α. The results of this test are displayed in the Table 6, where rejections of the null hypothesis are highlighted by the asterisks in the first column. The assumption of the independently distributed shocks is violated only in the case of the domestic investment efficiency shock and in the case of the foreign consumption preference shock (on the significance level $\alpha = 0.05$). We can see that the autocorrelation does not pose a serious problem for most shocks.

Figure 6: Smoothed Shocks



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Appendix - Predictions vs. Observations

Quality of the model performance can be evaluated by comparison of the k-step-ahead forecast of the observed variables with the actual realization of the observed variables. Figure 7 displays one-step-ahead forecasts (green line) and the observed values (blue line) for each observed variable. Figures 8 and 9 display 4-step-ahead forecasts (dashed lines) and observations (blue solid line) of the most important observed variables: output, inflation and interest rate.

From the eyeball test, we can see that the model is best in predicting interest rates. The model also does a "good job" in explaining the movements in consumption, investment and GDP. On the other hand, real wage, inflation, and internal exchange rate seem to have the worst fit among the time series. It seems that the model does not fit these series quite well.

However, it is also possible to compare the model one-step-ahead predictions with the naïve predictions where the prediction is equal to the last observed value. I can calculate the measure of fit of the predictions as the Mean Square Error (MSE)

$$MSE_M = \frac{\sum_{t=2}^{T}(x_t^{mf} - x_t^{obs})^2}{T-1} \quad \text{and} \quad MSE_N = \frac{\sum_{t=2}^{T}(x_t^{nf} - x_t^{obs})^2}{T-1},$$

where T is the number of observations, x_t^{mf} is the model one-step-ahead forecast for time t, x_t^{nf} is the naïve one-step-ahead forecast for time t, x_t^{obs} is the observed value in time t, and MSE_M (MSE_N) stand for Mean Square

Error of the model (naïve) one-step-ahead forecasts. If I divide these two measures I get

$$RFR = \frac{MSE_M}{MSE_N},$$

where RFP stands for the "relative forecast performance". The measure of relative forecast performance (RFP) give us the formal evaluation of the quality of the model forecast performance relatively to the performance of the naïve forecasts. If the RFP is below one it indicates that the model one-step-ahead forecast outperforms the naïve one-step-ahead forecast. On the other hand, if the RFP is higher than one it indicates that the model does a "poor job" in explaining the movement of the particular observable because the naïve one-step-ahead forecast outperforms the model one-step-ahead forecast.

Table 7: Forecast Performance of the Observed Variables

observable	MSE_M	MSE_N	RFP
int. exchange rate CZ	0.6226	1.1783	0.5284
int. exchange rate EA 12	0.3429	0.5850	0.5862
real wage EA 12	0.4765	0.7656	0.6224
interest rate CZ	0.0050	0.0079	0.6320
interest rate EA 12	0.0088	0.0125	0.7053
real wage CZ	1.8102	2.5514	0.7095
GDP EA 12	0.3284	0.4576	0.7176
HICP CZ	0.3786	0.4843	0.7816
consumption EA 12	0.0560	0.0671	0.8342
consumption CZ	0.2610	0.3033	0.8606
investment EA 12	1.5673	1.7729	0.8840
investment CZ	1.8860	2.0146	0.9362
GDP CZ	0.9836	0.9664	1.0178
HICP EA 12	0.1623	0.1568	1.0353

Table 7 displays calculated measures of the forecast performances. The observed variables are ordered from the lowest RFP to the highest. We can see that except for domestic output and foreign inflation, the model one-step-ahead forecast outperforms the naïve one-step-ahead forecast. The model is best in forecasting internal exchange rate, interest rate, real wage, and foreign output. In case of domestic output and foreign inflation, the naïve one-step-ahead forecast does a slightly better job than the model one-step-ahead forecast, however, the difference is almost negligible.

A formal evaluation of the model forecast performance brings slightly different results than which we can deduce from the eyeball test of Figure 7. It is so because the eyeball test does not take into account the different volatility of the observables. If some observed variable is highly volatile, then the naïve forecasts do not make much sense, and the model forecasts are likely to do a much better job. On the other hand, if some variable is not volatile too much, it is difficult for the model forecasts to outperform the naïve forecasts.

Figure 7: Observed Variables and One-step-ahead Forecasts, green line - one-step-ahead forecast, blue line - observations

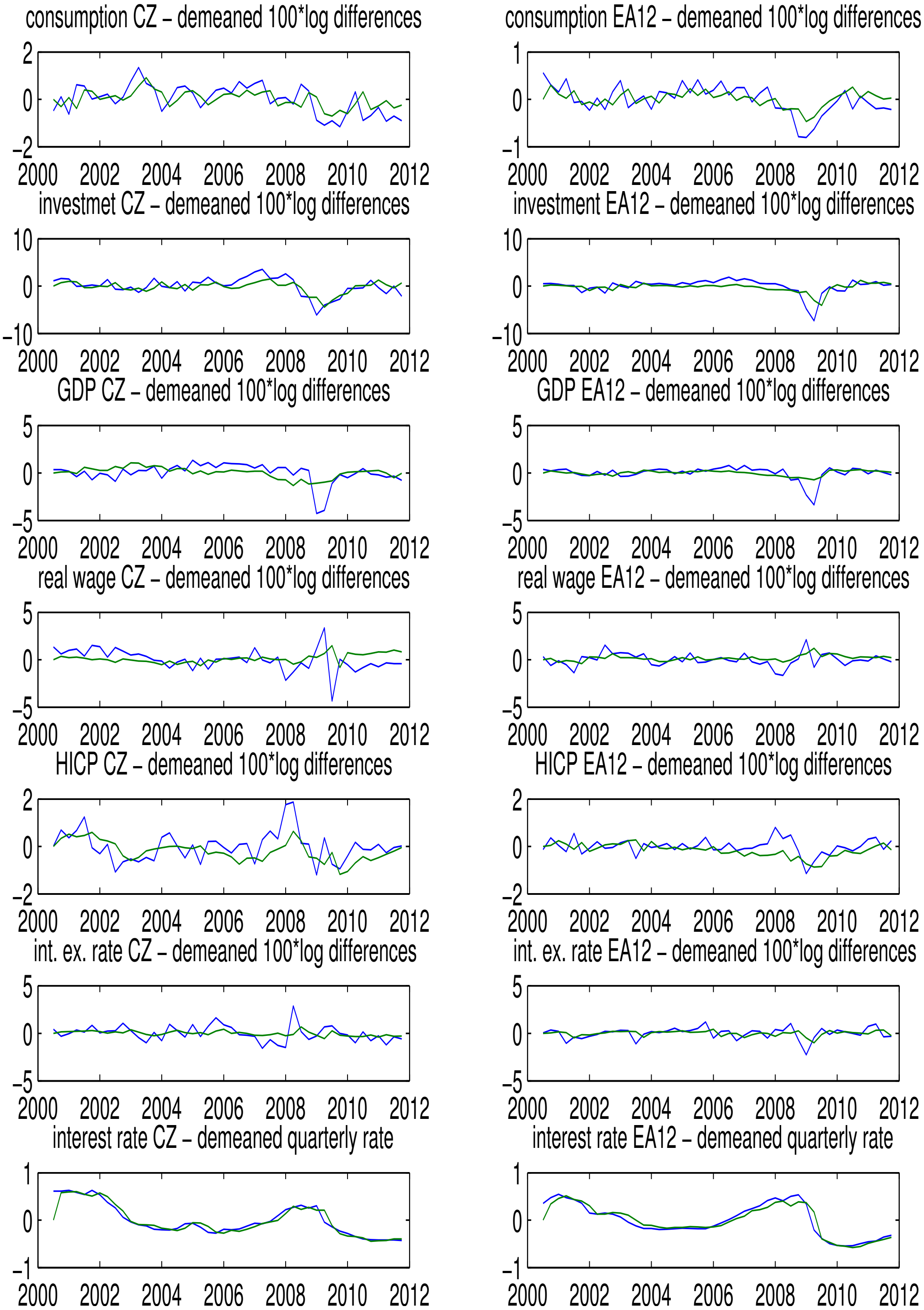


Figure 8: Observed Variables and 4-step-ahead Forecasts CZ, blue solid line - observations, dashed lines - 4-step-ahead forecasts

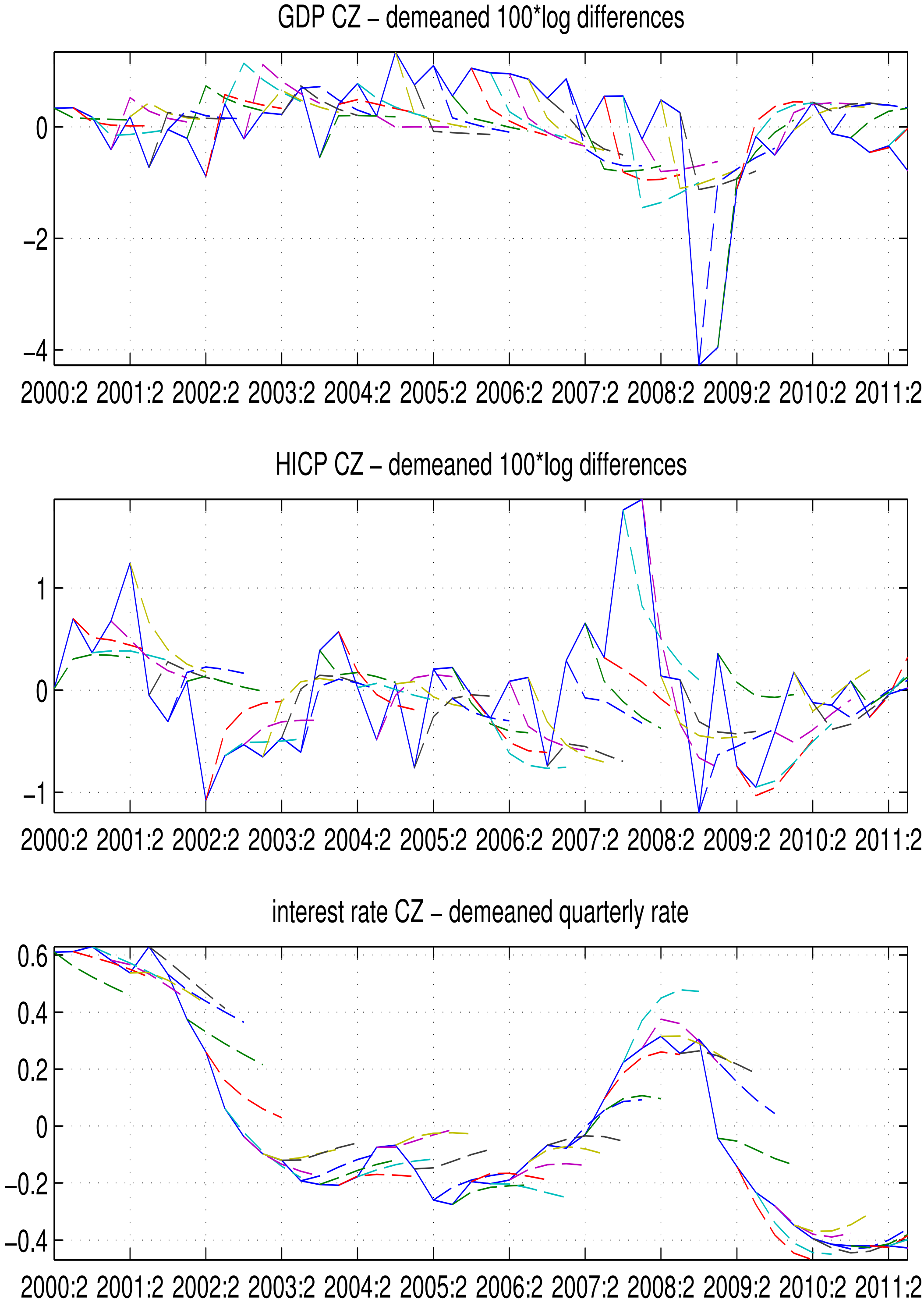
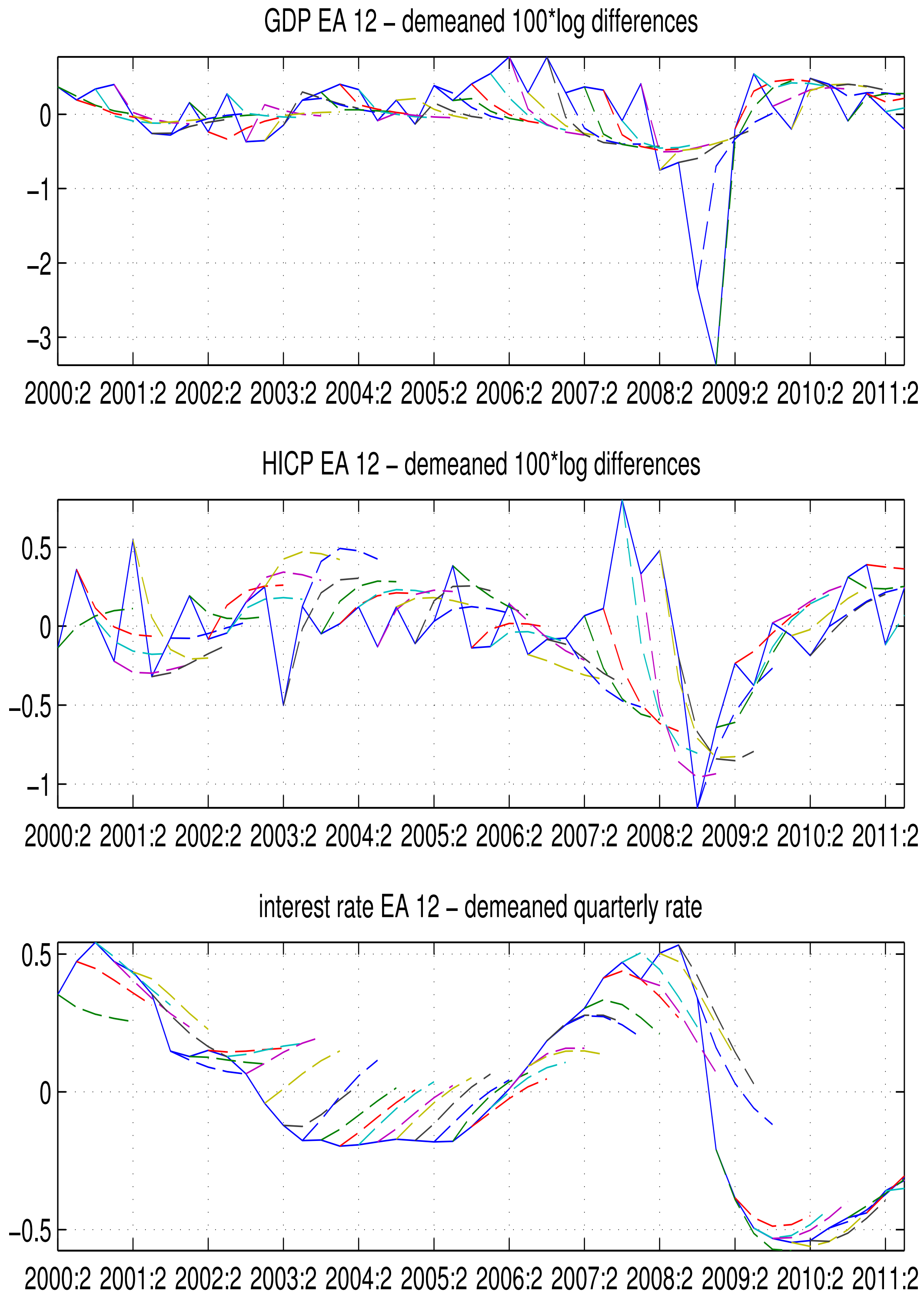


