

The Dynamics of Return Comovement and Spillovers Between the Czech and European Stock Markets in the Period 1997–2010

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Abstract

The paper examines the comovement and spillover dynamics between the returns of the Czech and some major European stock markets (namely, the Austrian, French, German, and UK markets, as well as the Central and Eastern European stock markets of Poland, Hungary, and Slovenia). By applying the Dynamic Conditional Correlation GARCH model and Granger causality tests on wavelet transformed returns series for the period April 1997–May 2010 the following specific questions are answered. Is the comovement (correlation) between the Czech and European stock markets time-varying? What effect did the financial crises in the period 1997–2010 and the accession of the Czech Republic to the European Union have on the comovement between the Czech and European stock markets investigated? We also investigate whether there were return spillovers between the markets and whether they depended on the horizon over which they are calculated (i.e., are they a multiscale phenomenon). We found that comovement between the Czech and other stock market returns is time-varying. Furthermore, we found significant return spillovers between the Czech and European stock markets in the observed period. The wavelet Granger causality tests show that return spillovers were a multiscale phenomenon.

1. Introduction

International stock market links are of great importance for the financial decisions of international investors. Since the seminal work of Markowitz (1958) and the empirical evidence of Grubel (1968) it has been recognized that international diversification reduces the total risk of a portfolio. This is due to non-perfect positive comovement between the returns on portfolio assets. Increased comovement between asset returns can therefore diminish the advantage of internationally diversified investment portfolios.

Modeling the comovement of stock market returns, however, is a challenging task. The conventional measure of interdependence, known as the Pearson correlation coefficient, is a symmetric, linear dependence metric, suitable for measuring dependence in multivariate normal distributions (Embrechts et al., 1999). The correlations, however, may be nonlinear and time-varying (Xiao and Dhesi, 2010; Égert and Kočenda, 2010). A better understanding of stock market interdependencies may be achieved by applying econometric methods: Vector Autoregressive (VAR) models (Gilmore and McManus, 2002; Tudor, 2010), cointegration analysis (Gerrits and Yuce, 1999; Patev et al., 2006), GARCH models (Tse and Tsui, 2002; Bae et al., 2003; Égert and Kočenda, 2010), and regime switching models (Schwender, 2010). A more novel approach is based on wavelet analysis (Ranta, 2010; Zhou, 2011). The existing empirical literature on the interdependence of the Czech and European

stock markets predominantly applies correlation analysis (Serwa and Bohl, 2005; Tudor, 2010; Harrison and Moore, 2009), Granger causality tests (Patev et al., 2006; Horobet and Lupu, 2009), cointegration analysis (Syllignakis and Kouretas, 2006; Patev et al., 2006) and GARCH modeling (Caporale and Spagnolo, 2010; Fedorova and Saleem, 2010).

In this paper, two modern techniques—the Dynamic Conditional Correlation GARCH (DCC-GARCH) model of Engle and Sheppard (2001) and MODWT Granger causality tests—are applied to answer the following questions. First, is the correlation (comovement) between the Czech stock market and the developed European stock markets of Austria, France, Germany, and the UK and between the stock markets of the Czech Republic and other Central and Eastern European (CEE) countries (Hungary, Poland, and Slovenia) time-varying? Second, what effect did the financial crises in the period 1997–2010 and the accession of the Czech Republic to the European Union have on the comovement between the Czech and European stock markets investigated? Third, were there return spillovers between the markets and did they depend on the horizon over which they are calculated (i.e., were they a multiscale phenomenon)?

Time-varying correlations between stock markets can be analyzed by multivariate GARCH models (Tse and Tsui, 2002; Bae et al., 2003; Égert and Kočenda, 2010; Xiao and Dhesi, 2010; Égert and Kočenda, 2010). DCC-GARCH models have gained popularity in stock market comovement research because they offer both the flexibility of univariate GARCH models and the simplicity of parametric correlation in the model and extend the CCC-GARCH (Constant Conditional Correlation GARCH) models (Silvennoinen and Teräsvirta, 2009).

Interdependencies between stock markets may not just be changing in time, but may also be scale-dependent (i.e., depend on the horizon over which they are investigated; see for example, Ranta, 2010, and Zhou, 2011). Candelon et al. (2008) argued that stock market comovement analyses should consider the distinction between short- and long-term investors. From a portfolio diversification point of view, short-term investors are more interested in stock market interdependencies over shorter time horizons (that is, at higher frequencies, or short-term movements), whereas long-term investors focus on lower-frequency interdependencies. As such, one has to resort to scale (frequency) domain analysis to obtain insights into the international interdependencies of stock markets at the scale level (Gençay et al., 2001a; Ranta, 2010). In such a context, given both the time horizon of economic decisions and the strength and direction of the economic relationships between the variables, which may differ according to the time scale of the analysis, wavelet analysis may be a useful analytical tool.

The Maximal Overlap Discrete Wavelet Transform (MODWT) possesses some features that render the tools of this wavelet very suitable for investigating interdependence between financial assets (for use of the MODWT variance, correlation, and wavelet cross-correlation tools, see, for example, Gençay et al. 2001a; In and Kim, 2006; Kim and In, 2007; Conlon et al., 2009; Ranta, 2010, or Zhou, 2011). Wavelet multiresolution analysis (MRA)—another MODWT tool—has been applied in Ramsey (1998 a,b), Gençay (2002), and Zhou (2011) to study Granger causal relationships between economic and financial variables on different time scales. They all confirm that Granger causal relationships are scale dependent. As Zhou (2011)

argued, this is an argument for investigating return spillovers not just in the time domain, but also in the time-frequency domain, on a scale-by-scale basis.

The present paper provides two contributions to the existing literature. To our knowledge, this is the first study to analyze the interdependence of the Czech and European stock markets in the time-frequency domain. Also, unlike existing studies, the present paper studies the effects of the global financial crisis of 2008–2009 on the comovement between the Czech and the European stock market returns. The paper is organized as follows. The next section describes the methodology. Section 3 describes the time series and empirical results, and Section 4 summarizes the main findings.

2. Methodology

2.1 The DCC-GARCH Model

The DCC-GARCH model of Engle and Sheppard (2001) assumes that returns from k assets are conditionally multivariate normal with zero expected value (r_t)¹ and covariance matrix $\mathbf{H}_t$. The returns of the asset (stock indices), given the information set available at time $t-1$, have the following distribution:²

$$r_t | \mathcal{I}_{t-1} \sim N(0, H_t) \quad (1)$$

and

$$\mathbf{H}_t \equiv \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \quad (2)$$

where $\mathbf{D}_t$ is the $k \times k$ diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{it}}$ on the i -th diagonal, and $\mathbf{R}_t$ is the time-varying correlation matrix.

The log-likelihood of this estimator is written as:

$$L = -\frac{1}{2} \sum_{t=1}^T \left(k \log(2\pi) + 2 \log(|H_t|) + \log(|R_t|) + \epsilon_t' R_t^{-1} \epsilon_t \right) \quad (3)$$

where $\epsilon_t \sim N(0, R_t)$ are the residuals standardized by their conditional standard deviation. The elements of the matrix $\mathbf{D}_t$ are given by the univariate GARCH(p,q) model (Engle and Sheppard, 2001):

$$h_{it} = \omega_i + \sum_{p=1}^{P_i} \alpha_{ip} r_{it-p}^2 + \sum_{q=1}^{Q_i} \beta_{iq} h_{it-q} \quad (4)$$

¹ The asset return series entering the DCC-GARCH model as an explanatory variable have to be filtered so that the expected (mean) value of the series is zero. Often, filtering is achieved by estimating a bivariate Vector Autoregressive (VAR) model for the return series to remove the potential linear structure between the variables of the model. The residuals of the VAR models are then used as inputs to the DCC-GARCH model (see Crespo-Cuaresma and Wójcik, 2006, or Ěgert and Kočenda, 2010).

² The description of the DCC-GARCH models is summarized from Engle and Sheppard (2001). We use the same notation as the authors.

where $i = 1, 2, \dots, k$ (variables), with the usual GARCH restrictions (for non-negativity and stationarity $\sum_{p=1}^{P_i} \alpha_{ip} + \sum_{q=1}^{Q_i} \beta_{iq} < 1$).

