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# Independent Spike Models: Estimation and Validation*

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*Abstract*
*We apply a class of Markov switching models (independent spike models) to six European electricity markets and two European gas markets. This paper extends the current framework by introducing Gamma distributed spikes, which improves the fit for most energy markets. The models are quite complex. The robustness of the estimates is therefore evaluated using three different estimation strategies: direct maximization of the likelihood function, the Expectation-Maximization algorithm, and Markov Chain Monte Carlo (MCMC). The seasonal variation is corrected for by using the month-ahead forward price as a predictor. The models provide good empirical results for most markets.*

## 1. Introduction

Energy prices are qualitatively different from many other commodity prices. They are often seasonal on a yearly, weekly, and daily time scale, and prices are mean-reverting. Other features are spikes and drops—sharp increases/decreases in the spot price followed by a return to the previous level a few days later (see Haugom, 2011).

We adjust for the seasonal variation by modeling the spread between spot prices and month-ahead forward prices. The daily data sets have therefore not been subject to any deseasonalization techniques other than on a weekly time frame, contrary to most other studies. The spread is modeled in a framework that can fit these extreme observations as well as other energy price characteristics. These models are known as Independent Spike models and have been successfully used in previous studies (see Janczura and Weron, 2010, for information on many of the recent developments).

The main methodological contribution of this paper is that we extend the independent spike model framework to include Gamma distributed spikes, which gives a better fit to the data than light-tailed (Gaussian) spikes and drops, while having lighter tails than log-normal spikes. This difference is important, as the conditional expected value of the spot price is undefined when spikes are log-normally distributed.

A second contribution of this paper is a robustness study. Independent spike models are complex with a high dimensional parameter vector, and this can cause practical problems when fitting the models. We have used three related estimation strategies—direct maximization of the likelihood function, the EM algorithm, and MCMC—to assess their practical performance on simulated data.

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The remainder of the paper is organized as follows. In Section 2, we introduce the most commonly used models for spot price dynamics, while Section 3 discusses independent spike models in detail. Section 4 presents the results when applying these models to market data, and Section 5 concludes.

## 2. Spot Price Dynamics

The spot price is determined by the equilibrium between supply and demand. These are often subject to various restrictions in most energy markets. The supply is capped by the maximum production capacity for the generator units and the production cost increases almost exponentially with higher output. There can be dramatic changes in supply caused by a production shortfall or integration with another market, which expands the production network. The demand is fairly inelastic; consumption is practically unaffected by the current price level and is strongly seasonal due to consumer and industrial use. A large amount of the change in demand for energy is caused by shifts in temperature throughout the year; warm summers increase demand due to the heavy use of air conditioning, while cold winters increase demand due to heating, especially in the Nordic countries. This seasonality in the demand leads to higher prices during the summer for many energy markets and during the winter for the Nordic countries (see Eydeland and Wolyniec, 2003; Benth et al., 2008). There is additional weekly and intraday temporal dependence. Industrial facilities increase the demand during business days, and there is also higher demand during business hours. This is reflected in price variations throughout the day, with higher spot prices during peak hours. These characteristics are not found for many other financial instruments.

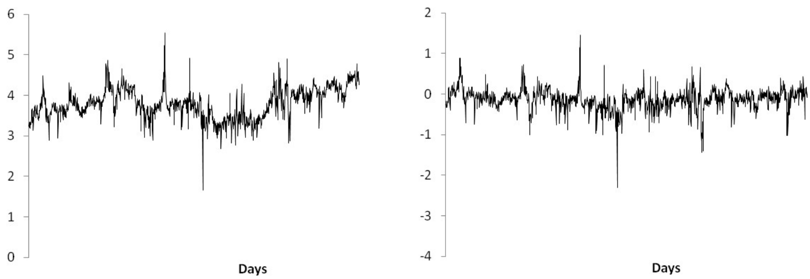
A prominent feature of energy spot prices is their tendency to return to the long-term level after a deviation. This effect is called mean-reversion and is not found for many other asset classes. Some energy markets are also characterized by volatility clustering. This effect can be directly observed as stressed periods when the price level varies much more than normal. This can be caused by some temporary disturbance of the market, congestion (see Haldrup and Nielsen, 2006) or uncertainty about the future.