Figure 9: Observed Variables and 4-step-ahead Forecasts EA 12, blue solid line - observations, dashed lines - 4-step-ahead forecasts



Appendix - Second Moments

I also evaluated the model performance by comparing selected second moments. I compared the second moments implied by the model with those implied by the data and those implied by the unrestricted VAR(1) model, which is treated as a benchmark. Figures 10-23 display calculated cross-correlations among observed variables, Figure 24 displays variance of the observed variables, and Figures 25-27 display auto-correlations of the observed variables up to the fourth order. Cross-correlations, variances, and auto-correlations implied by the models are calculated analytically, see Hamilton (1994, p. 264-266). Every subfigure display values calculated for the model (red line), for the data (green line), for the "original" VAR(1) model (blue line), and for the set of VAR(1) models (grey area) obtained from the "original" VAR(1) model by resampling residuals using wild bootstrap.[15] Loosely speaking, this set of resampled VAR(1) models serves as a confidence interval for the point estimate obtained by the "original" VAR(1) model. Ideally, we would like to see that the model point estimate (red line) and the VAR(1) point estimate (blue line) are close to the data estimate (green line).

We can see that the results for the data (green line) and for the VAR(1) point estimate (blue line) are almost identical, which is not very surprising. We can see that the model is able to replicate variance of most observed variables, however, it fails in replicating variance of both domestic and foreign inflation and variance of domestic interest rate and real wage. The model implies much higher volatility of these variables than it is in the data. There

[15] Each residual is randomly multiplied by a random variable with mean 0 and variance 1. This method assumes that the 'true' residual distribution is symmetric and can offer advantages over simple residual sampling for smaller sample sizes. See Wu (1986) and Liu (1988).

are also significant differences in autocorrelations of output and inflation. The model implies much lower autocorrelation of both domestic and foreign output than which can be found in the data, while for inflation the model implies much higher autocorrelation than which can be found in the data. The results for cross-correlations are, not surprisingly, quite unsuccessful. The model implies much lower cross-correlations for most variables than the data. However, this is a well-known weakness of New Keynesian DSGE models.