The dynamic correlation structure is defined by the following equations:

$$Q_t = \left(1 - \sum_{m=1}^M \alpha_m - \sum_{n=1}^N \beta_n \right) \bar{Q} + \sum_{m=1}^M \alpha_m (\epsilon_{t-m} \epsilon_{t-m}') + \sum_{n=1}^N \beta_n Q_{t-n} \quad (5)$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \quad (6)$$

where M is the length of the innovation term in the DCC estimator, and N is the length of the lagged correlation matrices in the DCC estimator ($\alpha_m \geq 0$, $\beta_n \geq 0$, $\sum_{m=1}^M \alpha_m + \sum_{n=1}^N \beta_n < 1$). $\bar{Q}$ is the unconditional covariance of the standardized residuals resulting from the first stage estimation and Q_t^* is a diagonal matrix composed of the square root of the diagonal elements of Q_t :

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11}} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{q_{22}} & 0 & \dots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \dots & \sqrt{q_{kk}} \end{bmatrix} \quad (7)$$

The elements of the matrix R_t are:

$$\rho_{ijt} = \frac{q_{ijt}}{\sqrt{q_{ii} q_{jj}}} \quad (8)$$

In order to estimate the DCC(m,n)-GARCH(p,q) model, we first pre-filter the return time series by estimating a VAR(p)³ model

$$\begin{aligned} r_{1,t} &= \mu_1 + \sum_{i=1}^p a_{1,i} r_{1,t-i} + \sum_{i=1}^p b_{1,i} r_{2,t-i} + \varepsilon_{1,t} \\ r_{2,t} &= \mu_2 + \sum_{i=1}^p a_{2,i} r_{2,t-i} + \sum_{i=1}^p b_{2,i} r_{1,t-i} + \varepsilon_{2,t} \end{aligned} \quad (9)$$

and then, with the residuals of the VAR model, we estimate a DCC(m,n)-GARCH(p,q) model. In the first stage the univariate GARCH(p,q) models are estimated for each residual series,⁴ and in the second stage, the residuals, transformed by their standard deviation estimated during the first stage, are used to estimate the parameters of the dynamic correlation.⁵

³ The optimal lag (p) in the VAR model is selected by the Schwarz Information Criterion (SIC).

⁴ We tested among univariate GARCH models (GARCH(1,1), GARCH(1,2), GARCH(2,1), and GARCH(2,2)) and, as proposed by Engle and Sheppard (2001), selected the one with the minimum Akaike information criteria.

In order to test whether a conditional correlation is time-varying, a test proposed by Engle and Sheppard (2001) is used:

$$H_0 : R_t = \bar{R} \quad (\forall t \in T)$$

against

$$H_1 : \text{vech}^u(R_t) = \text{vech}^u(R_t) + \beta_1 \text{vech}^u(R_{t-1}) + \dots + \beta_p \text{vech}^u(R_{t-p}) \quad (10)$$

where H_0 and H_1 are the null and the alternative hypothesis, and vech^u is a modified vech which only selects elements above the diagonal.

The testing procedure is as follows. First the univariate GARCH processes are estimated, and then the residuals are standardized. Then the correlation of the standardized residuals is estimated, and the vector of univariate standardized residuals is jointly standardized by the symmetric square root decomposition of the $\bar{R}$. Under the null of constant correlation, these residuals should be IID with a variance covariance matrix given by I_k . The artificial regressions are a regression of the outer products of the residuals on a constant and lagged outer products. The estimated vector autoregression is:

$$Y_t = \alpha + \beta_1 Y_{t-1} + \dots + \beta_s Y_{t-s} + \eta_t \quad (11)$$

where $Y_t = \text{vech}^u \left[\left(\bar{R}^{-\frac{1}{2}} D_t^{-1} \varepsilon_t \right) \left(\bar{R}^{-\frac{1}{2}} D_t^{-1} \varepsilon_t \right)' - I_k \right]$ and $\bar{R}^{-\frac{1}{2}} D_t^{-1} \varepsilon_t$ is a $k \times 1$ vector of residuals jointly standardized under the null hypothesis.

Under the null hypothesis the intercept and all of the lag parameters in the model should be zero. The test can then be conducted as $\frac{\hat{\delta} \mathbf{X}' \mathbf{X} \hat{\delta}}{\hat{\delta}^2}$, which is asymptotically $\chi^2_{(s+1)}$, where $\hat{\delta}$ are the estimated regression parameters and $\mathbf{X}$ is a matrix consisting of the regressors.

A significant rejection of the constant correlation hypothesis shows that DCC-GARCH is to be preferred to the constant conditional correlation GARCH model (CCC-GARCH) of Bollerslev (1990).

2.2 Maximal Overlap Discrete Wavelet Transform (MODWT)

Similar to Fourier analysis, wavelet analysis involves the projection of the original series onto a sequence of basis functions, which are known as wavelets. There are two basic wavelet functions: the father wavelet (called also a scaling function), ϕ , and the mother wavelet (called also a wavelet function), ψ , which can be scaled and translated to form a basis for the Hilbert space $L^2(\mathbb{R})$ of square integrable functions.

⁵ We estimated a variety of specifications for the DCC model, allowing for more lags in both the news term (α) and the decay term (β). More specifically we tested among the DCC(1,1), DCC(1,2), DCC(2,1), and DCC(2,2) models. As proposed by Engle and Sheppard (2001), we selected among them based on the LR test. In empirical applications, though, normally a bivariate DCC(1,1)-GARCH(1,1) model is estimated, with two financial assets, r_{1t} and r_{2t} (Engle, 2002; Égert and Kočenda, 2010).

The father and mother wavelets are defined by the functions:

$$\phi_{j,k}(t) = 2^{-\frac{j}{2}} \phi(2^{-j}t - k) \quad (12)$$

$$\psi_{j,k}(t) = 2^{-\frac{j}{2}} \psi(2^{-j}t - k) \quad (13)$$

where $j = 1, \dots, J$ is the scaling parameter in a J -level decomposition and k is a translation parameter ($j, k \in \mathbb{Z}$). The long-term trend of the time series is captured by the father wavelet, which integrates to 1, while the mother wavelet, which integrates to 0, describes fluctuations from the trend. The continuous wavelet transform of a square integrable time series $X(t)$ consists of scaling coefficients, $\alpha_{J,k}$, and wavelet coefficients, $\beta_{j,k}$ (Craigmile and Percival, 2002):

$$\alpha_{J,k}(t) = \int \phi_{J,k}(t) X(t) \quad (14)$$

and

$$\beta_{j,k}(t) = \int \psi_{j,k}(t) X(t) \quad (15)$$

It is possible to reconstruct $X(t)$ from these transform coefficients using:

$$X(t) = \sum_k \alpha_{J,k} \phi_{J,k}(t) + \sum_k \beta_{J,k} \psi_{J,k}(t) + \sum_k \beta_{J-1,k} \psi_{J-1,k}(t) + \dots + \sum_k \beta_{1,k} \psi_{1,k}(t) \quad (16)$$

In practice we observe a time series at a finite number of regularly spaced times, so we can make use of the discrete wavelet transform (DWT)⁶ or the maximal overlap discrete wavelet transform (MODWT). The MODWT is a linear filtering operation that transforms a series into coefficients related to variations over a set of scales. It is similar to the discrete wavelet transform, but it gives up the orthogonality property of the DWT to gain other features that render the MODWT more suitable for the aims of our study, namely (Percival and Mojfeld, 1997): i) the ability to handle any sample size regardless of whether or not the series is dyadic (that is, of size 2^{J_0} , where J_0 is a positive integer number); ii) increased resolution at higher scales, as the MODWT oversamples the data; iii) translation-invariance, which ensures that the MODWT wavelet coefficients do not change if the time series is shifted in a “circular” fashion; iv) a more asymptotically efficient wavelet variance estimator than the DWT.