The frequent occurrence of large changes in prices is unique to energy spot markets due to consumers' inability to react to changes in price levels and capped production levels. Periods of high demand have much higher price sensitivity to disruptions, as excess supply is scarce and expensive to produce, leading to a large increase in the price, which is reversed when the disruption is gone (see Benth et al., 2008).

Not only do upward spikes occur in energy markets, but there are also large downward movements called drops. Large drops are caused by unexpected events, either decreased demand or (more likely) increased supply in combination with the need for real-time balancing; neither an excess nor a lack of energy can exist in the spot markets. This has recently led to some occurrences of negative electricity prices in the hour-ahead market in both the German and Danish electricity spot markets (see Janczura and Weron, 2010). The production of renewable energy (primarily wind power) is very volatile.

There is now a consensus that a realistic model for electricity prices must include seasonality, mean-reversion, varying volatility, and jumps (see Escribano et al., 2011, for a recent analysis of eight different markets).

**Figure 1 The Log Spot Price of the EEX Market (left panel) and the Spread between the Log Spot Price and the Log Month-Ahead Forward Price (right panel)**




## 3. Independent Spike Models

In energy spot price modeling there is some concern about what to model due to the close relation between spot and forward prices and their seasonal variation. Almost all previous studies use deseasonalized data; techniques vary from using a sinusoidal function with a linear trend (see Pilipovic, 2007) to wavelet decomposition (see Weron et al., 2004) and hybrid methods. The purpose of the deseasonalization is to work with stationary time series data. The different techniques for removing seasonal dependence complicates comparisons between published models.

We use the framework introduced in De Jong and Schneider (2009), letting the electricity prices mean-revert to the market-quoted monthly forward price. The log spot price and the spread between the log spot and log forward price for the EEX market are presented in *Figure 1*.

This will implicitly correct the spot price for exogenous information and yearly seasonal effects, as the same factors influence the forward price. Much of that information is discarded when using statistically deseasonalized data. The weekday dependence is not incorporated into the monthly forward price and this is adjusted for in the model by rescaling the daily prices with their inverted mean prices.

It can be seen in *Figure 1* that the spread switches between a normal state and one or several extreme states where the spread is very large (positive or negative). This observation is the basis for our modeling approach.

### 3.1 Markov Regime Switching

Independent Spike models are attractive as a complex model is derived from a regime switching mixture of simple models. The mixture is governed by a Markov chain, i.e., a complex model is derived as a mixture of simple models. The general idea is to use one model for a normally functioning market and other, independent model(s) for disruptions in the market. The first paper discussing Markov switching models that we are aware of is Lindgren (1978). We begin by defining the regime variable.

A Markov chain is a process where the future state depends only on the current state and the probability of a particular value is given by

$$P\left(R_t = j \mid R_{t-1} = i\right) = \pi_{ij}$$

The transition probabilities must fulfill the condition that they sum to unity. It is often enough to have two or three states for modeling energy prices. This is interpreted as a normal state and one or two spike regimes. To ease the visual interpretation of the model we will use *B*, *S* or *B*, *S*, *D* as the sets of possible regimes, where *B* indicates the base regime, *S* the spike or up-spike regime, and *D* the drop or down-spike regime.

### 3.2 Base Regime Dynamics

Mean-reversion of prices is one of the most important features of energy spot price models (see Section 2). Many energy spot price models are defined using a mean-reverting stochastic differential equation driven by standard Brownian motion or a Levy process (see Andreasen and Dahlgren, 2006).

The models in this paper are primarily used for forecasting, which is why we resort to discrete time models. A discrete time version of the Vasicek model (see Vasicek, 1977) forms the basis for our extensions:

$$\Delta y_{t+1} = \alpha\left(\mu_t - y_t\right) + \sigma\varepsilon_t$$

written in terms of the log spot price. The model can be applied once we have an expression for the mean-reversion level that is consistent with the seasonal patterns (see Botterud et al., 2011). We defined the mean-reversion level as a function of the month-ahead logarithmic forward price

$$\mu_t = g\left(f_t\right) = \mu_0 f_t$$

This model is still limited by having constant volatility, even though the volatility has been observed to vary between time periods in some markets. We therefore adopt, as proposed by Janczura and Weron (2010, 2011) and Regland and Lindström (2010) in the setting of independent spike models, the use of a Constant Elasticity of Volatility (CEV) process in the base regime. This generalization accounts for varying volatility depending on the price level and is described as follows:

$$\Delta y_{t+1} = \alpha\left(\mu_t - y_t\right) + \sigma y_t^{\gamma}\varepsilon_t$$

where $\gamma$ is henceforth called the CEV parameter. The model reduces to the Vasicek model for $\gamma = 0$ and to the Cox, Ingersoll, and Ross model (see Cox et al., 1985) for $\gamma = 0.5$. The CEV model is able to capture the skew of volatility versus price as stated in Eydeland and Wolyniec (2003) and the inverse leverage effect persistent in some energy markets, which states higher volatility for higher price levels.