Figure 10: Cross-Correlations of Domestic Output

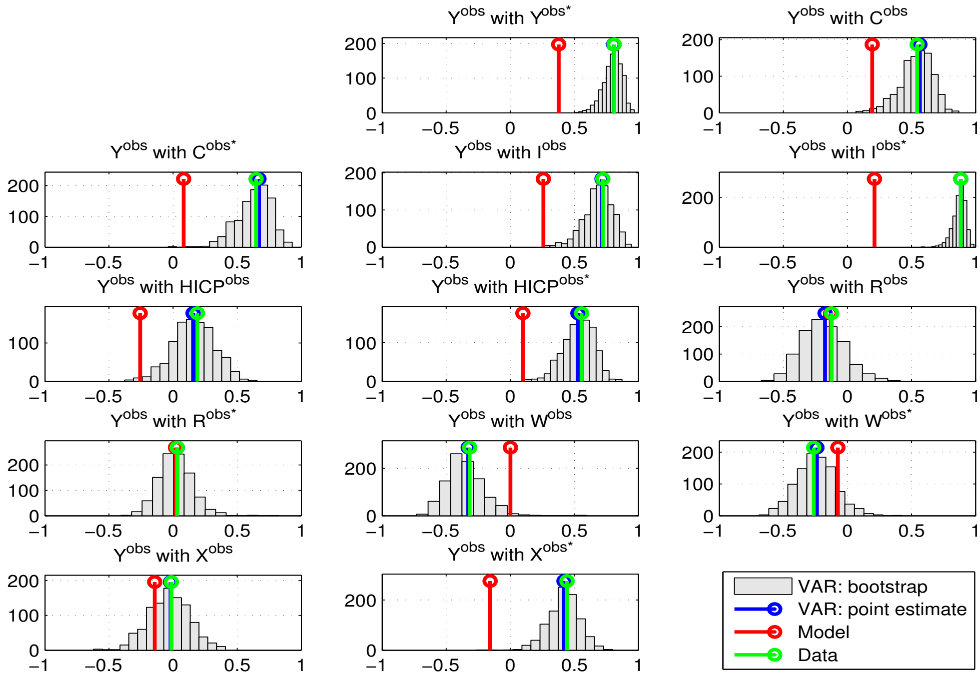


Figure 11: Cross-Correlations of Foreign Output

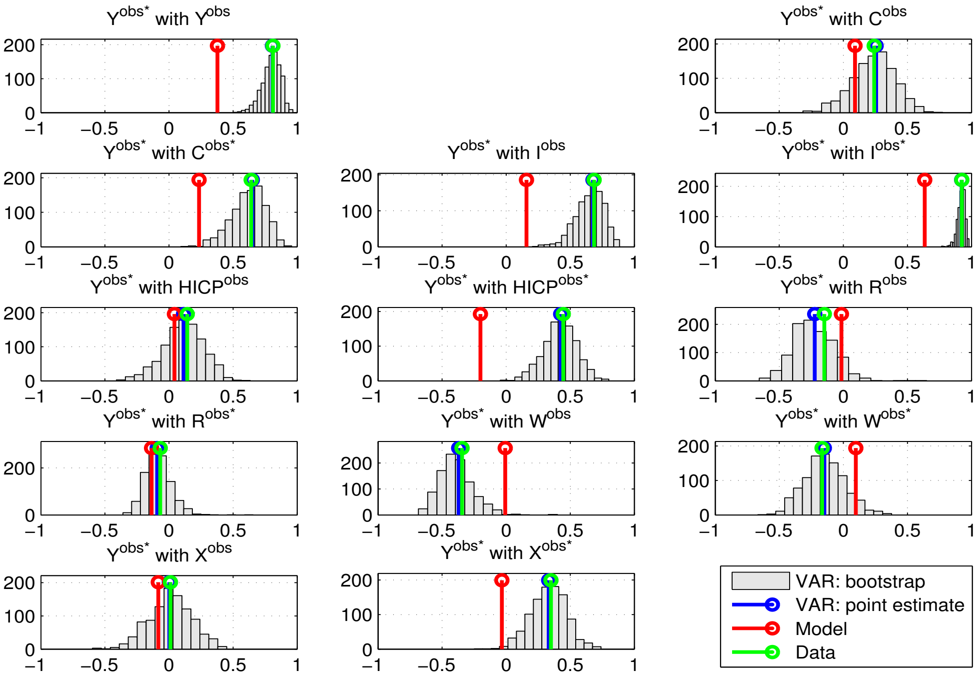


Figure 12: Cross-Correlations of Domestic Consumption

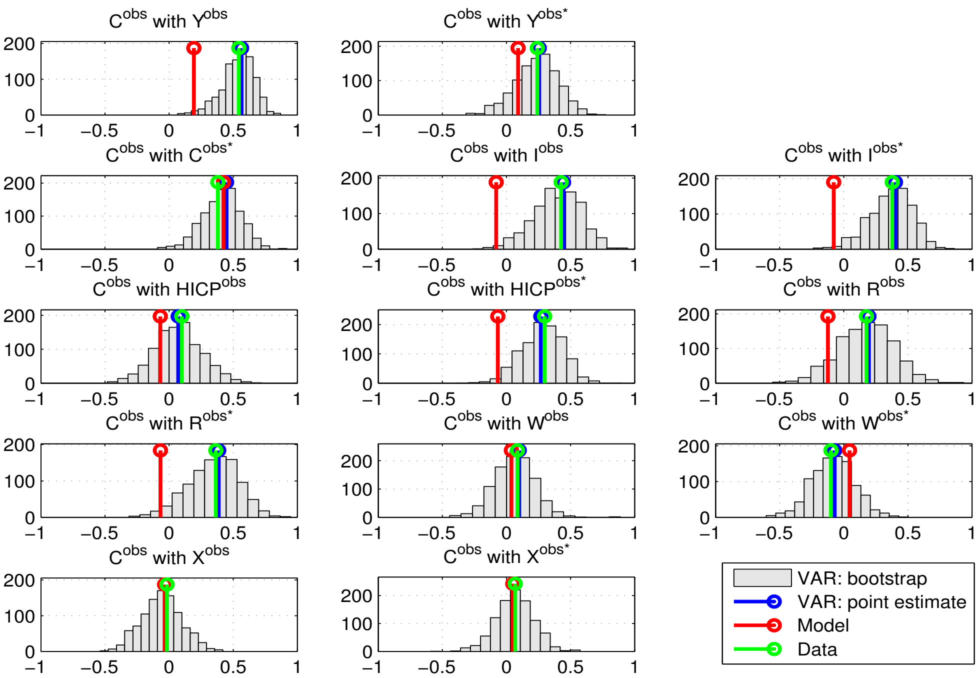


Figure 13: Cross-Correlations of Foreign Consumption

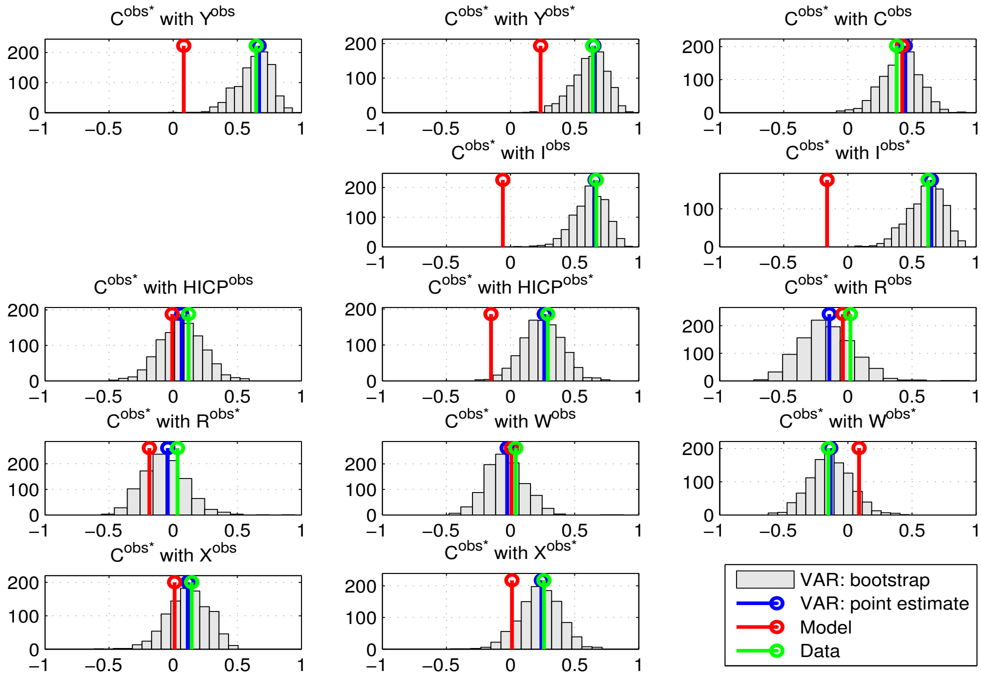


Figure 14: Cross-Correlations of Domestic Investment

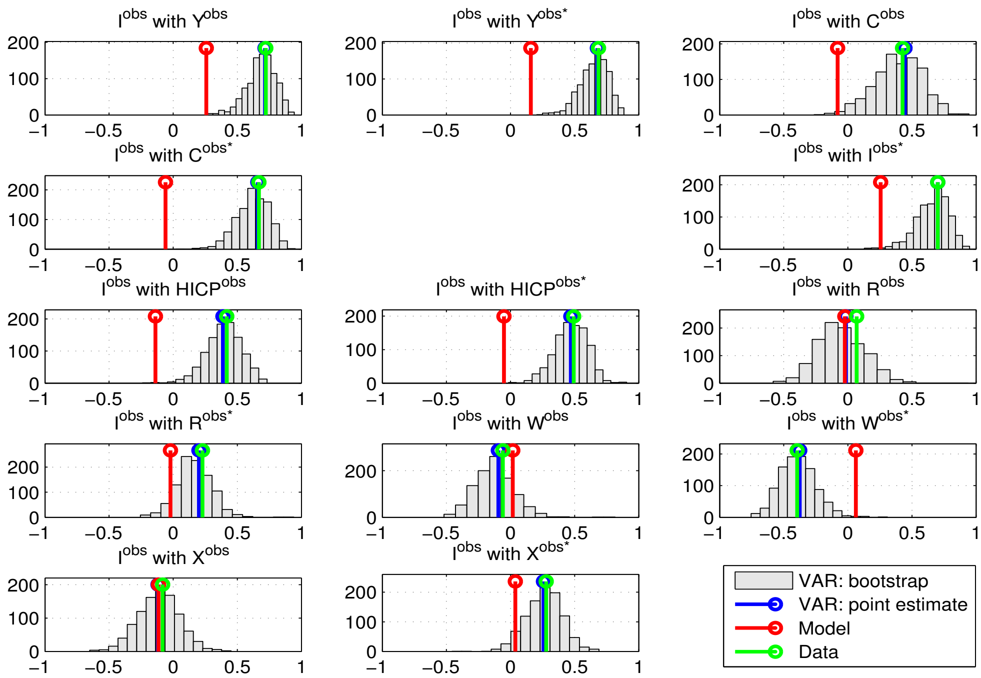


Figure 15: Cross-Correlations of Foreign Investment

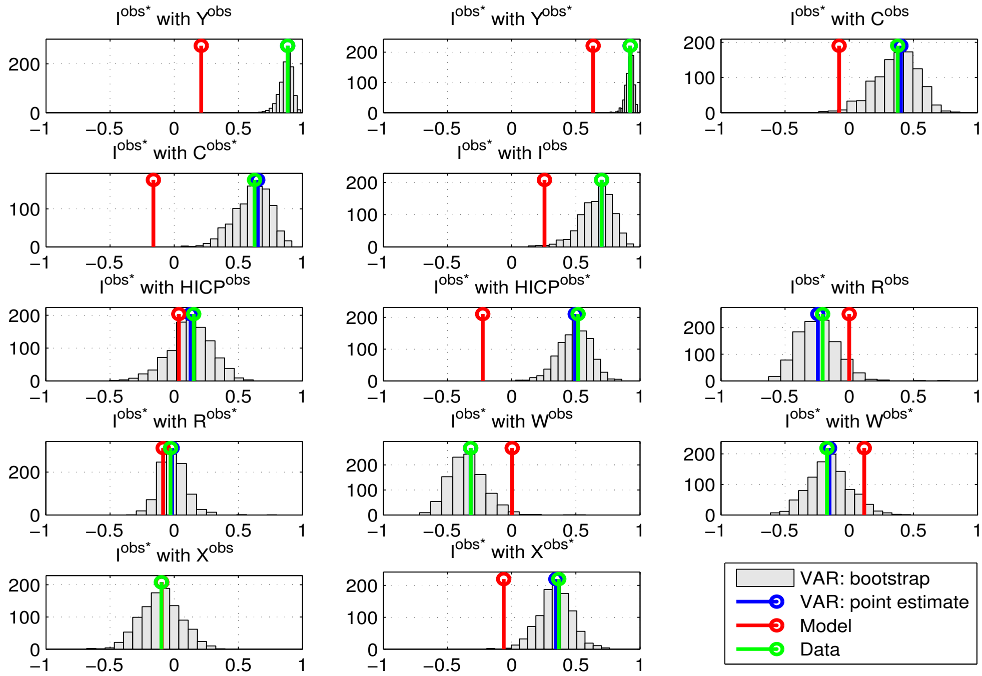


Figure 16: Cross-Correlations of Domestic Inflation

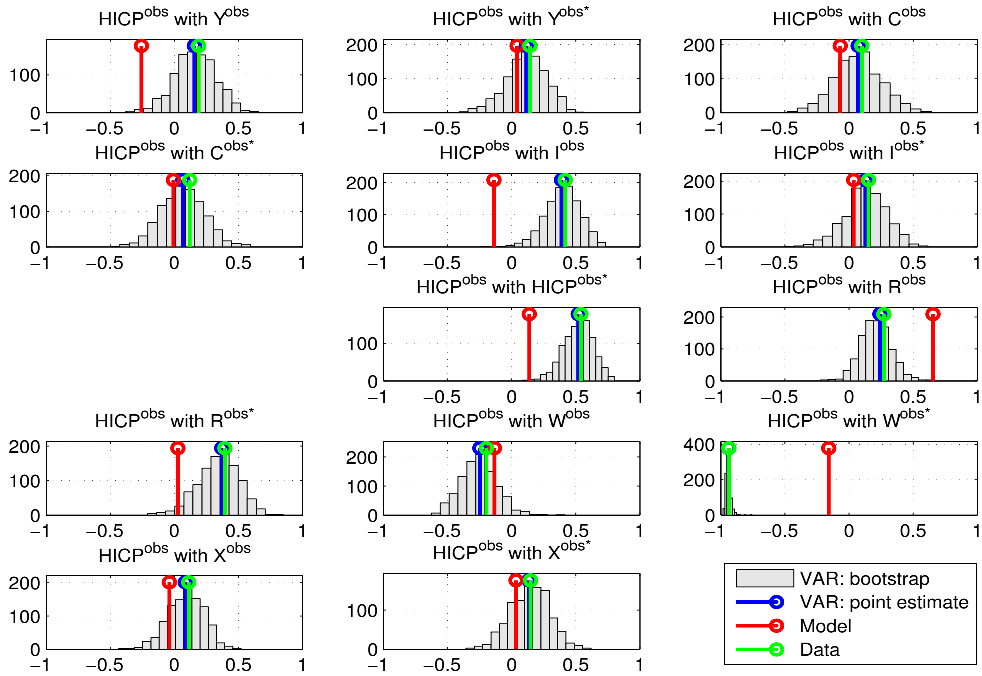


Figure 17: Cross-Correlations of Foreign Inflation

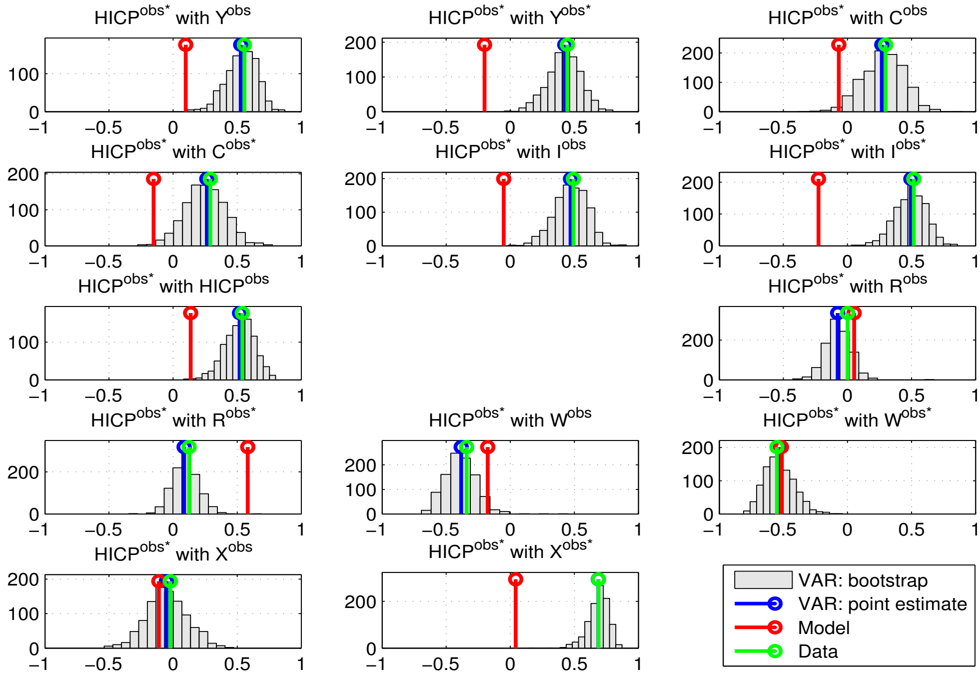


Figure 18: Cross-Correlations of Domestic Interest Rate

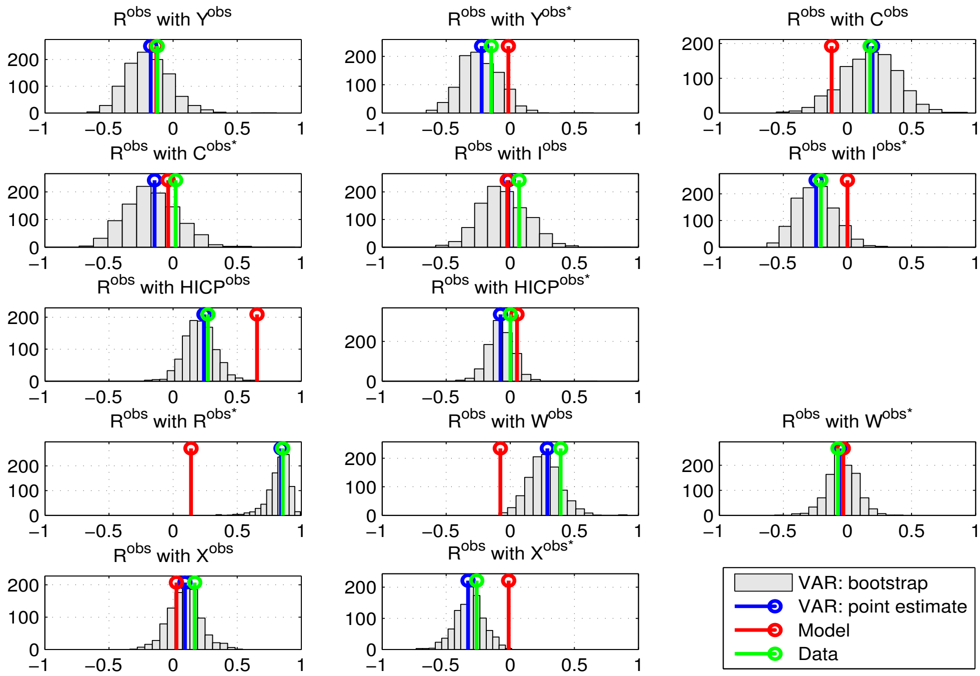


Figure 19: Cross-Correlations of Foreign Interest Rate