The MODWT is defined in the following way. Let⁷ $\mathbf{x}$ be an N dimensional vector whose elements represent the real-valued time series $\{X_t : t = 0, \dots, N-1\}$. For any positive integer, J_0 , the level J_0 MODWT of $\mathbf{x}$ is a transform consisting of

⁶ For a presentation of the DWT, please refer to Percival and Walden (2000).

⁷ Concepts and notations as in Percival and Walden (2000) are used. In the paper only a brief description of the MODWT is given. For an elaborate description, refer to Percival and Walden (2000) or Gençay et al. (2002).

the $J_0 + 1$ vectors $\tilde{\mathbf{w}}_1, \dots, \tilde{\mathbf{w}}_{J_0}$ and $\tilde{\mathbf{v}}_{J_0}$, all of which have dimension N . The vector $\tilde{\mathbf{w}}_j$ contains the MODWT wavelet coefficients associated with changes on a scale of $\tau_j = 2^{j-1}$ (for $j = 1, \dots, J_0$), while $\tilde{\mathbf{v}}_{J_0}$ contains the MODWT scaling coefficients associated with averages on a scale of $\lambda_{J_0} = 2^{J_0}$.⁸ Based upon the definition of the MODWT coefficients we can write (Percival and Walden, 2000):

$$\tilde{\mathbf{w}}_j = \tilde{\mathbf{W}}_j \mathbf{x} \quad (17)$$

and

$$\tilde{\mathbf{v}}_{J_0} = \tilde{\mathbf{V}}_{J_0} \mathbf{x} \quad (18)$$

where $\tilde{\mathbf{W}}_j$ and $\tilde{\mathbf{V}}_{J_0}$ are $N \times N$ matrices.

By definition, the elements of $\tilde{\mathbf{w}}_j$ and $\tilde{\mathbf{v}}_{J_0}$ are outputs obtained by filtering $\mathbf{X}$, namely:

$$\tilde{W}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l \bmod N} \quad (19)$$

and

$$\tilde{V}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l \bmod N} \quad (20)$$

for $t = 0, \dots, N-1$, and where $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ are the j th-level MODWT wavelet and scaling filters defined in terms of the j th-level equivalent wavelet and scaling filters.

The MODWT treats the series as if it were periodic, whereby the unobserved samples of the real-valued time series $X_{-1}, X_{-2}, \dots, X_{-N}$ are assigned the observed values at $X_{N-1}, X_{N-2}, \dots, X_0$.

The MODWT coefficients are thus given by:

$$\tilde{W}_{j,t} = \sum_{l=0}^{N-1} \tilde{h}_{j,l}^\circ X_{t-l \bmod N} \quad (21)$$

and

$$\tilde{V}_{j,t} = \sum_{l=0}^{N-1} \tilde{g}_{j,l}^\circ X_{t-l \bmod N} \quad (22)$$

for $t = 0, \dots, N-1$; $\tilde{h}_{j,l}^\circ$ and $\tilde{g}_{j,l}^\circ$ are periodization of $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ to circular filters of length N .

This periodic extension of the time series is known as analyzing $\{X_t\}$ using “circular boundary conditions” (Percival and Walden, 2000; Cornish et al., 2006).

⁸ Percival and Walden (2000) denote scales for wavelet coefficient as τ and scales for scaling coefficients as λ . We use these notations as well.

There are $L_j - 1$ wavelet and scaling coefficients that are influenced by the extension (“the boundary coefficients”). Since L_j increases with j , the number of boundary coefficients increases with scale. Exclusion of boundary coefficients in the wavelet variance, wavelet correlation, and covariance provides unbiased estimates (Cornish et al., 2006).

One of the useful characteristics of the MODWT is the additive decomposition of the signal (time series), as $\mathbf{X}$ can be recovered from its MODWT via (Percival and Walden, 2000):

$$\mathbf{X} = \sum_{j=1}^{J_0} \tilde{\mathbf{W}}_j^T \tilde{\mathbf{w}}_j + \tilde{\mathbf{V}}_{J_0}^T \tilde{\mathbf{v}}_{J_0} = \sum_{j=1}^{J_0} \tilde{\mathbf{d}}_j + \tilde{\mathbf{s}}_{J_0} \quad (23)$$

which defines an MODWT-based multiresolution analysis (MRA) of $\mathbf{X}$ in terms of j th level MODWT details $\tilde{\mathbf{d}}_j = \tilde{\mathbf{W}}_j^T \tilde{\mathbf{w}}_j$ (which capture local fluctuations over the whole period of a time series on each scale) and the J_0 level of the MODWT smooths $\tilde{\mathbf{s}}_{J_0} = \tilde{\mathbf{V}}_{J_0}^T \tilde{\mathbf{v}}_{J_0}$ (which provides the “smooth” or overall “trend” of the original signal). Adding $\tilde{\mathbf{d}}_j$ to $\tilde{\mathbf{s}}_{J_0}$, for $j=1, 2, \dots, J_0$, gives an increasingly accurate approximation of the original signal. Using the “circular boundary condition” the elements of $\tilde{\mathbf{d}}_j$ that are affected by circularity (and have to be excluded to obtain unbiased estimates, based on the MODWT detail and smooth time series) are those with time indices satisfying either $t = 0, \dots, L_j - 2$ or $t = N - L_j + 1, \dots, N - 1$.

3. Empirical Results

3.1 Data

The stock index returns data are calculated as the differences of the logarithmic daily closing values of the indices $(\ln(P_t) - \ln(P_{t-1}))$, where P_t is the day t closing index value). The following indices were considered: PX (for the Czech Republic), ATX (for Austria), CAC40 (for France), DAX (for Germany), FTSE100 (for the UK), BUX (for Hungary), LJSEX (for Slovenia), and WIG20 (for Poland). The first day of observation is April 1, 1997, and the last day is May 12, 2010. Days of no trading on any of the observed stock markets were removed. The total number of observations amounts to 3,060 days. The data sources for the closing unadjusted (for dividends) values of the PX, BUX, LJSEX, and WIG20 indices are their respective stock exchanges, and the data source for the closing unadjusted (for dividends) values⁹ of the ATX, CAC40, DAX, and FTSE100 indices is Yahoo! Finance.¹⁰

⁹ This approach has been used in several studies (e.g. Baumöhl and Výrost, 2010; Ülkü, 2011; Forbes and Rigobon, 2002).

¹⁰ There are no major differences in the trading hours of the stock markets investigated. The trading hours are the following: 9.15 a.m.–4.20 p.m. on the Prague Stock Exchange, 8.55 a.m.–5.35 p.m. on the Vienna Stock Exchange, 9.00 a.m.–5.30 p.m. on the NYSE Euronext Stock Exchange in Paris (CAC40 index), 9.00 a.m.–5.30 p.m. on the Frankfurt Stock Exchange, 9.00 a.m.–5.30 p.m. on the London Stock Exchange, 9.00 a.m.–5.00 p.m. on the Budapest Stock Exchange, 9.00 a.m.–12.50 p.m. on the Ljubljana Stock Exchange, and 9.00 a.m.–4.20 p.m. on the Warsaw Stock Exchange.

Table 1 Descriptive Statistics of Return Series of Stock Market Indices

	Min	Max	Mean	Std. deviation	Skewness	Kurtosis
PX	-0.1990	0.2114	0.0002595	0.01667	-0.29	24.62
ATX	-0.1637	0.1304	0.0002515	0.01558	-0.40	14.91
CAC40	-0.0947	0.1059	0.0001206	0.01628	0.09	7.83
DAX	-0.0850	0.1080	0.0002071	0.01756	-0.06	6.58
FTSE100	-0.0927	0.1079	0.0000774	0.01361	0.09	9.30
BUX	-0.1803	0.2202	0.0004859	0.02021	-0.30	15.90
LJSEX	-0.1285	0.0768	0.0003521	0.01062	-0.87	20.19
WIG20	-0.1066	0.1344	0.0001517	0.01967	-0.03	6.53

Table 2 Jarque-Bera, Ljung-Box, and ARCH Effect Tests

	Jarque-Bera statistics	Ljung-Box Q^2 statistics ($Q^2(10)$)	ARCH effect (5)
PX	59,654.93***	1,695.84***	677.47***
ATX	18,153.48***	2,713.71***	747.94***
CAC40	2,982.52***	1,499.74***	448.72***
DAX	1,635.47***	1,509.54***	447.35***
FTSE100	5,069.61***	1,922.94***	561.27***
BUX	21,260.91***	927.74***	329.46***
LJSEX	38,073.93***	1,377.33***	544.36***
WIG20	1,584.98***	840.27***	306.21***

Notes: Jarque-Bera statistics: *** (**) indicates that the null hypothesis (of normal distribution) is rejected at the 1% (5%) significance level. Ljung-Box Q^2 statistics ($Q^2(10)$) report the values of the statistics at the 10th lag: *** indicates that the null hypothesis of no serial correlation can be rejected at the 1% significance level for up to the 10th lag. Engle's ARCH test reports the values of the LM test statistics at the 5 lags included: *** indicates that the null hypothesis of no ARCH effects can be rejected at the 1% significance level for up to the 5th lag.