It may be worthwhile to consider that the stochastic differential equation that was discretized in order to arrive at the model may not have a solution for all values of $\gamma$, e.g. when $\gamma < 0$.

### 3.3 Spike/Drop Regime Distributions

The fact that spikes are rare events makes them difficult to model and even more difficult to fit to data. Previous studies have approached this problem by modeling the spikes with a white noise process. This simplifies the statistical analysis.

We continue this tradition and use several distributions in our analysis. Previous studies have used the Gaussian, log-normal (see Weron et al., 2004), Gaussian Jump-Diffusion (see De Jong, 2006), and even Pareto distributions (see Weron, 2009) to model the spikes and drops. The Gaussian Jump-Diffusion distribution is a random sum of Gaussian normal variables where the number of variables is governed by a Poisson process. Let $X$ be distributed as the compound Poisson-driven jump diffusion:

$$\begin{aligned} X &= \sum_{i=1}^{N+1} Z_i \\ Z_i &\in N\left(\mu, \sigma^2\right) \\ N &\in Po(\lambda), \lambda > 0 \end{aligned}$$

This distribution simplifies to a Gaussian if $\lambda = 0$, is skewed when $\mu \neq 0$, $\lambda > 0$, and has heavy tails when $\lambda > 0$.

### 3.3.1 Gamma Distribution

We also used the Gamma distribution for the spikes/drops. The Gamma distribution can be interpreted as a sum of independent exponential random variables. More precisely, $\alpha$ is the number of exponential variables with intensity $\beta$. The probability density is given by:

$$p(x;\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp^{-\beta x}, \alpha > 0, \beta > 0, x > 0$$

where $\alpha$ is the shape parameter, $\beta$ is the inverse scale parameter (also known as the rate parameter), and $\Gamma$ is the Gamma function.

The distribution has fewer parameters than the Gaussian Jump diffusion, heavier tails than the Gaussian distribution (which is often found to be too light tailed), and lighter tails than the log-normal and Pareto distribution (which is so heavy tailed that the expected value of the spot price is undefined, making any attempt at using Monte Carlo simulations to evaluate portfolio strategies void). This problem was acknowledged by Weron (2009), who reverted to modeling prices rather than log-prices in order to use heavy-tailed spike distributions.

Another nice feature of the Gamma distribution is that it is part of the exponential family, making fast and robust inference using the Expectation-Maximization (EM) algorithm feasible.

## 3.4 Full Model Description

The complete independent spike model with three regimes, where the base regime is given by a CEV model and where spikes and drops have distribution $F$, is given by:

$$\begin{aligned} y_{t+1}^{B} &= y_t^{B} + \alpha\left(\mu_t - y_t\right) + \sigma y_t^{\gamma} \varepsilon_t \\ y_{t+1}^{S} &= \mu_t + Z_t^{S}, Z^{S} \in F\left(\mu_S, \sigma_S\right) \\ y_{t+1}^{D} &= \mu_t - Z^{D}, Z^{D} \in F\left(\mu_D, \sigma_D\right) \end{aligned}$$

The regimes are switched by a Markov chain governed by the transition matrix:

$$\mathbf{\Pi} = \begin{bmatrix} 1-\pi_{BS}-\pi_{BD} & \pi_{BS} & \pi_{BD} \\ \pi_{SB} & 1-\pi_{SB} & 0 \\ \pi_{DB} & 0 & 1-\pi_{DB} \end{bmatrix}$$

where we have restricted the Markov chain so that it cannot go from spikes to drops (or vice versa) directly without passing the base regime. This reduces the number of parameters and eliminates parameters that are really difficult to estimate.

### 3.5 Parameter Estimation Methods

We used three related estimation strategies to estimate the parameters and latent regimes in independent spike models: direct maximization of the likelihood function (see De Jong, 2006), the Expectation Maximization algorithm (see Janczura and Weron, 2010), and Markov Chain Monte Carlo (MCMC). All of these methods are closely related, ensuring that the maximum likelihood optimality properties (e.g. Cramer-Rao) hold for all estimates.