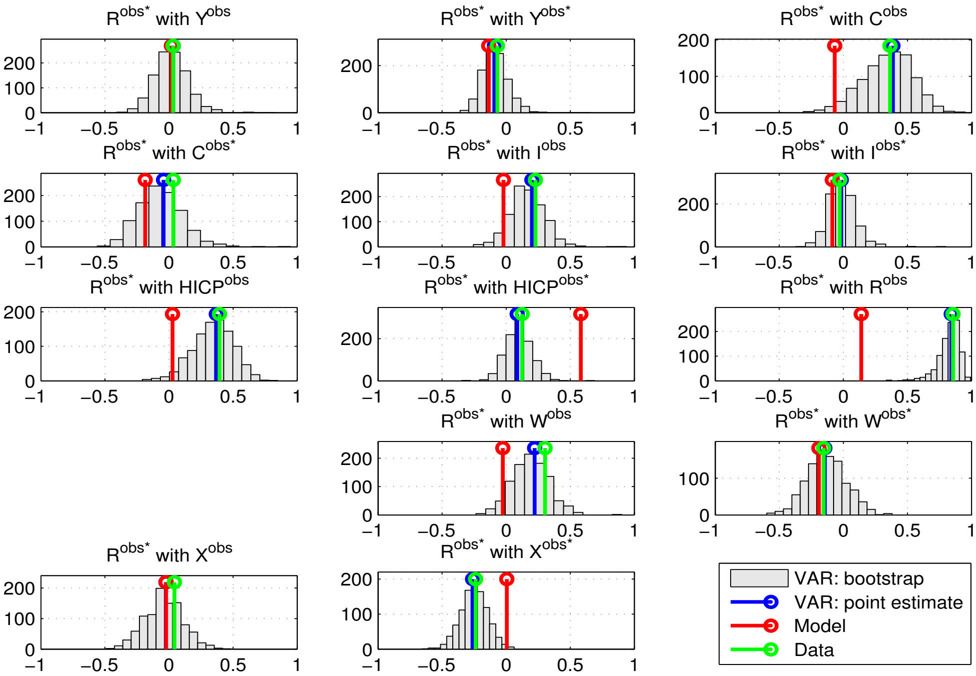


Figure 20: Cross-Correlations of Domestic Real Wage

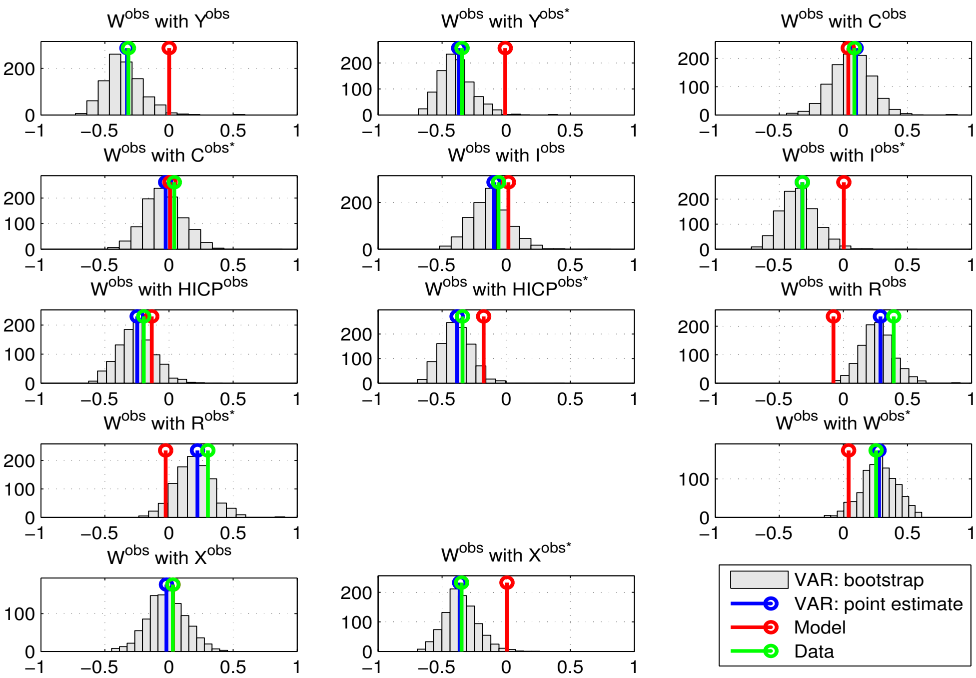


Figure 21: Cross-Correlations of Foreign Real Wage

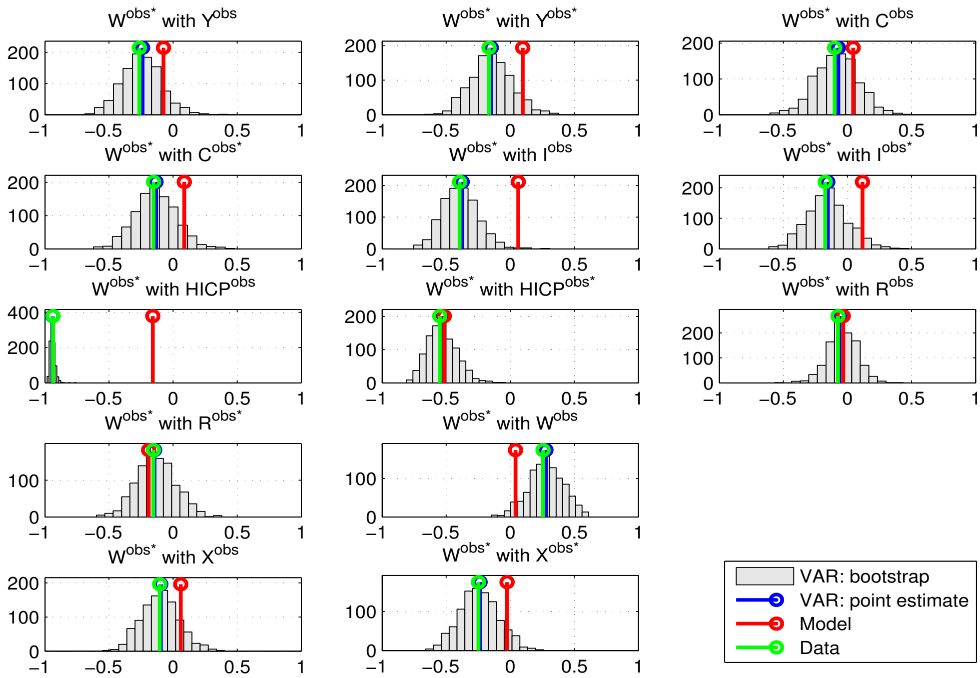


Figure 22: Cross-Correlations of Domestic Internal Exchange Rate

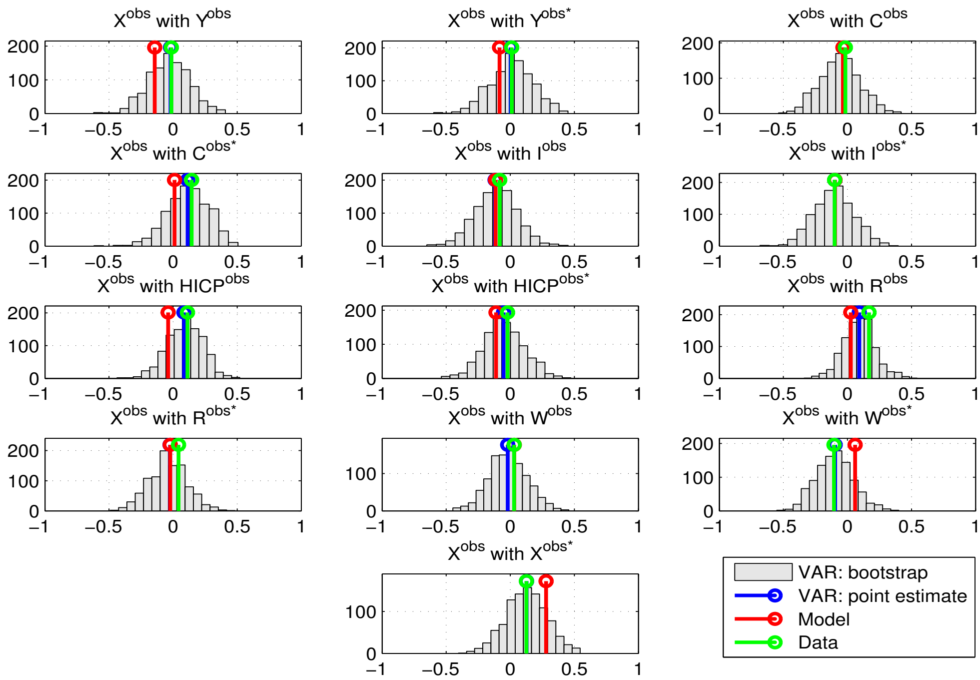


Figure 23: Cross-Correlations of Foreign Internal Exchange Rate

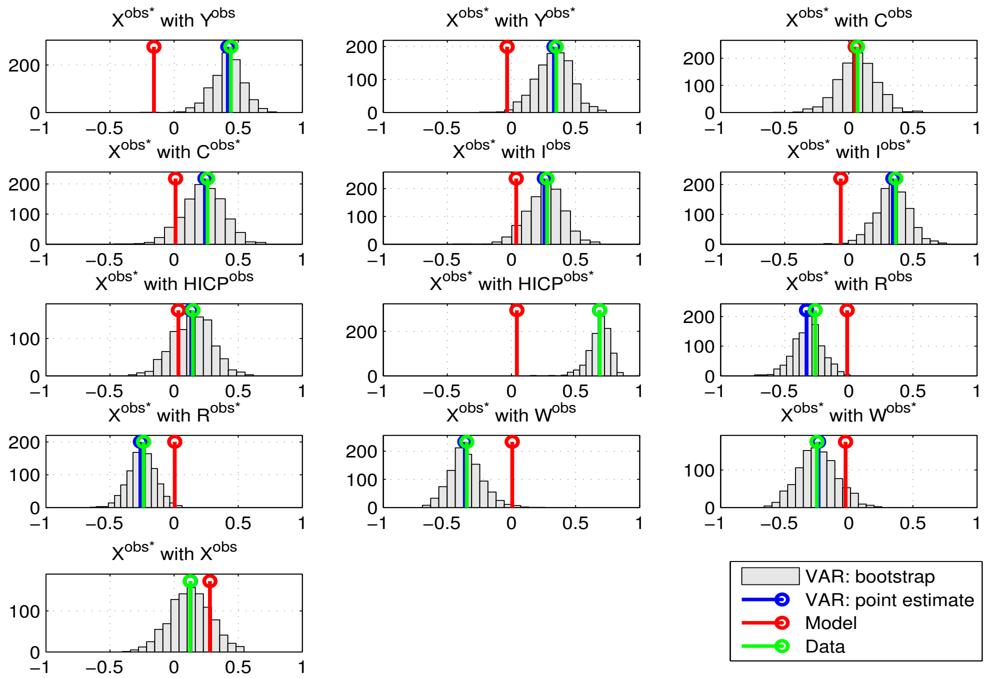


Figure 24: Variance of Observables

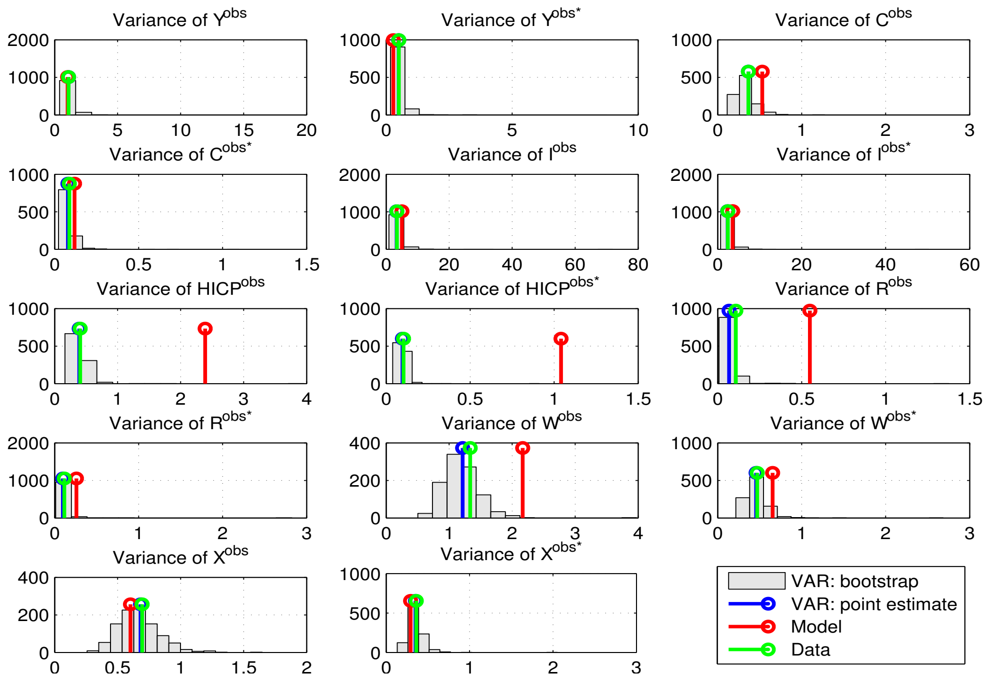


Figure 25: Auto-Correlation Functions 1

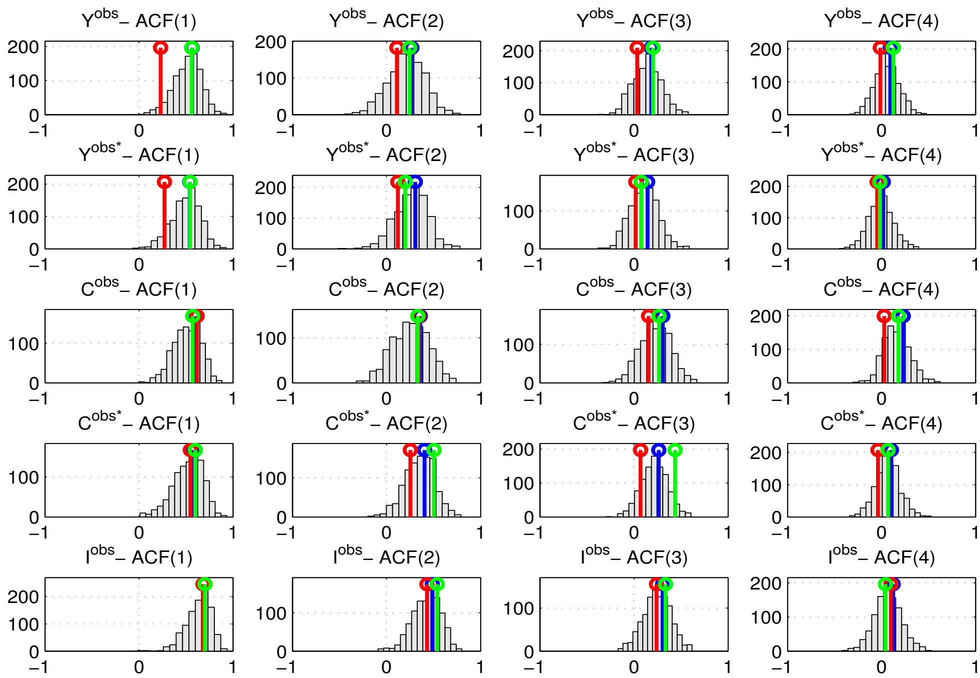


Figure 26: Auto-Correlation Functions 2

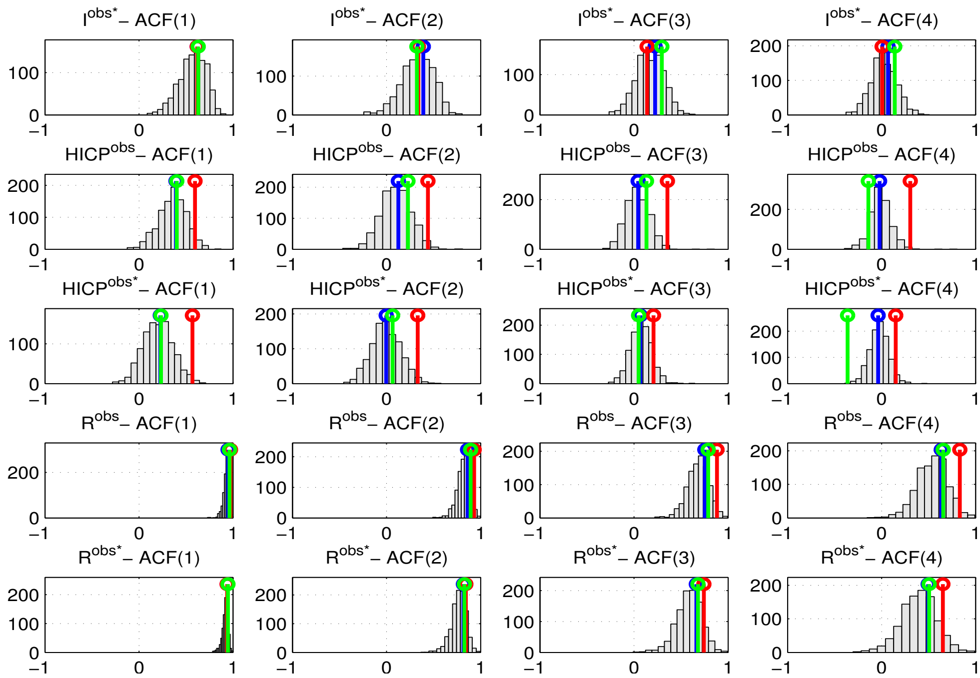
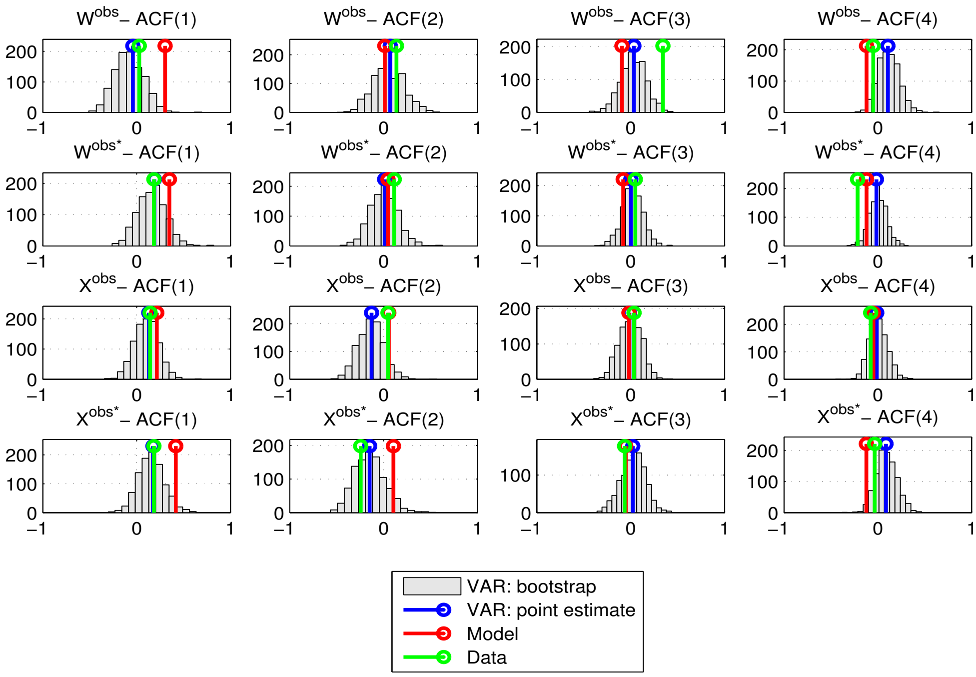


Figure 27: Auto-Correlation Functions 3



References

[1] Hamilton JJ (1994): *Time Series Analysis*. Princeton University Press, Princeton.