Table 1 presents some descriptive statistics of the data. We observe a higher spread between the maximum and minimum daily returns in the PX and BUX indices than with the other ones. The standard deviation of the daily returns is the smallest with the LJSEX index.

Table 2 shows that the Jarque-Bera test rejects the hypothesis of normally distributed observed time series; all indices are asymmetrically (left) distributed around the sample mean, and kurtosis is greater than for normally distributed time series. The Ljung-Box Q-statistics reject the null hypothesis of no serial correlation in the squared residuals (of the VAR models) of the stock index returns for all the stock indices. Because we use the GARCH process to model the variance in the asset returns, we also tested for the presence of the ARCH effect. The null hypothesis of no ARCH effects is rejected at the 1% significance level.

The stationarity of the stock index return time series was tested using a combination of the Augmented Dickey-Fuller (ADF), Phillips-Peron (PP), and Kwiatkowski-Philips-Schmidt-Shin (KPSS) tests.¹¹ The results¹² show that all of the time series are stationary.

¹¹ It must be noted that Dickey-Fuller tests tend to over-reject the null hypothesis in the presence of GARCH errors (Kim and Schmidt, 1993), especially when the condition $\alpha + \beta < 1$ is not fulfilled. For the KPSS test, Carrasco and Chen (2002) proved that the limit distribution of the KPSS statistic under stable GARCH(1,1) errors remains valid.

¹² The results are not presented here, but can be obtained from the author.

Table 3 Results of Bivariate VAR Models for Stock Index Returns

	PX-ATX	PX-CAC40	PX-DAX	PX- FTSE100	PX-BUX	PX-LJSEX	PX-WIG20
A constant	0.00026 (0.85)	0.00026 (0.87)	0.00026 (0.85)	0.00027 (0.89)	0.00024 (0.81)	0.00029 (0.95)	0.00026 (0.87)
PX (lag1)	-0.02905 (-1.29)	-0.04373** (-2.08)	-0.03832* (-1.88)	-0.05441** (-2.57)	0.05837*** (3.27)	0.02106 (1.11)	-0.03249 (-1.49)
Other index in pair (lag1)	0.06008** (2.49)	0.09570*** (4.44)	0.08667*** (4.47)	0.13688*** (5.28)	-0.03457 (-1.60)	-0.08498*** (-2.85)	0.05630*** (3.06)
	ATX-PX	CAC40-PX	DAX-PX	FTSE100- PX	BUX-PX	LJSEX-PX	WIG20-PX
A constant	0.00025 (0.88)	0.00014 (0.49)	0.00022 (0.69)	0.00010 (0.39)	0.00048 (1.30)	0.00026 (1.41)	0.00016 (-1.46)
PX (lag1)	-0.02167 (-1.03)	-0.06460*** (-3.14)	-0.03586* (-1.66)	-0.07281*** (-4.20)	0.01559 (0.72)	0.04968*** (4.22)	-0.03753 (0.29)
Other index in pair (lag1)	0.04421 (1.96)**	0.00003 (0.00)	-0.02694 (-1.32)	0.00476 (0.22)	0.00497 (0.00)	0.20015*** (10.84)	0.00630 (0.44)

Notes: In parenthesis under the parameter estimation, *t*-statistics are given. *** (**/*) denote rejection of the null hypothesis that parameter is equal zero at 1% (5%/10%) significance level. The first index (for example PX in PX-ATX pair) in the indices pairs represents dependent variable in a bivariate VAR model regression equation.

3.2 DCC-GARCH Time-Varying Conditional Correlation Analysis Results

Initially, a bivariate Vector Autoregressive (VAR) model with one lag¹³ of the returns was used to remove the potential linear structure between the pairs of stock index returns. Then the residuals of the VAR model were used as inputs for the DCC-GARCH model.

The results of the VAR model (*Table 3*) show that the lagged returns of ATX, CAC40, DAX, FTSE100, LJSEX, and WIG20 are statistically significant, meaning that they influence the PX returns. The lagged PX returns also significantly (at least at the 5% significance level) explain the CAC40, FTSE100, and LJSEX returns. It follows that return innovations in ATX and DAX spill-over to the PX returns, whereas return spillovers are bi-directional between PX and CAC40, between PX and FTSE100, and between PX and LJSEX.

The null hypothesis of constant correlation was rejected for all stock index pairs (*Table 4*). We thus proceeded with selecting the appropriate DCC(*m,n*)-GARCH(*p,q*) specification. Based on the Akaike information criteria for the different GARCH(*p,q*) ($p \leq 2, q \leq 2$) models, the GARCH(1,1) specification was chosen. The lag structure of the DCC (*m,n*) parameters was selected by the LR (likelihood ratio) test.¹⁴ DCC(1,1) (the restricted model in the LR test) was not rejected in favor of the DCC models with a higher lag structure for stock index pairs PX-ATX, PX-CAC40, PX-DAX, PX-FTSE100, and PX-WIG20. The correlation between PX and these stock indices was thus estimated by the DCC(1,1)-GARCH(1,1) model. For the pairs PX-BUX and

¹³ We used the SIC (Schwarz Information Criterion) criteria to select the optimal lag length of the VAR model because Ashgar and Abdi (2007) showed that it performs better than other information criteria in this task. The results are not presented here, but can be obtained from the author.

¹⁴ The results of the Akaike information criteria and the LR test are not presented here, but can be obtained from the author.

Table 4 Constant Correlation Test for Stock Indices Paired with PX

Parameter	PX-ATX	PX-CAC40	PX-DAX	PX-FTSE100	PX-BUX	PX-LJSEX	PX-WIG
χ^2	121.4063	38.1977	47.2672	55.9997	59.2594	33.7127	75.4326
p-value	0.0000***	0.0000***	0.0000***	0.0000**	0.0000****	0.0004***	0.0078***

Notes: A constant correlation model test of Engle and Sheppard (2001) with 10 lags is estimated. The test statistic is χ^2 with 10 +1 degrees of freedom. *** denote rejection of the null hypothesis of constant correlation at 1% significance (** at 5% significance, and * at 10% significance) level.