Direct maximization can be tricky, as the likelihood function is multi-modal. The likelihood was maximized using the derivative-free Nelder-Mead algorithm. It is often argued that maximizing the likelihood by maximizing the full information likelihood as part of the EM algorithm is more robust. An obvious advantage of the EM algorithm is that the value of the likelihood function increases monotonously, but even the EM algorithm can get stuck in a local maximum.

MCMC has not been previously considered for independent spike models. MCMC is based on Markov Chain sampling from the posterior distribution, i.e., the joint distribution of latent states and parameters conditional on observations. We refer to Robert and Casella (1999) for details of the method. MCMC will converge regardless of the initial parameters used (the initial part of the simulated chain is discarded as burn-in) if a sufficient number of samples from the Markov chain are generated (though this can very time-consuming in practice).

We follow the implementation of an MCMC scheme for a CEV process from Eraker (2001), which uses a uniform prior distribution for the parameters known as Jeffrey's prior, and least squares estimation of parameters to sample from an Inverse Gamma (IG) distribution. The CEV parameter $\gamma$ does not have any explicit solution and the sampling is therefore implemented using the Metropolis-Hastings algorithm.

The implementation for the spike distributions is straightforward for the Gaussian distribution, as we are using conjugate priors. The log-normal spikes are updated using the same method after the observations are transformed.

We follow the adoption from Son and Oh (2006) for the Gamma distributed spikes, where the inverted scale parameter $\beta$ has an inverted gamma distribution and the shape parameter $\alpha$ has a non-standard distribution with a probability density function. We sample from this density using the Adaptive Rejection Sampling (ARS) method suggested by Son and Oh (2006).

The practical performance of different estimation techniques for regime-switching models was evaluated in Ryden (2008), who compared the EM and

**Table 1 Simulation Study Using Three Regime-Independent Spike Models with a CEV Base Regime and Log-Normal Spikes. With 5,000 Observations All the Methods Perform Well and Yield Almost the Same Estimates**

| | Initial Par. | True Par. | ML | EM | MCMC |
|---|---|---|---|---|---|
| $\alpha$ | 0.300 | 0.250 | 0.224 | 0.229 | 0.251 |
| $\mu$ | 1.000 | 0.950 | 0.953 | 0.953 | 0.953 |
| $\sigma$ | 0.100 | 0.080 | 0.081 | 0.081 | 0.081 |
| $\mu_S$ | 0.000 | -1.000 | -0.999 | -0.997 | -1.021 |
| $\sigma_S$ | 0.500 | 0.300 | 0.297 | 0.297 | 0.345 |
| $\mu_D$ | 0.000 | -1.000 | -0.985 | -1.007 | -1.038 |
| $\sigma_D$ | 0.500 | 0.300 | 0.309 | 0.313 | 0.371 |
| $\pi_{BS}$ | 0.050 | 0.015 | 0.016 | 0.016 | 0.019 |
| $\pi_{BD}$ | 0.500 | 0.700 | 0.670 | 0.681 | 0.733 |
| $\pi_{SB}$ | 0.050 | 0.015 | 0.014 | 0.014 | 0.017 |
| $\pi_{DB}$ | 0.500 | 0.700 | 0.713 | 0.717 | 0.761 |
| $\gamma$ | 0.000 | 0.500 | 0.522 | 0.549 | 0.498 |
| Log-Likelihood | | | 0.958 | 0.958 | 0.956 |

MCMC techniques. The findings were inconclusive in the sense that no method was uniformly better than the others regardless of the model, but he found that the EM often is the fastest method if point estimates are the only quantity of interest. However, the computational demands even out for more complex models and when the bootstrap is used for computing confidence intervals.

### 3.5.1 Simulation Study

We simulated 5,000 synthetic observations using CEV dynamics and Log-Normal spikes. The results are shown in *Table 1*. Direct maximization of the likelihood function and the Expectation Maximization algorithm perform very well, while MCMC estimates the parameters well except for the transition probabilities. Some remarks on the study: the EM algorithm was iterated 50 times and MCMC was run 20,000 times, with the first 20% discarded as burn-in.

The results for the ML and EM estimators are similar, while additional iterations are needed for the MCMC to converge.