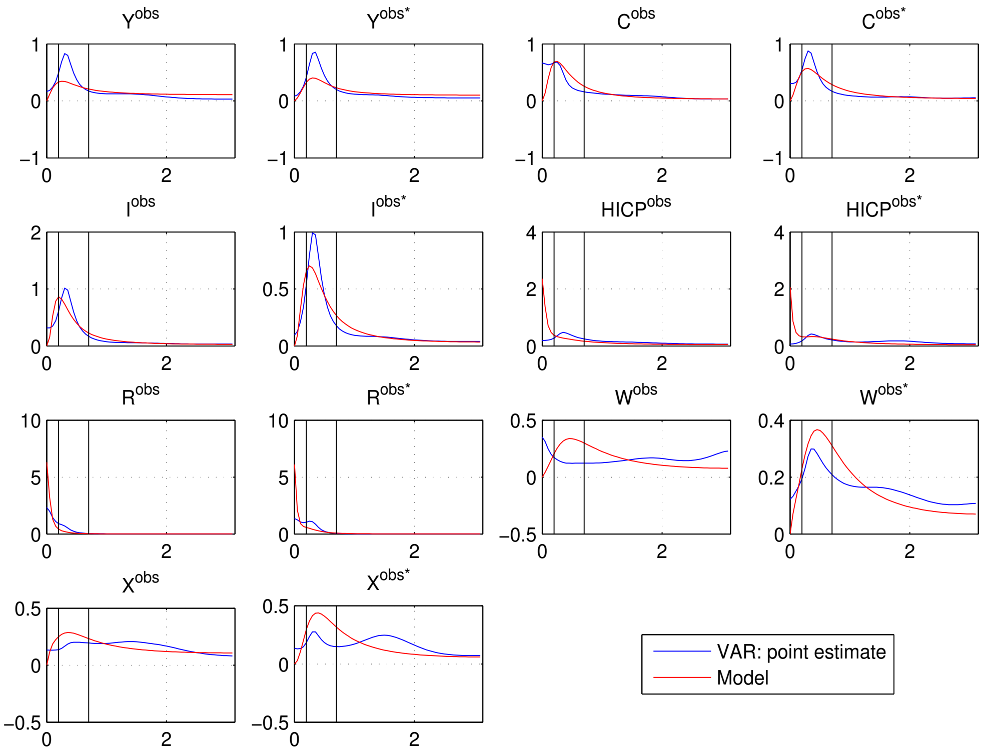
[2] Liu RY (1988): Bootstrap Procedures under some Non-I.I.D. Models. *Annals of Statistics*, 16(4):1696-1708.

[3] Wu CFJ (1986): Jackknife, Bootstrap and Other Resampling Methods in Regression Analysis. *Annals of Statistics*, 14(4):1261-1350.

Spectral Analysis

Figure 28 displays spectrum density functions of the observed variables for the model and for the VAR(1) model. The vertical lines highlight the frequencies which correspond to periods of 1 quarter and 10 quarters.

Figure 28: Spectrum Density Functions



Appendix - Shock Decomposition

Shock decomposition displays individual contributions of the shocks to the deviations of the examined variable from its respective steady state. Evolution of the examined variable is in every period decomposed into the contributions of the structural shocks.