Table 5 Results of a DCC(1,1)-GARCH (1,1) Model for Indices in Pair with PX

Parameter	PX-ATX	PX-CAC40	PX-DAX	PX-FTSE100	PX-WIG
ω PX—other index	7.36469e-06 *** (4.2997)	7.58322e-06 *** (4.32)	7.83881e-06*** (4.33)	7.79969e-06*** (4.38)	7.49563e-06*** (4.33)
α PX—other index	0.13570*** (8.51)	0.13418*** (8.36)	0.13212*** (8.37)	0.13418*** (8.59)	0.13407*** (8.47)
β PX—other index	0.84025*** (58.94)	0.84046*** (57.42)	0.84094*** (57.20)	0.83931*** (58.02)	0.84075*** (58.03)
Ljung-Box $Q^2(10)$ statistics	9.20	10.63	13.09*	11.85*	8.67
ω other index—PX	3.49478e-06 *** (3.76)	2.30619e-06*** (2.73)	3.26284e-06 *** (3.02)	1.28874e-06*** (3.15)	5.02609e-06*** (2.42)
α other index—PX	0.11833*** (5.82)	0.09035*** (6.84)	0.11332*** (6.51)	0.09321*** (6.28)	0.06170*** (5.56)
β ther index—PX	0.86824*** (43.52)	0.90493*** (67.35)	0.88120*** (53.11)	0.90341*** (82.0)	0.92639*** (65.04)
Ljung-Box $Q^2(10)$ statistics	11.66	7.75	10.40	11.10	6.44454
α	0.01546 *** (2.37)	0.02345*** (4.24)	0.02440*** (4.16)	0.02381*** (4.21)	0.02458*** (2.73)
β	0.98226 *** (127.05)	0.96756*** (123.37)	0.96396*** (99.77)	0.96581*** (110.98)	0.96453*** (60.77)

Notes: ω PX—other index, α PX—other index, β PX—other index are estimated parameters of a univariate GARCH (1,1) model, with input that are residuals of the estimated bivariate Vector Autoregressive (VAR) model with PX returns as a dependent variable and the other index return series in the bivariate VAR as an explanatory variable. ω other index—PX, α other index—PX, β ther index—PX are the estimated parameters of a univariate GARCH (1,1) model, with residuals input from the estimated bivariate Vector Autoregressive (VAR) model with PX returns as an explanatory variable and the other index return series in the bivariate VAR as a dependent variable. In parentheses under the parameter estimation, t -statistics are given: *** (**/*) denote rejection of the null hypothesis that parameter is zero at a 1% (5%/10%) significance level. Ljung-Box $Q^2(10)$ statistics reports the value of the statistics at lag 10: ***(**/*) indicate that the null hypothesis of no serial correlation in squared residuals of estimated GARCH model can be rejected at 1% (5%/10%) significance level for lags up to 10.

PX-LJSEX the DCC(2,2)-GARCH(1,1) model was chosen, as the LR showed that the restricted models (i.e., those with parameters $m < 2$ and $n < 2$) can be rejected in favor of the unrestricted DCC(2,2)-GARCH(1,1) model.

The results of the DCC(1,1)-GARCH(1,1) model are presented in Table 5 and the results of the DCC(2,2)-GARCH(1,1) model in Table 6. The DCC parameters α and β are significant for all stock index pairs and $\beta > \alpha$ for all pairs, so the current variances of the returns are more affected by the magnitude of past variances than by past return innovations. A high persistence is observed in the series of correlations R_t (as β is close to 1). The sum of the DCC parameters ($\alpha + \beta$) is very close to 1 in all cases, indicating that the conditional variances are highly persistent and only slowly mean-reverting. The results of the Ljung-Box statistics do not reject the null hypo-

Table 6 Results of a DCC(2,2)-GARCH (1,1) Model for Indices in Pair with PX

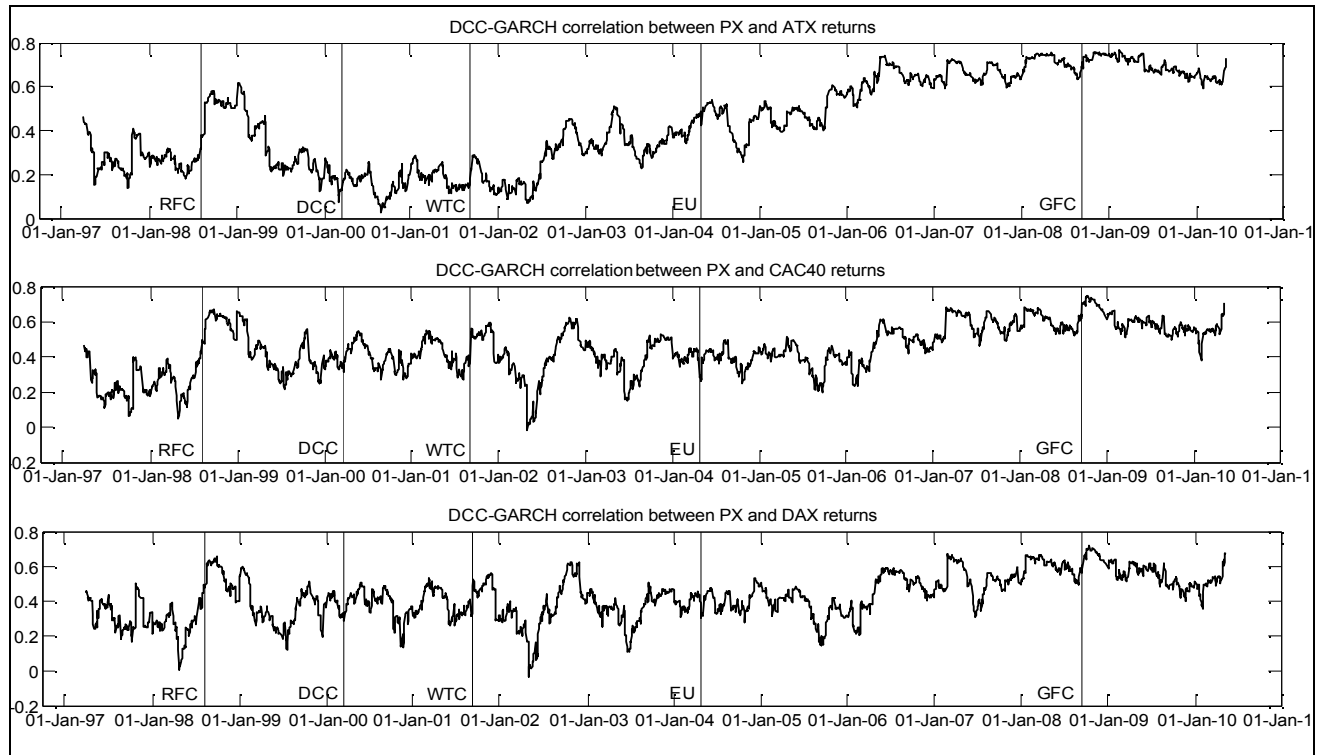
Parameter	PX-BUX	PX-LJSEX
ω PX—other index	7.36470e-06*** (4.30)	7.54622e-06*** (4.39)
α PX—other index	0.13570*** (8.62)	0.13885*** (8.64)
β PX—other index	0.84025*** (59.36)	0.83667*** (57.58)
Ljung-Box $Q^2(10)$ statistics	9.37	11.77*
ω other index—PX	1.56495e-05 (2.12)	4.36966e-06*** (3.44)
α other index—PX	0.15924*** (2.74)	0.35710*** (6.08)
β other index—PX	0.80785*** (12.58)	0.64290*** (12.24)
Ljung-Box $Q^2(10)$ statistics	4.90	12.68
α_1	0.01379 (0.77)	0.04902*** (2.41)
α_2	0.06246*** (3.59)	2.000000e-06 (0.00)
β_1	0.06351 (0.81)	0.15822 (0.61)
β_2	0.79890*** (9.83)	0.67585*** (3.73)

Notes: See notes for Table 5.

thesis of no serial correlation in the squared residuals of the estimated models, suggesting that the models are appropriately specified.