### 3.6 Model Validation

The distributions, size of the regime variable, and type of base model form a large set of possible models. This emphasizes the need for appropriate measures for selecting and validating models. We discarded models where the parameters attain unrealistic values, e.g. when the $\gamma$ parameter was estimated such that the corresponding SDE did not have a solution.

A tool we use is the Schwartz (or Bayesian Information) Criterion. It measures the fit of models using the negative log-likelihood and penalizes it by the number of parameters times the logarithm of the number of observations. The evaluation is ranking-based; the lowest value indicates the best model.

**Table 2 Installed Capacity for Germany, France, the UK, the Netherlands, the Nord Pool System, and Nord Pool Sweden. Germany Has Four Transmission System Operators and Only 79% of the Total Installed Capacity Is Published.**

| | Germany | France | UK | Netherlands | Nordic | Sweden |
|---|---|---|---|---|---|---|
| Date | 2010 | 2009 | 2009 | 2010 | 2009 | 2009 |
| Coal | 37% | 7% | 33% | 25% | 21% | 15% |
| Gas | 11% | 3% | 40% | 69% | 4% | 5% |
| Hydro | 7% | 19% | 1% | — | 52% | 47% |
| Nuclear | 21% | 64% | 19% | 3% | 12% | 26% |
| Other | 24% | 7% | 7% | 3% | 11% | 7% |

The basic distribution test we use is the *Kolmogorov-Smirnov* (K-S) test, also adopted for electricity prices in Janczura and Weron (2010), which compares the empirical cumulative distribution function with the theoretical cumulative density function (see Robert and Casella, 1999). While the K-S test is suitable for many distributions, specific tests have better reliability. We also use the Lilliefors test to assess if the residuals are Gaussian when possible.

The chosen model was finally validated by visually comparing the simulated trajectory from the estimated model with the data. We are especially interested in whether the models can reproduce the qualitative features of the data.

## 4. Analysis of European Energy Markets

Six electricity and two gas markets are analyzed. The data consists of daily one-day-ahead spot prices and month-ahead forward contract prices from 2005 to 2010.

We applied a large number of models to each market, estimating each of them using Maximum likelihood, EM, and MCMC. The preferred model is then selected on the basis of the model selection and validation criteria in Section 3.6. The preferred model is also inspected visually in order to see if the qualitative features of the data are captured by the model.

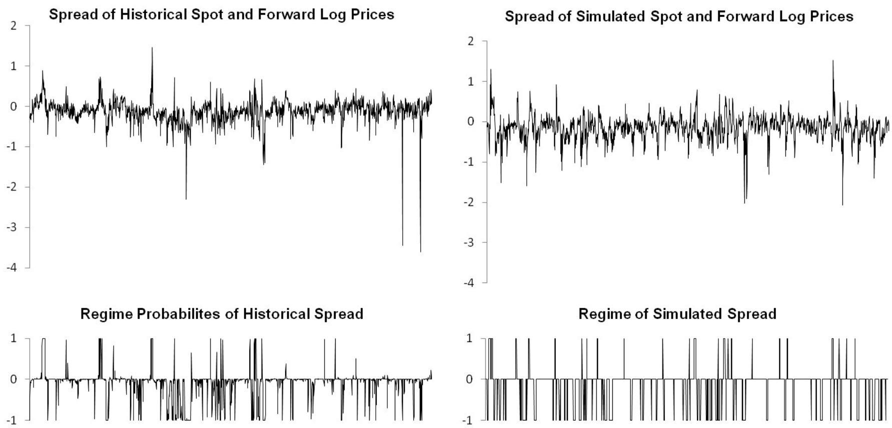
### 4.1 Electricity Markets

The composition of power generation from different energy sources can explain the qualitative behavior in markets. The composition of (net) installed capacity is shown in *Table 2*, where we can see large differences between markets (see EEX, 2009; NationalGrid, 2009; Nordel, 2008; RTE, 2010; TenneT, 2010). It can be seen that in the UK and the Netherlands power is generated predominantly using gas and coal as fuel, while French power generation is dominated by nuclear power. Note that the installed capacity does not have to reflect the actual generation of power, only the possible capacity for the entire power generation system.