Figure 29: Shock Decomposition of Observables - Domestic Output

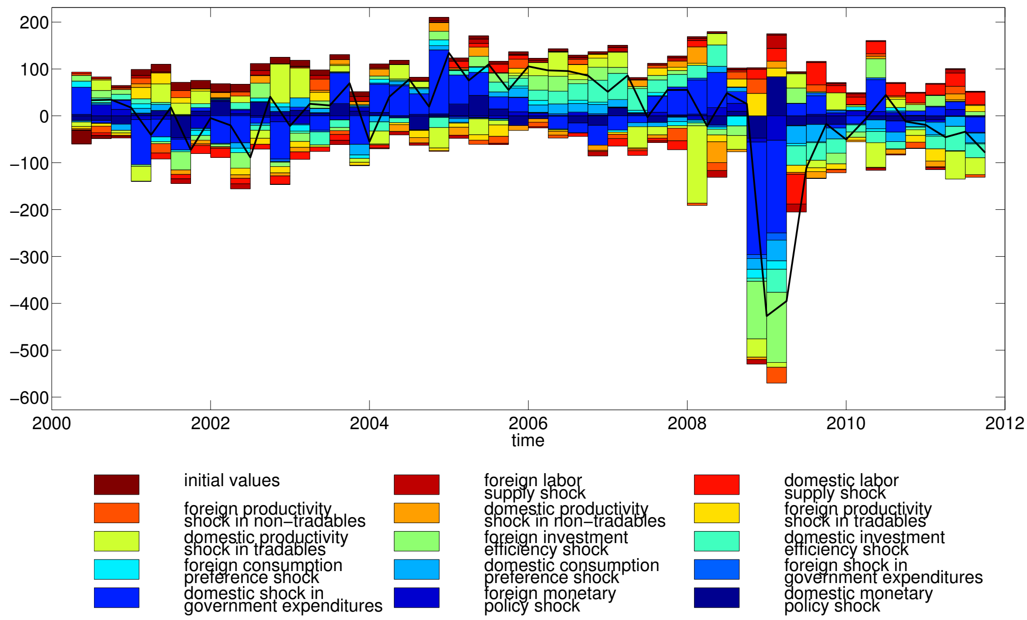


Figure 30: Shock Decomposition of Observables - Foreign Output

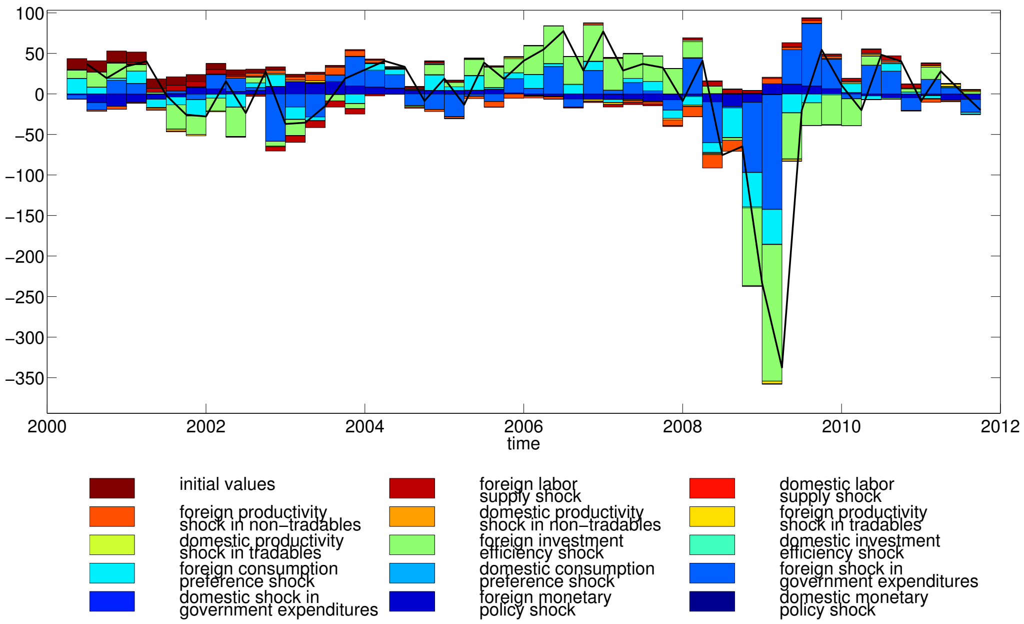


Figure 31: Shock Decomposition of Observables - Domestic Consumption

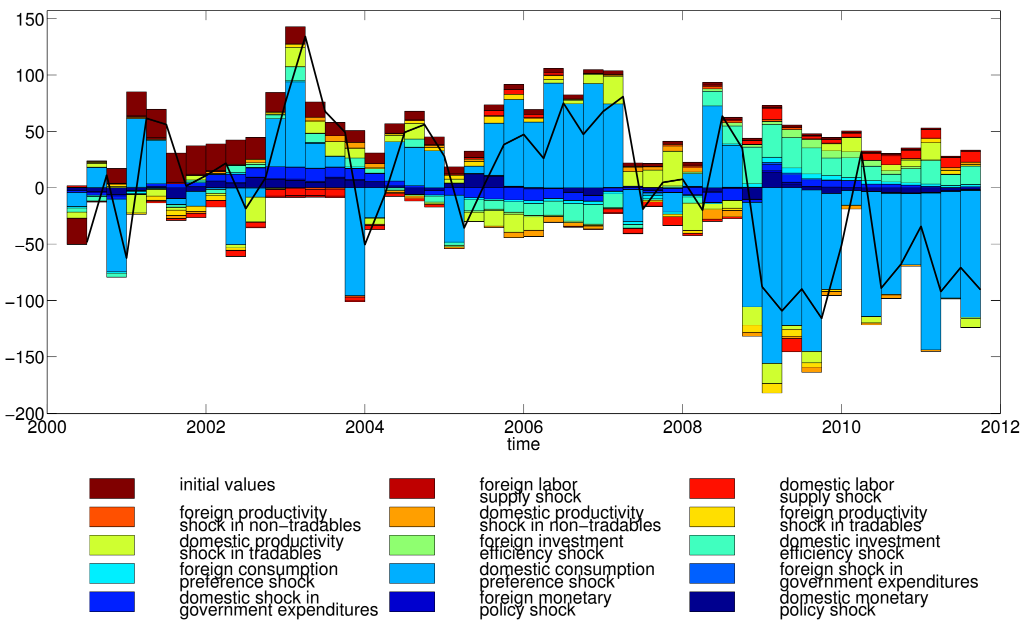


Figure 32: Shock Decomposition of Observables - Foreign Consumption

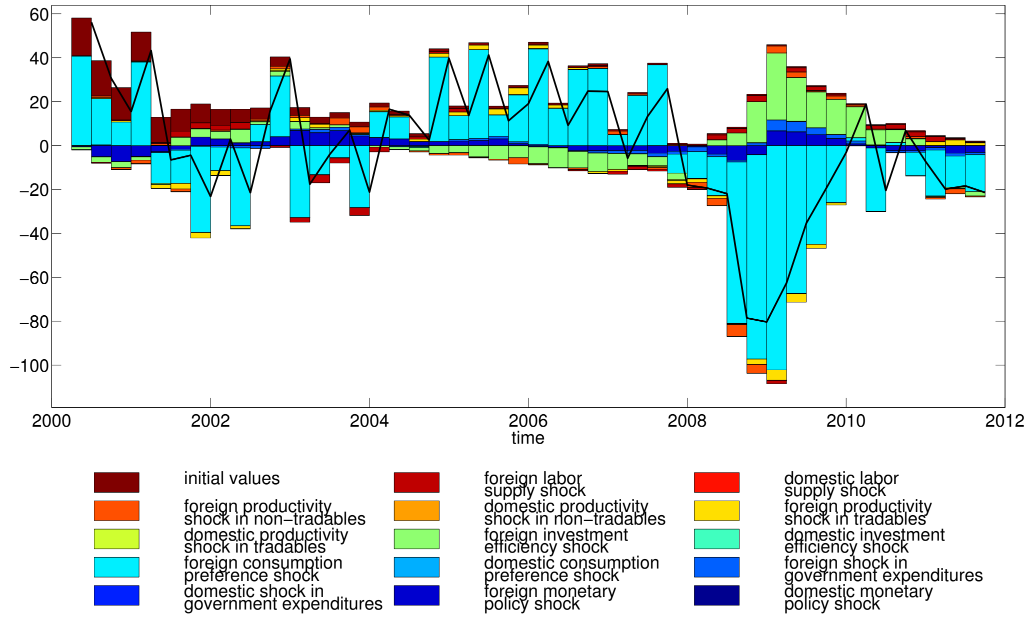


Figure 33: Shock Decomposition of Observables - Domestic Investment

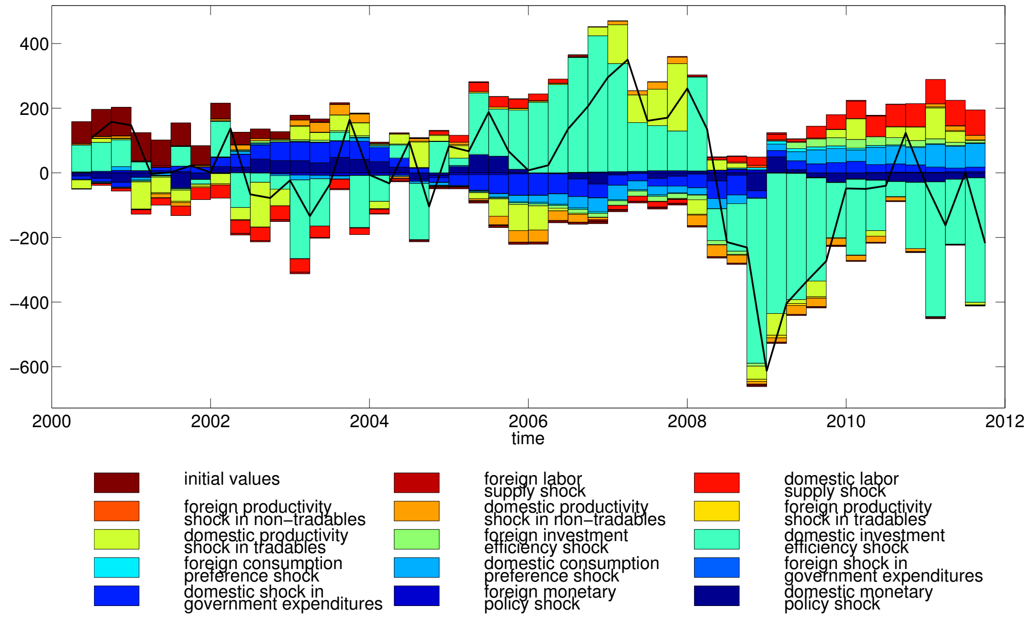


Figure 34: Shock Decomposition of Observables - Foreign Investment

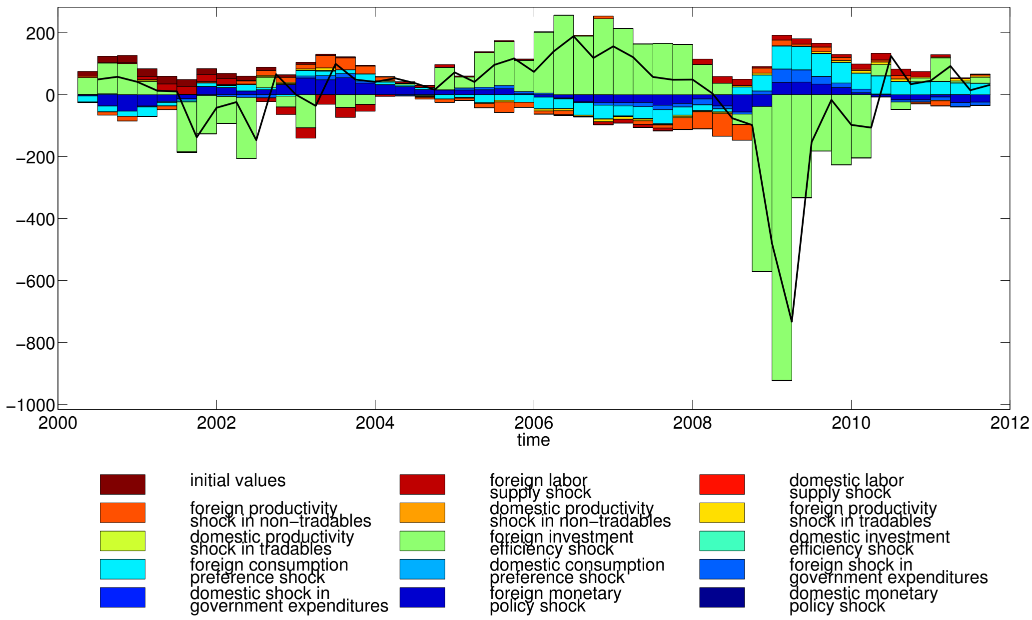


Figure 35: Shock Decomposition of Observables - Domestic Interest Rate