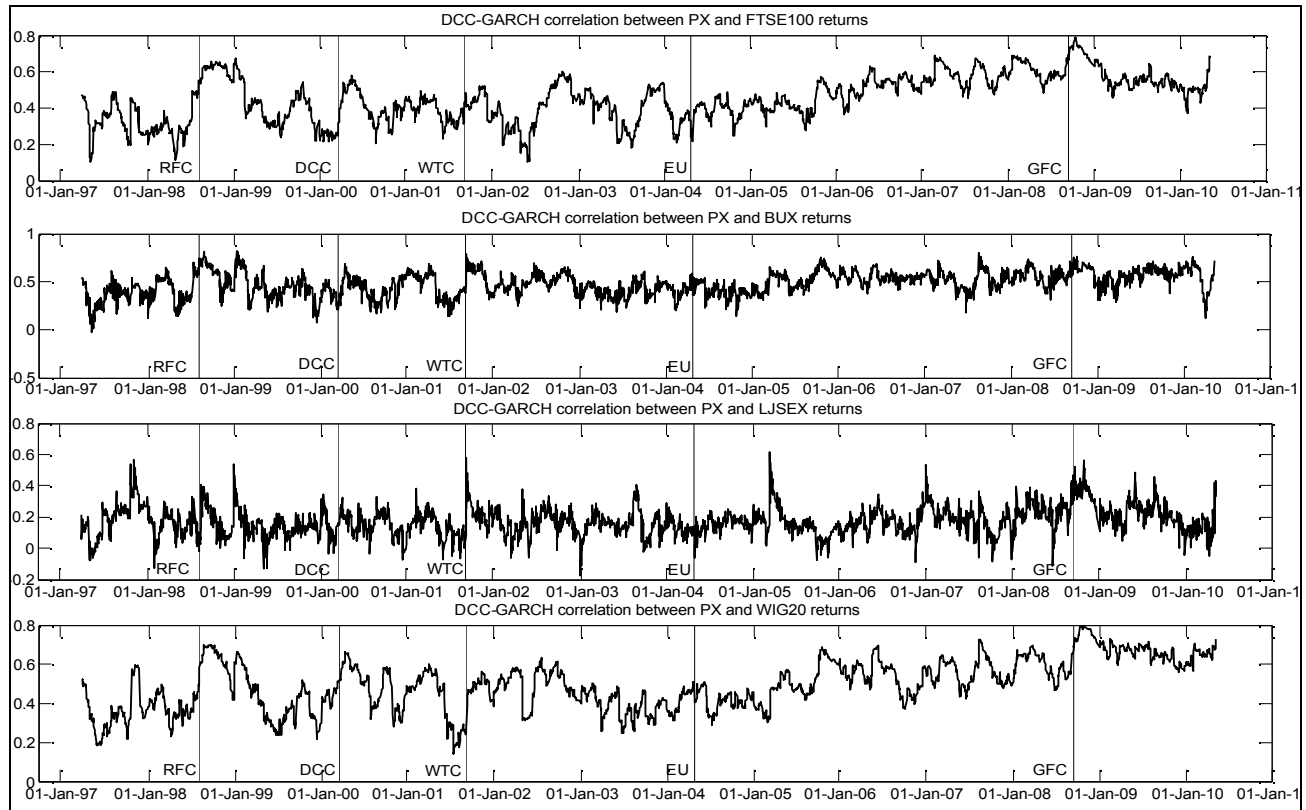
As can be seen in *Figures 1a* and *1b* (and also Table 7 in continuation), the PX returns correlate most with the ATX, WIG20, and CAC40 returns and are least correlated with the LJSEX returns. For investors already present in the Czech stock market, the greatest diversification benefits are thus achievable by investing in LJSEX-indexed instruments. It is of great interest to international investors to determine if comovement increases in the short or long term. The long-term comovement dynamics can be observed from the trend analysis. In *Figures 1a* and *1b*, we observe a rising trend of pair-wise correlations between the returns of the Czech and other European stock markets. The rising correlation trend is more evident for some stock index pairs (namely, between PX and ATX and CAC40 and FTSE100) and less evident—but still rising—for others (namely, between PX and LJSEX), suggesting that comovement between stock markets increased in the observed period. This confirms the empirical evidence in the existing literature (for example, Serwa and Bohl, 2005; Tudor, 2010; Harrison and Moore, 2009) of rising comovement between the Czech and other European stock markets in the last decade.¹⁵

¹⁵ In the empirical literature on stock market comovements, more factors have been determined to influence the level of long-term comovement of stock markets. Forbes and Chinn (2004) found that direct trade flows have a positive effect on cross-country stock market correlations, while competition in third markets tends to have a negative effect. Quinn and Voth (2008) provide evidence that more open countries face higher stock market correlations with those abroad relative to closed economies. On investigating factors of international stock market comovement during the recent global financial crisis, Didier et al. (2011) found evidence that stock market liquidity can also significantly explain stock market comovements.

Figure 1a Pair-Wise Conditional Correlation between PX and Other Stock Indices Returns

Notes: On the time axis the financial crises are denoted: RFC = Russian financial crisis (outbreak on August 13, 1998), DCC = Dot-Com crisis (the date, March 24, 2000, is taken, when the peak of S&P500 was reached, before the dot-com crisis began), WTC = attack on WTC in New York (September 11, 2001), EU = the date when Czech Republic joined the European Union (May 1, 2004), GFC = Global financial crisis (September 16, 2008). The vertical dotted lines indicate these events.

Figure 1b *Pair-Wise Conditional Correlation between PX and Other Stock Indices Returns*



The rising trend of the DCC-GARCH conditional correlation is especially noticeable after 2002. Real and financial integration in the European Union had already taken place before de facto accession to the European Union. There is plenty of evidence in the empirical literature that European integration has increased interdependence among financial markets.¹⁶

Clearly, stock return comovement is time-varying. This finding is in accordance with the recent literature on the measurement of different stock market comovements (Forbes and Rigobon, 2002; Syriopoulos, 2007; Gilmore et al., 2008; Kizys and Pierdzioch, 2009). As Malkiel (2003) and Zhou (2011) argued, shorter-term links between markets are to a great extent influenced by sporadic events, market sentiment, and psychological factors, which can cause short-term changes in market behavior. Financial crises are events that can disrupt the links in international financial markets in the short or long term.

The existing literature (Longin and Solnik, 1995; Ang and Bekaert, 2002; Baele, 2005) presents evidence that correlations among international markets tend to increase when stock returns fall precipitously. The main financial market disruptions during the period April 1997–May 2010 (the Russian financial crisis, the attack on the World Trade Center in New York, the dot-com crisis, and the global financial market crisis) are indicated in *Figures 1a* and *1b*. These events, as evident from *Figures 1a* and *1b*, led to an increase in correlation that lasted from 100 to 400 days. The global financial crisis, with the collapse of Lehman Brothers on 16 September 2008 taken as the major event that spread the financial crisis from the U.S. to world financial markets, had a similar impact on the Czech stock market's comovement with European stock markets as the earlier financial crises noted.

To examine statistically the effect of European Union integration and the global financial crisis of 2008–2009 on the comovement between the Czech and European stock markets investigated, we split the total period covered in the present study into three sub-periods and examined the regression model.¹⁷

$$\rho_{ij,t} = \mu_{ij} + \sum_{p=1}^P \phi_p \rho_{ij,t-p} + \sum_{k=1}^2 a_k DV_{k,t} + e_{ij,t} \quad (24)$$

where μ_{ij} is a regression constant, $\rho_{ij,t}$ is the pair-wise correlation between the PX ($i = \text{PX}$) and the other stock index returns ($j = \text{ATX, CAC40, DAX, FTSE100, BUX, LJSEX, WIG20}$), obtained from the DCC-GARCH model estimated above, $D_{k,t}$ is the dummy variable for the second and the third sub-periods (treated as an exogenous variable in the VAR model), and $e_{ij,t}$ is the error term. The optimal lag length (p) is determined by the SIC criteria. The three sub-periods are:

1. April 1, 1997–April 30, 2004 (i.e., from the start of the sample observation until EU enlargement).

¹⁶ Empirical studies of the effects of European integration on the interdependence of the developed European stock markets which confirm this assumption include, for example, Longin and Solnik (1995), and Bessler and Yang (2003). For the CEE stock markets, this assumption was confirmed by the studies of Syllignakis and Kouretas (2006), Harrison and Moore (2009), and Caporale and Spagnolo (2010).

¹⁷ The regression model is very similar to the one suggested by Chiang et al. (2007).