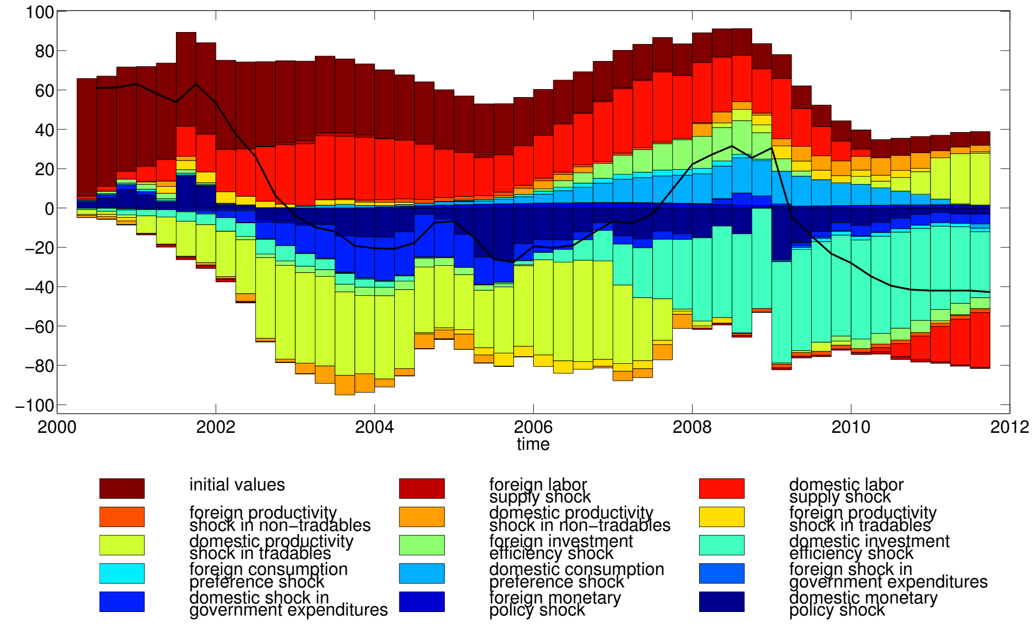


Figure 36: Shock Decomposition of Observables - Foreign Interest Rate

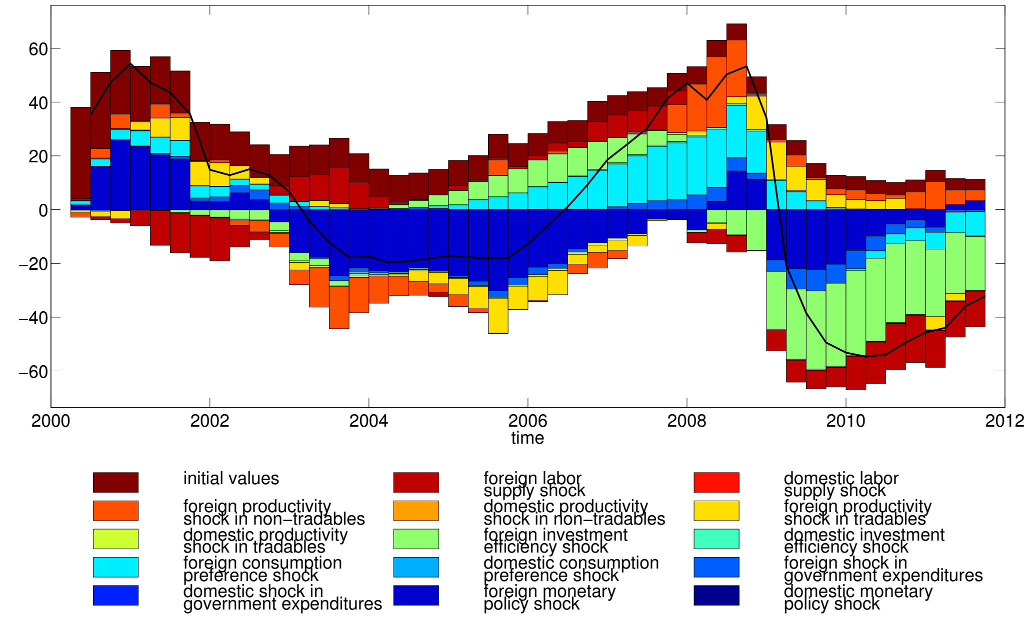


Figure 37: Shock Decomposition of Observables - Domestic Inflation

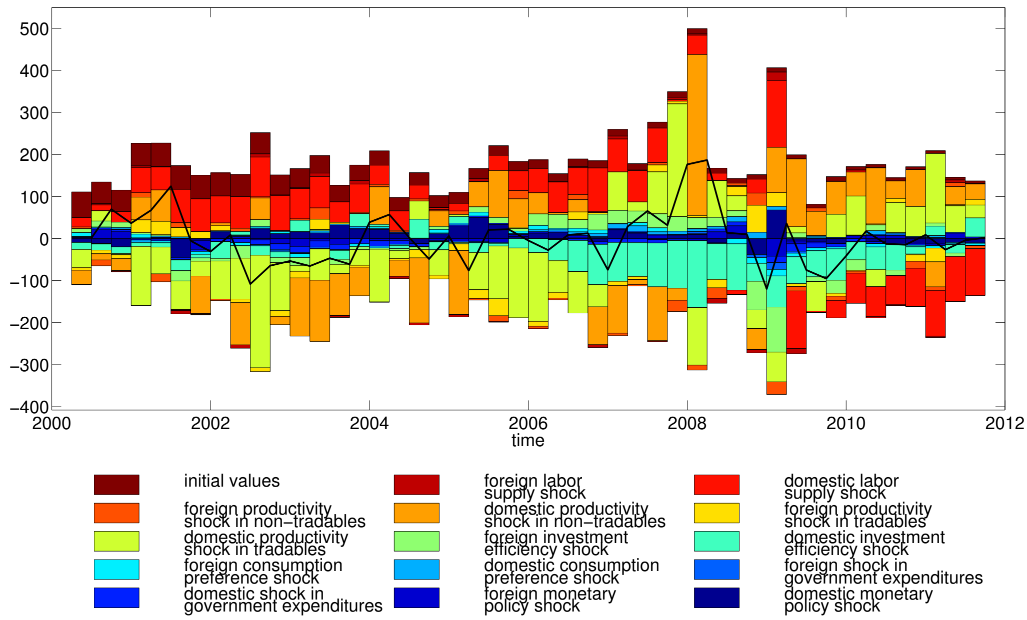


Figure 38: Shock Decomposition of Observables - Foreign Inflation

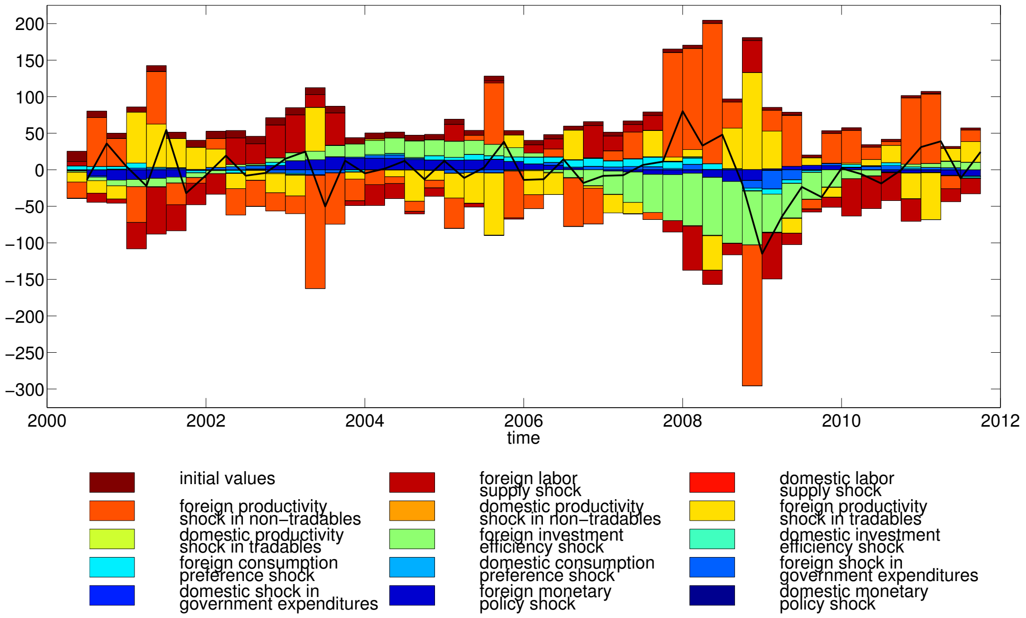


Figure 39: Shock Decomposition of Observables - Domestic Real Wage

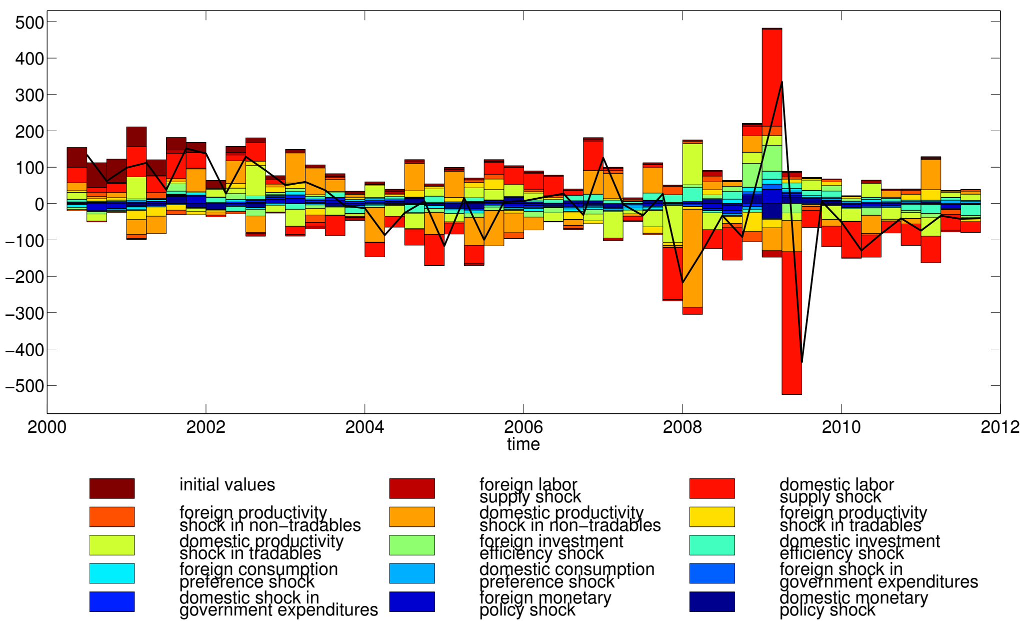


Figure 40: Shock Decomposition of Observables - Foreign Real Wage

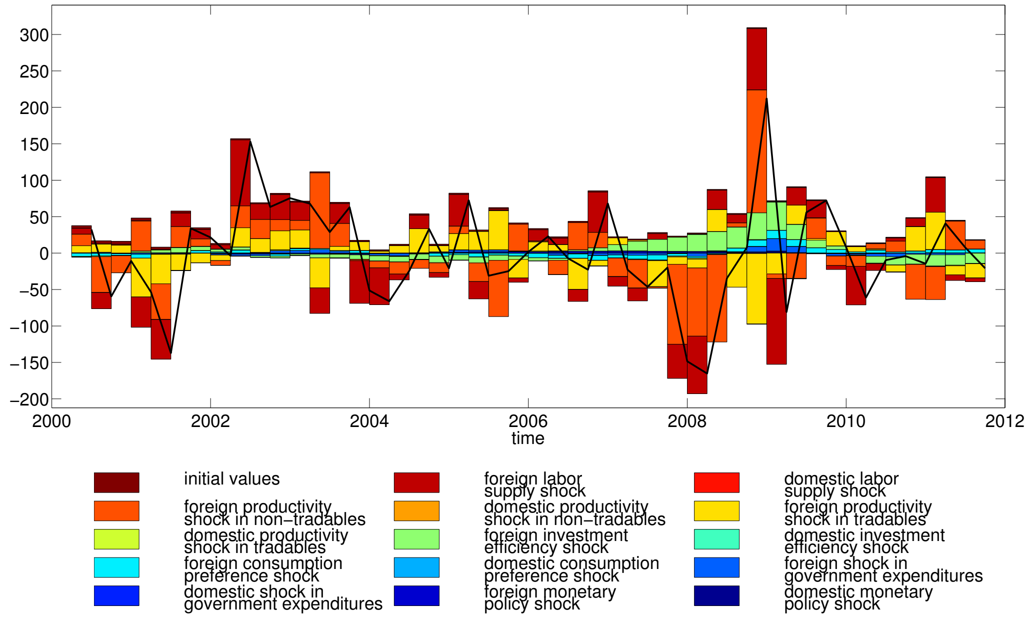


Figure 41: Shock Decomposition of Observables - Domestic Internal Exchange Rate

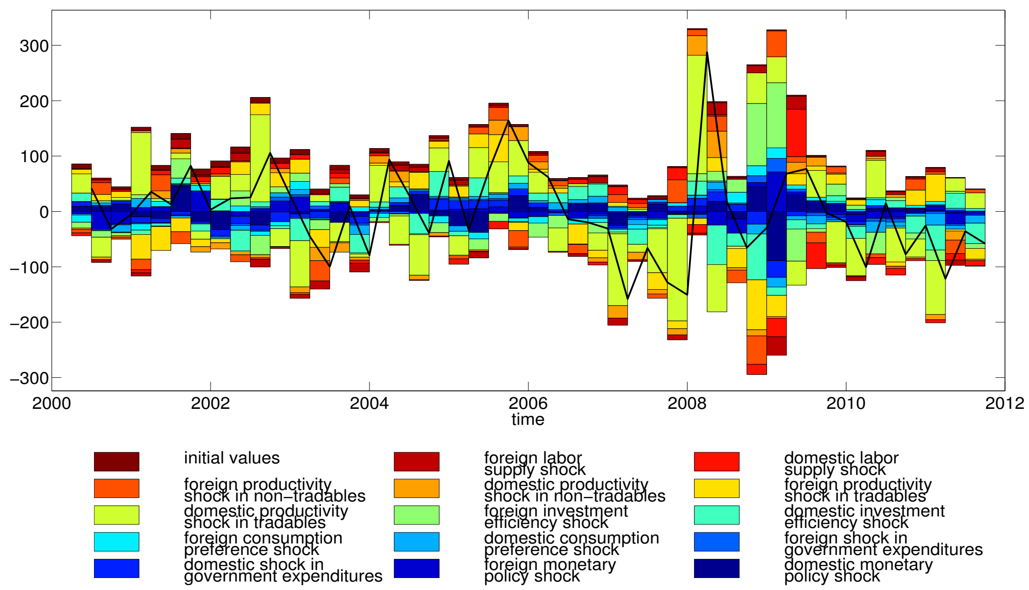
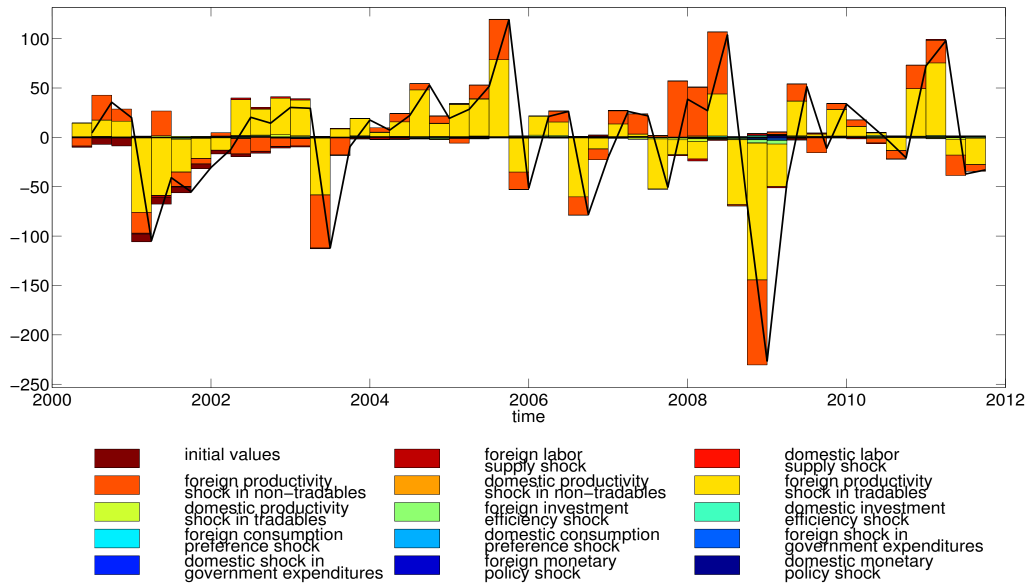