Table 7 Results of the Tests in Changes of Correlation During the Three Sub-Periods

Pair-wise correlations	Average (mean) correlation in sub-period 1	Average (mean) correlation in sub-period 2	Average (mean) correlation in sub-period 3	μ_{ij}	$\rho_{ij, t-1}$	Dummy variable for the second sub-period	Dummy variable for the third sub-period
PX and ATX	0.2975	0.5851	0.6897	0.00212*** (3.00)	0.99246*** (446.49)	0.00248*** (2.83)	0.00325*** (2.73)
PX and CAC40	0.3846	0.4810	0.5898	0.00601*** (4.52)	0.98410*** (308.70)	0.00188** (2.11)	0.00379*** (2.86)
PX and DAX	0.3709	0.4656	0.5567	0.00759*** (5.20)	0.97933*** (268.55)	0.00222** (2.33)	0.00429*** (3.06)
PX and FTSE100	0.3891	0.5032	0.5705	0.00756*** (5.17)	0.98023*** (277.86)	0.00270*** (2.97)	0.00402*** (3.10)
PX and BUX	0.4424	0.5109	0.5706	0.09963*** (18.54)	0.77460*** (67.68)	0.01542*** (5.08)	0.02934*** (6.80)
PX and LJSEX	0.1588	0.1751	0.2242	0.05228*** (19.51)	0.67047*** (49.89)	0.00499* (1.89)	0.02239*** (5.97)
PX and WIG20	0.4386	0.5330	0.6689	0.00903*** (5.42)	0.97936*** (270.15)	0.00180** (2.06)	0.00526*** (3.74)

Notes: In the Table 6 the regression estimates of the equation (24) are given. Average correlation is calculated as the arithmetic average of correlation in the sub-period. The optimal lag for all pair-wise correlation time series as determined by SIC criteria is 1. ***, **, and * indicate the significance level of the t-statistics. The results are robust across the sub-samples.

2. May 1, 2004–September 16, 2008 (i.e., after Czech Republic entered the European Union until the start of the global financial crisis).
3. September 17, 2008–May 12, 2010 (i.e., from the start of the global financial crisis until the end of the entire observation period).

Significance of the estimated coefficients of the dummy variables would indicate structural changes in the pair-wise correlation coefficients in the second and the third sub-periods. The results are presented in Table 7.

The dummy variable for the third sub-period is positive and highly significant for all pair-wise correlation time series, whereas for the second sub-period the dummy variable is not significantly different from zero (at least at the 5% significance level) for the pair of PX and LJSEX returns. Based on this evidence, we can argue that EU integration led to more correlated movements in all stock markets in Europe except Slovenia. Since the start of the global financial crisis, the comovement has increased further.

3.3 Multiscale Granger Causality Test of Spillovers Between Stock Market Returns

To study the multiscale return spillovers between the Czech and the European stock markets, we resort to the Granger causality test.¹⁸ More specifically, we perform the modified Granger causality test of Granger (1969) and Granger and Morgenstern (1970), running the following bi-variate Vector Autoregression (VAR) models:

¹⁸ The strength of comovement (i.e., correlation analysis) is another interesting issue that could be investigated at the multiscale level. This, however, would exceed the limits of this paper and is left for further research.

$$1. \quad D_{j,t}^{PX} = \mu_1 + \sum_{i=1}^p \alpha_{1,i} D_{j,t-i}^{oi} + \sum_{i=1}^p \beta_{1,i} D_{j,t-i}^{PX} + \varepsilon_{j,t} \quad (25)$$

where $D_{j,t}^{PX}$ is the MODWT wavelet detail series of the PX returns on scale τ_j ($j = 1, \dots, 7$),¹⁹ $D_{j,t-i}^{oi}$ is the wavelet detail series of the other stock index returns on scale τ_j , t is a time index, and p is the number of time lags. The null hypothesis is

$$H_0 : \sum_{i=1}^p \alpha_{1,i} = 0 ; \text{ and} \quad (26)$$

$$2. \quad D_{j,t}^{oi} = \mu_2 + \sum_{i=1}^p \alpha_{2,i} D_{j,t-i}^{oi} + \sum_{i=1}^p \beta_{2,i} D_{j,t-i}^{PX} + \varepsilon_{j,t}$$

with the null hypothesis $H_0 : \sum_{i=1}^p \beta_{2,i} = 0$.

The hypotheses are tested using the F-test and the interpretation of the results is as follows: if the null $H_0 : \sum_{i=1}^p \alpha_{1,i} = 0$ (or $H_0 : \sum_{i=1}^p \beta_{2,i} = 0$, respectively) is rejected

and, at the same time, hypothesis $H_0 : \sum_{i=1}^p \beta_{2,i} = 0$ (or $H_0 : \sum_{i=1}^p \alpha_{1,i} = 0$, respectively) is

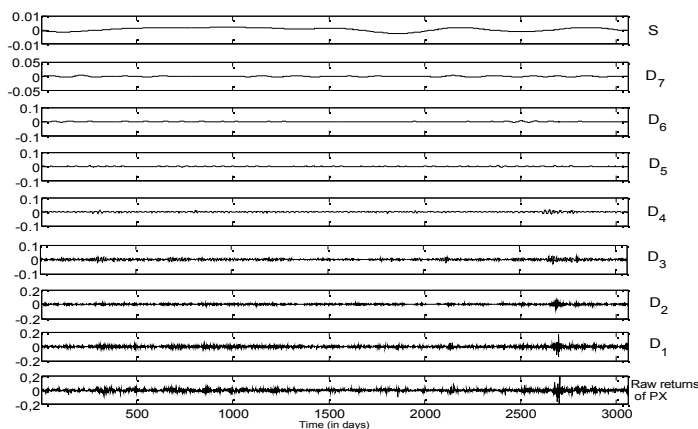
not rejected, we conclude that the foreign (from the Czech perspective) stock index returns (or the PX returns, respectively) are Granger-causing the PX returns (or foreign index returns, respectively) on scale τ_j . If both of the null hypotheses are rejected, then a feedback mechanism exists between the pair of index returns on scale τ_j . If none of the hypotheses are rejected, no Granger causality exists between the variables on the particular scale. A one-way causal relationship is confirmed only if for one VAR equation the null hypothesis is rejected and for the other VAR equation the null hypothesis is not rejected.²⁰

The MODWT MRA analysis of the index return series is performed by using a Daubechies least asymmetric filter with a wavelet filter length of 8 (LA8).²¹ This is a common wavelet filter used in other empirical studies on financial market interdependencies (Gençay et al., 2001b; Ranta, 2010). The maximum level of the MODWT is 7 ($J_0 = 7$) in order to also investigate the longer-term relationship between the stock markets.²² A “circular boundary condition” is applied; therefore, only

¹⁹ Scale τ_1 (or scale 1, as $\tau_1 = 2^{1-1} = 1$) measures the dynamics of the returns over 2–4 days; scale τ_2 (scale 2, as $\tau_2 = 2^{2-1} = 2$) over 4–8 days; scale τ_3 (scale 4, as $\tau_3 = 2^{3-1} = 4$) over 8–16 days; scale τ_4 (scale 8, $\tau_4 = 2^{4-1} = 8$) over 16–32 days; scale τ_5 (or scale 16) over 32–64 days; and scale τ_6 (or scale 32) over 64–128 days.

²⁰ When the GARCH effect is present in the time series, the Granger causality test leads to over-rejection of the true null hypothesis. Instead, the wavelet (MODWT) pre-filtering is robust to GARCH errors (Månsson, 2012).

Figure 2 MRA of PX Daily Returns



Notes: The raw return series presents the returns of the original data (PX daily returns). D_j ($j = 1, \dots, 7$) are the return series of the scale j details obtained by the MRA analysis. S is the smooth series.

details not affected by the boundary condition are captured in the Granger analysis. The MRA analysis for the PX return series is presented in *Figure 2*.

To obtain unbiased estimates in the Granger causality test, only non-boundary wavelet details may be considered. *Tables 8a* and *8b* report the results of the multi-scale Granger causality tests.