Figure 42: Shock Decomposition of Observables - Foreign Internal Exchange Rate



Appendix - Dynare Code

```
//DYNARE CODE
//Kolasa(2009) - demeaned log differences
//CZ and EA12, 2Q 2000 - 3Q 2011

clc;
close all;

var
c,c_star,        //consumption
x,x_star,        //internal terms of trade
s,               //external terms of trade
i,i_star,        //investment
y_H,y_F_star,    //product in tradable sector
y_N,y_N_star,    //product in non-tradable sector
y,y_star,        //product
r,r_star,        //nominal interest rate
q,               //real exchange rate
pi,pi_star,      //inflation
pi_T,pi_T_star, //inflation of tradable goods
k,k_star,        //capital
r_K,r_K_star,    //rental rate of capital
w,w_star,        //real wages
q_T,q_T_star,    //tobin's Q
l,l_star,        //labor
mrs,mrs_star,    //marginal rate of substitution
```

```
pi_H,pi_F_star, //inflation of raw tradable goods
mc_H,mc_F_star, //real marginal costs in tradable sector
pi_N,pi_N_star, //inflation of non-tradable goods
mc_N,mc_N_star, //real marginal costs in non-tradable sector

//Observables
y_obs, y_star_obs,
c_obs, c_star_obs,
i_obs, i_star_obs,
r_obs, r_star_obs,
pi_obs, pi_star_obs,
w_obs, w_star_obs,
x_obs, x_star_obs,

//AR processes for shocks
epsilon_g,epsilon_g_star,
epsilon_d,epsilon_d_star,
epsilon_i,epsilon_i_star,
epsilon_a_H,epsilon_a_F_star,
epsilon_a_N,epsilon_a_N_star,
epsilon_l,epsilon_l_star;

varexo
epsilon_m,epsilon_m_star,
mu_g,mu_g_star,
mu_d,mu_d_star,
mu_i,mu_i_star,
mu_a_H,mu_a_F_star,
mu_a_N,mu_a_N_star,
mu_l,mu_l_star;

parameters
beta, beta_star,       //discount factor
```

```
gamma_c,gamma_c_star,  //share of tradable goods in consumption
gamma_i,gamma_i_star,  //share of tradable goods in investment
alpha,alpha_star,      //share of domestic tradable goods
omega,omega_star,      //share of distribution costs
n,                     //relative size of economies
h,h_star,              //habit formation
sigma,sigma_star,      //inv. elasticity of intertemporal subs.
tau,tau_star,          //depreciation rate
S_prime,S_prime_star,  //adjustment costs
eta,eta_star,          //elasticity of output wrt capital
theta_W,theta_W_star,  //Calvo parameters for households
delta_W,delta_W_star,  //wage indexation
phi_W,phi_W_star,      //elasticity of subst. among labor types
phi,phi_star,          //inv. elasticity of labor supply
delta_H,delta_F_star,  //indexation of tradables
theta_H,theta_F_star,  //Calvo parameters for tradables
delta_N,delta_N_star,  //indexation of non-tradables
theta_N,theta_N_star,  //Calvo parameters for non-tradables
rho,rho_star,          //interest rate smoothing
psi_y,psi_y_star,      //elasticity of interest rate to output
psi_pi,psi_pi_star,    //elast. of interest rate to inflation

//persistence of shocks
rho_a_H, rho_a_F_star,
rho_a_N, rho_a_N_star,
rho_d, rho_d_star,
rho_l, rho_l_star,
rho_g, rho_g_star,
rho_i, rho_i_star,

//shares of variables in steady-state
ss_C_Y,
ss_C_star_Y_star,
```

```
ss_I_Y,
ss_I_star_Y_star,
ss_G_Y,
ss_G_star_Y_star,
ss_Y_N_Y,
ss_Y_N_star_Y_star,
ss_Y_H_Y,
ss_Y_F_star_Y_star;

//calibration of the model
//calibrated parameters
beta = 0.9975; beta_star = 0.9975;
gamma_c = 0.5424; gamma_c_star = 0.5014;
gamma_i = 0.4956; gamma_i_star = 0.4219;
alpha = 0.28; alpha_star = 0.989;
omega = 1; omega_star = 1;  //omega = 0; omega_star = 0;
n = 0.0135;
tau = 0.025; tau_star = 0.025;
phi_W = 3; phi_W_star = 3;
eta = 0.3868; eta_star = 0.3654;
delta_H = 0; delta_F_star = 0;
delta_N = 0; delta_N_star = 0;
delta_W = 0; delta_W_star = 0;

//calibrated shares of variables in steady-state
ss_C_Y = 0.4946;
ss_C_star_Y_star = 0.5691;
ss_I_Y = 0.2620;
ss_I_star_Y_star = 0.2036;
ss_G_Y = 1 - ss_C_Y - ss_I_Y;
ss_G_star_Y_star = 1 - ss_C_star_Y_star - ss_I_star_Y_star;

ss_Y_N_Y = ss_C_Y*(1 + omega - gamma_c)/(1 + omega) + ss_G_Y +
```

```
    ss_I_Y*(1 - gamma_i);

ss_Y_N_star_Y_star=ss_C_star_Y_star*(1+omega_star-gamma_c_star)
    /(1+omega_star)+ss_G_star_Y_star+ss_I_star_Y_star *
    (1 - gamma_i_star);

ss_Y_H_Y = 1 - ss_Y_N_Y;
ss_Y_F_star_Y_star = 1 - ss_Y_N_star_Y_star;

model(linear);

// 1.-6. Market Clearing Conditions
y_H = ss_C_Y/ss_Y_H_Y*gamma_c*alpha/(1+omega)*(c+(1-gamma_c)*x+
    (1-alpha)*s)+ss_C_star_Y_star/ss_Y_H_Y*(1-n)/n*gamma_c_star
    *(1-alpha_star)/(1+omega_star)*(c_star+(1-gamma_c_star)*
    x_star+alpha_star*s) + ss_I_Y/ss_Y_H_Y * gamma_i*alpha *
    (i + (1-gamma_i)*(1+omega)*x+(1-alpha)*s)+ss_I_star_Y_star/
    ss_Y_H_Y*(1-n)/n*gamma_i_star*(1-alpha_star)*(i_star +
    (1-gamma_i_star)*(1+omega_star)*x_star + alpha_star*s);

y_F_star = ss_C_star_Y_star/ss_Y_F_star_Y_star*gamma_c_star*
    alpha_star/(1+omega_star)*(c_star+(1-gamma_c_star)*x_star -
    (1-alpha_star)*s)+ss_C_Y/ss_Y_F_star_Y_star * n/(1-n) *
    gamma_c*(1-alpha)/(1+omega)*(c+(1-gamma_c)*x - alpha*s) +
    ss_I_star_Y_star/ss_Y_F_star_Y_star*gamma_i_star*alpha_star
    *(i_star + (1-gamma_i_star)*(1+omega_star)*x_star -
    (1-alpha_star)*s) +  ss_I_Y/ss_Y_F_star_Y_star * n/(1-n) *
    gamma_i*(1-alpha) * (i + (1-gamma_i)*(1+omega)*x -alpha*s);

y_N = ss_C_Y/ss_Y_N_Y*((1-gamma_c)*(c-gamma_c*x)+gamma_c*omega/
    (1+omega)*(c+(1-gamma_c)*x)) + ss_I_Y/ss_Y_N_Y*(1-gamma_i)*
    (i-gamma_i*(1+omega)*x) + ss_G_Y/ss_Y_N_Y * epsilon_g;
```

```
y_N_star=ss_C_star_Y_star/ss_Y_N_star_Y_star*((1-gamma_c_star)
    *(c_star-gamma_c_star*x_star)+gamma_c_star*omega_star/
    (1+omega_star)*(c_star+(1-gamma_c_star)*x_star))+
    ss_I_star_Y_star/ss_Y_N_star_Y_star*(1-gamma_i_star)*
    (i_star-gamma_i_star*(1+omega_star)*x_star)+
    ss_G_star_Y_star/ss_Y_N_star_Y_star * epsilon_g_star;

y = ss_Y_H_Y * y_H + ss_Y_N_Y * y_N;

y_star = ss_Y_F_star_Y_star*y_F_star+
    ss_Y_N_star_Y_star*y_N_star;

// 7.-8. Euler Equations
c-h*c(-1) = c(+1) - h*c - (1-h)/sigma * (r - pi(+1)) +
    (1-h)/sigma * (epsilon_d - epsilon_d(+1));

c_star-h_star*c_star(-1)=c_star(+1)-h_star*c_star-(1-h_star)/
    sigma_star*(r_star - pi_star(+1)) + (1-h_star)/sigma_star *
    (epsilon_d_star - epsilon_d_star(+1));

// 9. International Risk Sharing Condition
q = epsilon_d_star - epsilon_d - sigma_star/(1-h_star)*
    (c_star - h_star * c_star(-1)) + sigma/(1-h)*(c - h*c(-1));

// 10.-11. Law for Capital Acumulation
k = (1-tau)*k(-1) + tau*(i + epsilon_i);

k_star=(1-tau_star)*k_star(-1)+tau_star*(i_star +
    epsilon_i_star);

// 12.-13. Real Marginal Costs
r_K = w + l - k(-1);
```

```
r_K_star = w_star + l_star - k_star(-1);

// 14.-15. Investment Demand
i - i(-1) = beta *(i(+1)-i) + 1/S_prime *(q_T + epsilon_i) -
    (gamma_i*(1+omega)-gamma_c)/S_prime * x;

i_star - i_star(-1) = beta_star*(i_star(+1)-i_star)+
    1/S_prime_star*(q_T_star+epsilon_i_star)-
    (gamma_i_star*(1+omega_star)-gamma_c_star)/
    S_prime_star * x_star;

// 16.-17. Price of the Capital
q_T = beta*(1-tau)*q_T(+1)-(r - pi(+1)) +
    (1-beta*(1-tau))*r_K(+1);

q_T_star = beta_star *(1-tau_star) * q_T_star(+1) - (r_star -
    pi_star(+1)) + (1-beta_star*(1-tau_star))* r_K_star(+1);

// 18.-19. Labor Input
l = eta * (r_K - w) + ss_Y_H_Y * (y_H - epsilon_a_H) +
    ss_Y_N_Y * (y_N - epsilon_a_N);

l_star = eta_star * (r_K_star - w_star) + ss_Y_F_star_Y_star *
    (y_F_star - epsilon_a_F_star) + ss_Y_N_star_Y_star *
    (y_N_star - epsilon_a_N_star);

//20.-21. Real Wage Rate
w-w(-1)=(1-theta_W)*(1-beta*theta_W)/(theta_W*(1+phi_W*phi))*
     (mrs-w)+ beta*(w(+1)-w)+beta*(pi(+1)-delta_W*pi) -
     (pi - delta_W*pi(-1));

w_star-w_star(-1)=(1-theta_W_star)*(1-beta_star*theta_W_star)/
     (theta_W_star*(1+phi_W_star*phi_star))*(mrs_star-w_star)+
```

```
     beta_star*(w_star(+1)-w_star)+beta_star*(pi_star(+1)-
     delta_W_star*pi_star)-(pi_star-delta_W_star*pi_star(-1));

//22.-23. Marginal Rate of Substitution
mrs = epsilon_l + phi*l - epsilon_d + sigma/(1-h)*(c-h*c(-1));

mrs_star = epsilon_l_star + phi_star*l_star - epsilon_d_star +
      sigma_star/(1-h_star)*(c_star - h_star*c_star(-1));

//24.-25. PC for Raw Tradables
pi_H - delta_H*pi_H(-1) = (1-theta_H)*(1-beta*theta_H)/theta_H*
      mc_H+beta *(pi_H(+1) - delta_H*pi_H);

pi_F_star - delta_F_star*pi_F_star(-1) = (1-theta_F_star)*
      (1-beta_star*theta_F_star)/theta_F_star * mc_F_star +
      beta_star *(pi_F_star(+1) - delta_F_star*pi_F_star);

//26.-27. PC for Non-tradables
pi_N - delta_N*pi_N(-1) = (1-theta_N)*(1-beta*theta_N)/theta_N*
      mc_N + beta * (pi_N(+1)- delta_N * pi_N);

pi_N_star-delta_N_star*pi_N_star(-1)=(1-theta_N_star)*
      (1-beta_star*theta_N_star)/theta_N_star * mc_N_star +
      beta_star * (pi_N_star(+1)- delta_N_star * pi_N_star);

//28.-29. Real Marginal Costs in Tradable Sector
mc_H = (1-eta)*w + eta * r_K - epsilon_a_H + (1-alpha)*s +
      (1+omega-gamma_c)*x;

mc_F_star=(1-eta_star)*w_star+eta_star*r_K_star-
       epsilon_a_F_star-(1-alpha_star)*s+
       (1+omega_star-gamma_c_star)*x_star;
```

```
//30.-31. Real Marginal Costs in Non-tradable Sector
mc_N = (1-eta)*w + eta * r_K - epsilon_a_N - gamma_c*x;

mc_N_star = (1-eta_star)*w_star + eta_star * r_K_star -
       epsilon_a_N_star - gamma_c_star*x_star;

//32.-33. Internal Exchange Rate
x - x(-1) = pi_N - pi_T;

x_star - x_star(-1) = pi_N_star - pi_T_star;

//34.-35. Inflation of Tradables
pi_T = 1/(1+omega)*(pi_H+(1-alpha)*(s-s(-1))+omega*pi_N);

pi_T_star = 1/(1+omega_star)*(pi_F_star-(1-alpha_star)*
       (s - s(-1))+omega_star * pi_N_star);

//36.-37. CPI Inflation
pi = gamma_c * pi_T + (1 - gamma_c) * pi_N;

pi_star = gamma_c_star*pi_T_star + (1-gamma_c_star)*pi_N_star;

//38. Real Exchange Rate
q = (alpha + alpha_star-1)*s + (1 + omega_star - gamma_c_star)*
        x_star - (1 + omega - gamma_c) * x;

//39.-40. Monetary Policy Rule
r = rho*r(-1)+(1-rho)*(psi_y*y(+1)+psi_pi*pi(+1))+epsilon_m;

r_star=rho_star*r_star(-1)+(1-rho_star)*(psi_y_star*y_star(+1)+
        psi_pi_star * pi_star(+1)) + epsilon_m_star;

//41.-42. Productivity Shock in Tradable Sector
```

```
epsilon_a_H = rho_a_H * epsilon_a_H(-1) + mu_a_H;

epsilon_a_F_star=rho_a_F_star*epsilon_a_F_star(-1)+mu_a_F_star;

//43.-44. Productivity Shock in Non-tradable Sector
epsilon_a_N = rho_a_N * epsilon_a_N(-1) + mu_a_N;

epsilon_a_N_star=rho_a_N_star*epsilon_a_N_star(-1)+mu_a_N_star;

//45.-46. Preference Shock
epsilon_d = rho_d * epsilon_d(-1) + mu_d;

epsilon_d_star=rho_d_star*epsilon_d_star(-1)+mu_d_star;

//47.-48. Labor Supply Shock
epsilon_l = rho_l * epsilon_l(-1) + mu_l;

epsilon_l_star=rho_l_star*epsilon_l_star(-1)+mu_l_star;

//49.-50. Shock in Government Expenditures
epsilon_g = rho_g * epsilon_g(-1) + mu_g;

epsilon_g_star=rho_g_star*epsilon_g_star(-1)+mu_g_star;

//51.-52. Investment Efficiency Shock
epsilon_i = rho_i * epsilon_i(-1) + mu_i;

epsilon_i_star=rho_i_star*epsilon_i_star(-1)+mu_i_star;

//Linking Observables to Model Variables
y_obs = y - y(-1);
y_star_obs = y_star - y_star(-1);
```

```
pi_obs = pi;
pi_star_obs = pi_star;

x_obs = x - x(-1);
x_star_obs = x_star - x_star(-1);

r_obs = r;
r_star_obs = r_star;

w_obs = w - w(-1);
w_star_obs = w_star - w_star(-1);

c_obs = c - c(-1);
c_star_obs = c_star - c_star(-1);

i_obs = i - i(-1);
i_star_obs = i_star - i_star(-1);
end;

//Model is in the gap form, therefore steady state for all
//variables is 0.
initval;
c = 0; c_star = 0;
x = 0; x_star = 0;
s = 0;
i = 0; i_star = 0;
y_H = 0; y_F_star = 0;
y_N = 0; y_N_star = 0;
y = 0; y_star = 0;
r = 0; r_star = 0;
q = 0;
pi = 0; pi_star = 0;
k = 0; k_star = 0;
```

```
r_K = 0; r_K_star = 0;
w = 0; w_star = 0;
q_T = 0; q_T_star = 0;
l = 0; l_star = 0;
mrs = 0; mrs_star = 0;
pi_H = 0; pi_F_star = 0;
mc_H = 0; mc_F_star = 0;
pi_N = 0; pi_N_star = 0;
mc_N = 0; mc_N_star = 0;
epsilon_g = 0; epsilon_g_star = 0;
epsilon_d = 0; epsilon_d_star = 0;
epsilon_i = 0; epsilon_i_star = 0;
epsilon_a_H = 0; epsilon_a_F_star = 0;
epsilon_a_N = 0; epsilon_a_N_star = 0;
epsilon_l = 0; epsilon_l_star = 0;
end;

//Estimated Parameters and their Priors
estimated_params;
h,              0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
h_star,         0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
sigma,          1.0, 1E-5, 10,      gamma_pdf,1,0.7;
sigma_star,     1.0, 1E-5, 10,      gamma_pdf,1,0.7;
phi,            1.0, 1E-5, 10,      gamma_pdf,1,0.7;
phi_star,       1.0, 1E-5, 10,      gamma_pdf,1,0.7;
S_prime,        4.0, 1E-5, 10,      normal_pdf,4,1.5;
S_prime_star,   4.0, 1E-5, 10,      normal_pdf,4,1.5;

//delta_H,      0.5, 1E-5, 0.9999,  beta_pdf,0.5,0.2;
//delta_F_star, 0.5, 1E-5, 0.9999,  beta_pdf,0.5,0.2;
//delta_N,      0.5, 1E-5, 0.9999,  beta_pdf,0.5,0.2;
//delta_N_star, 0.5, 1E-5, 0.9999,  beta_pdf,0.5,0.2;
//delta_W,      0.5, 1E-5, 0.9999,  beta_pdf,0.5,0.2;
```

```
//delta_W_star, 0.5, 1E-5, 0.9999,  beta_pdf,0.5,0.2;
theta_H,        0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.05;
theta_F_star,   0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.05;
theta_N,        0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.05;
theta_N_star,   0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.05;
theta_W,        0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.05;
theta_W_star,   0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.05;

rho,            0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.15;
rho_star,       0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.15;
psi_y,          0.25, 1E-5, 2,      gamma_pdf,0.25,0.1;
psi_y_star,     0.25, 1E-5, 2,      gamma_pdf,0.25,0.1;
psi_pi,         1.3, 1E-5, 5,       gamma_pdf,1.3,0.15;
psi_pi_star,    1.3, 1E-5, 5,       gamma_pdf,1.3,0.15;

rho_a_H,        0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_a_F_star,   0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_a_N,        0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_a_N_star,   0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_d,          0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_d_star,     0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_l,          0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_l_star,     0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_g,          0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_g_star,     0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_i,          0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;
rho_i_star,     0.7, 1E-5, 0.9999,  beta_pdf,0.7,0.1;

stderr mu_a_H,          inv_gamma_pdf,2,inf;
stderr mu_a_F_star,     inv_gamma_pdf,2,inf;
stderr mu_a_N,          inv_gamma_pdf,2,inf;
stderr mu_a_N_star,     inv_gamma_pdf,2,inf;
stderr mu_d,            inv_gamma_pdf,6,inf;
```

```
stderr mu_d_star,        inv_gamma_pdf,6,inf;
stderr mu_l,             inv_gamma_pdf,10,inf;
stderr mu_l_star,        inv_gamma_pdf,10,inf;
stderr mu_g,             inv_gamma_pdf,3,inf;
stderr mu_g_star,        inv_gamma_pdf,3,inf;
stderr mu_i,             inv_gamma_pdf,6,inf;
stderr mu_i_star,        inv_gamma_pdf,6,inf;
stderr epsilon_m,        inv_gamma_pdf,0.3,inf;
stderr epsilon_m_star,   inv_gamma_pdf,0.3,inf;

corr mu_a_H, mu_a_F_star,       0, -1, 1, normal_pdf,0,0.4;
corr mu_a_N, mu_a_N_star,       0, -1, 1, normal_pdf,0,0.4;
corr mu_d, mu_d_star,           0, -1, 1, normal_pdf,0,0.4;
corr mu_l, mu_l_star,           0, -1, 1, normal_pdf,0,0.4;
corr mu_g, mu_g_star,           0, -1, 1, normal_pdf,0,0.4;
corr mu_i, mu_i_star,           0, -1, 1, normal_pdf,0,0.4;
corr epsilon_m, epsilon_m_star, 0, -1, 1, normal_pdf,0,0.4;
end;

varobs  y_obs, y_star_obs, c_obs, c_star_obs, i_obs,
        i_star_obs, r_obs, r_star_obs, pi_obs, pi_star_obs,
        w_obs, w_star_obs, x_obs, x_star_obs;

estimation(datafile=data_cz_eu_2Q2000_3Q2011, mh_nblocks=4,
        mh_replic=2000000, mh_drop=0.75, mh_jscale=0.25,
        mode_check, smoother);

shock_decomposition y_obs y_star_obs c_obs c_star_obs i_obs
        i_star_obs r_obs r_star_obs pi_obs pi_star_obs w_obs
        w_star_obs x_obs x_star_obs;
```