²¹ A reasonable choice of filter must consider the specific analysis goal we want to achieve (such as isolation of transient events in a time series, analysis of variance, multiresolution analysis, etc.) and the properties we need in the filter to achieve that goal (Percival and Walden, 2000). Choosing the wavelet filter of the shortest width ($L = 2, 4, 6$) can sometimes introduce undesirable artifacts into the resulting analyses. Alternatively, while wavelet filters with a large L can be a better match to the characteristic features in a time series, their use can result in more coefficients (or details) being influenced by boundary conditions and an increase in the computational burden. Percival and Walden (2000) suggest a strategy of using the smallest L that gives reasonable results. The Daubechies class of wavelets possesses appealing regularity characteristics and produces transforms that are effectively localized differences of adjacent weighted averages. The least asymmetric (LA) subclass, known as symmlets, has approximate linear phase and exhibits near symmetry about the filter midpoint. This linear phase property means that events and sinusoidal components in the wavelet and scaling coefficients at all levels can be aligned with the original time series. For the MODWT, this alignment is achieved by circularly shifting the coefficients (or details) by an amount dictated by the phase delay properties of the basic filter (Cornish et al., 2006). LA filters are available in even widths L . A wider filter is smoother in appearance and reduces the possible appearance of artifacts in a multiresolution analysis due to the filter shape. It also results in stronger uncorrelatedness between wavelet coefficients across scales for certain time series, which is useful for deriving confidence bounds from certain wavelet-based estimates. Taking all these considerations into account, the LA(8) filter is an appropriate choice (Percival and Walden, 2000), as it yields coefficients that are approximately uncorrelated between scales while having a filter width short enough such that the impact of boundary conditions is tolerable.

²² $J_0 = 7$ is the maximum MODWT level we can use, as the time series (of 3,060 days) is too short to obtain MODWT details unaffected by the boundary condition for $J_0 = 8$ (the elements of $\tilde{D}_8$ affected by the boundary condition would be the first 1,784 elements, i.e., the elements with indices $t = 0, \dots, L_j - 2 = 0, \dots, 1785$, and the elements with time indices $t = 3060 - 1785 + 1, \dots, N - 1 = 1276, \dots, 3059$).

Table 8a Multiscale Granger Causality Test Results (scales τ_1 to τ_4)

A pair of indices	Direction of causality	Raw return series	Scale τ_1	Scale τ_2	Scale τ_3	Scale τ_4
PX-ATX	PX→ATX	1.06	4.39**	9.27***	4.62**	0.28
	ATX→PX	6.22**	7.43***	3.40*	3.61*	0.66
PX-CAC40	PX→CAC40	9.86***	15.42***	67.50***	20.66***	21.66***
	CAC40→PX	19.72***	32.34***	54.15***	26.96***	22.44***
PX-DAX	PX→DAX	2.77*	14.57***	35.81***	14.85***	1.97
	DAX→PX	20.02**	15.82***	40.11***	19.77***	2.31
PX-FTSE100	PX→FTSE100	17.68***	18.96***	119.95***	40.60***	13.17***
	FTSE100→PX	27.84***	32.12***	101.41***	41.65***	13.96***
PX-BUX	PX→BUX	0.04	2.12	22.68***	29.04***	43.47***
	BUX→PX	10.69***	0.22	207.19***	44.92***	41.13***
PX-LJSEX	PX→LJSEX	17.82***	66.96***	2.69	199.16***	48.57***
	LJSEX→PX	8.15***	92.41***	7.83***	188.97***	61.50***
PX-WIG20	PX→WIG20	2.14	6.09**	20.18***	26.76***	72.07***
	WIG20→PX	9.35***	7.99***	9.95***	34.89***	66.50***

Notes: The table reports the F-statistics of the Granger causality tests for the full sample and also for different time scales. The lag length used in the Granger test is determined by Schwarz Information Criterion (SIC). The optimal lag for all pair-wise correlation time series as determined by SIC criteria is 1. The notation A→B means that A Granger causes B. *** denotes 1 percent significance, ** denotes 5 % significance level and * a 10% significance level of rejection of the null hypothesis.

Table 8b Multiscale Granger Causality Test Results (scales τ_5 to τ_8)

A pair of indices	Direction of causality	Scale τ_5	Scale τ_6	Scale τ_7
PX-ATX	PX→ATX	4.59**	88.27***	40.82**
	ATX→PX	4.44**	56.14***	44.01***
PX-CAC40	PX→CAC40	2.64	36.10***	17.98***
	CAC40→PX	2.33	46.34***	16.59***
PX-DAX	PX→DAX	3.50*	45.13***	4.27**
	DAX→PX	3.04*	50.81***	3.43*
PX-FTSE100	PX→FTSE100	2.12	5.14**	0.01
	FTSE100→PX	1.92	12.98***	0.29
PX-BUX	PX→BUX	0.01	21.31***	88.49***
	BUX→PX	0.19	22.04***	100.76***
PX-LJSEX	PX→LJSEX	88.23***	37.43***	1.48
	LJSEX→PX	69.52***	34.94***	3.36*
PX-WIG20	PX→WIG20	0.00	251.09***	7.20***
	WIG20→PX	0.05	269.53***	5.38**

Notes: See notes for Table 8a.

The results show that there are significant Granger causal relationships (return spillovers) between the Czech and European stock markets, mostly bi-directional—from the Czech market to other European stock markets and the reverse—indicating a feedback Granger causal mechanism between the stock markets' returns. The results are in line with the conclusions drawn from the research of Égert and Kočenda (2010), Patev et al. (2006), and Horobert and Lupu (2009), who show that not only do the stock returns in the developed European stock markets (of Austria, France, Germany, and the UK) Granger-cause the stock returns of CEE stock markets, but also the CEE stock market returns may influence the stock returns in developed markets.

The strength (as measured by F-statistics) of the causal relationships may vary across scales. This finding has been confirmed by other multiscale Granger causality test studies (Ramsey, 1998a,b; Gençay, 2002; Zhou, 2011). For example, we can find that on scales τ_1 to τ_4 and again on scale τ_6 a strong bi-directional Granger causal relationship exists between the PX and FTSE100 returns. No Granger causal relationship, however, is found on scales τ_5 and τ_7 .

The strength and direction of the causal relationship for the raw return series may change when specific time scales are examined. For example, we observe that for the raw return series (untransformed daily return series) the ATX returns Granger-cause the PX returns; however, the scale analysis indicates that a bi-directional Granger causal relationship exists between the stock markets' returns.²³ As Zhou (2011) has also argued, this finding indicates that the results from the raw returns data, which averages all time scales, may be inappropriate for time-scale-conscious investors.

4. Conclusion

We studied the comovement and spillover dynamics between the returns of the Czech stock market and other major European stock markets (the developed stock markets of Austria, France, Germany, and the UK) and the stock markets of CEE countries (namely, Hungary, Slovenia, and Poland) in the period April 1997–May 2010. A DCC-GARCH analysis is applied to show that the correlation between the Czech and European stock markets is time-varying. This is clearly demonstrated by financial market crises (the Russian financial crisis, the dot-com crisis, and the global financial crisis), which led to short-term increases in correlation between the Czech and other stock market returns investigated.

The returns of the PX index during the period April 1997–May 2010 correlated most with the ATX, WIG20, and CAC40 returns and least with the LJSEX returns. For investors already present in the Czech stock market, the greatest diversification benefits are thus achievable by investing in LJSEX-indexed instruments. The DCC-GARCH correlation analysis also showed a rising trend in the pairwise correlations between the returns of the Czech and European stock markets. The rising correlation trend is more evident for some stock index pairs (namely, between PX and ATX and CAC40 and FTSE100) and less evident for others (namely, between PX and LJSEX). By splitting the entire observation period into three sub-periods, the present paper has proved, based on econometric evidence, that European Union integration led to more correlated movements in all stock markets in Europe with the exception of Slovenia. Since the start of the global financial crisis the comovement has increased further.

The paper aimed also to investigate the return spillovers between the Czech and European stock markets and whether they depend on the horizon over which they are calculated (i.e., whether they are a multiscale phenomenon). The DCC-GARCH model results show that there were significant return spillovers between the stock markets in the period April 1997–May 2010. The multiscale return spillovers, analyzed using a multiscale Granger causality test, indicate that the Granger causal

²³ The strength of the comovement (i.e., correlation analysis) is another interesting issue that could be investigated on the multiscale level. This, however, would exceed the limits of the present paper.

relationships between the Czech and European stock markets over the entire period investigated were mostly bi-directional—from the Czech to other European stock markets and the reverse—indicating a feedback Granger causal mechanism. The strength and direction of the causal relationship for the raw return series may change when specific time scales are examined.

The results of the present study are important for international investors seeking diversification benefits in European stock markets. The results suggest that such investors should consider the time-varying comovement between stock market returns and investigate scale-based return spillovers.

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