#### 4.1.1 European Energy Exchange

The European Energy Exchange (EEX) is the power market for the German power grid. It is the most volatile of the markets considered, and it also has the largest number of observed drops; 11% of the observations are estimated to be lower than

**Figure 2 Historical and Simulated Spread for the EEX Market. The Lower Left Box Presents the Posterior Regime Probabilities and the Lower Right Box the Corresponding Regime Probabilities for the Simulated Data.**




normal. We believe that this is due to the large amount of (volatile) renewable energy in this market. There are even negative prices in this data set. These prices have been truncated to the smallest positive price observed in the market.

The preferred model for EEX was a three-regime model with log-normal spikes and the Vasicek model for the base regime. However, the model does not fully explain the negative skewness and kurtosis of the data even though the qualitative features of the data are captured by the model. Another limitation is the truncation of the data, in combination with the logarithmic transformation, making further model development necessary.

*Figure 2* shows the historical log-spread versus a simulated log-spread. The posterior regime probabilities of the historical spread are also compared with the posterior regimes for the simulated spread with satisfactory results.

### 4.1.2 PowerNext

PowerNext is the French power market. There are relatively few spikes and drops compared to the other markets, but the historical spread is quite volatile, with a volatility of around 15%. A plausible explanation is that power generation dominated by nuclear power is fully adequate for the French market.

The best-fitting model for the spread on the PowerNext market is a three-regime model with log-normal spikes and CEV dynamics. In *Figure 3* we see good similarity between the historical and simulated spreads and posterior regime probabilities.

### 4.1.3 UKSPOT

Power generation in the United Kingdom is heavily dependent on gas and coal. The spot price volatility is low, but the frequency of spikes is rather high (4% of the observations are classified as spikes). The size of these is significantly smaller

**Figure 3 Historical and Simulated Spread for the Powernext Market. The Lower Left Box Presents the Posterior Regime Probabilities and the Lower Right Box the Corresponding Regime Probabilities for the Simulated Data.**

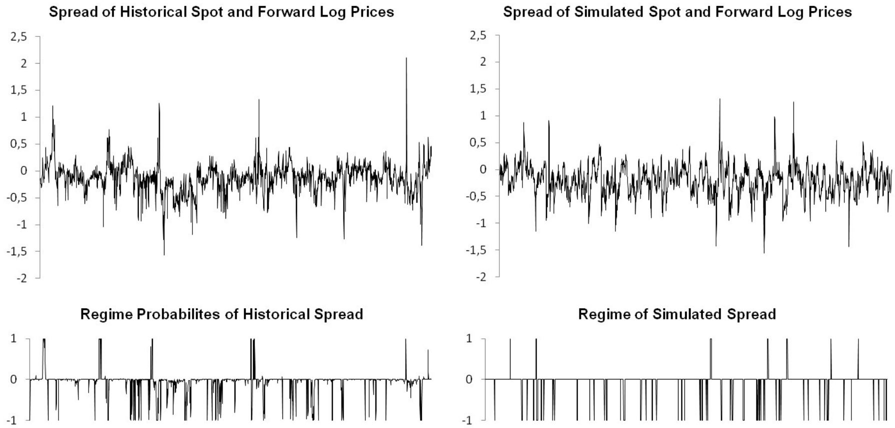
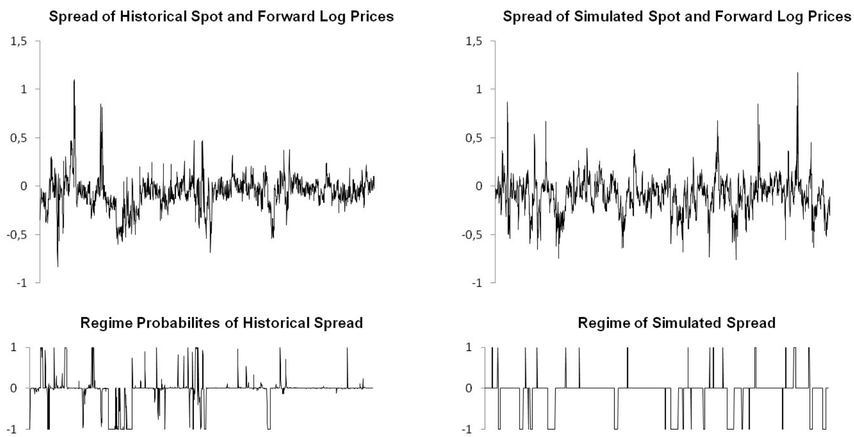


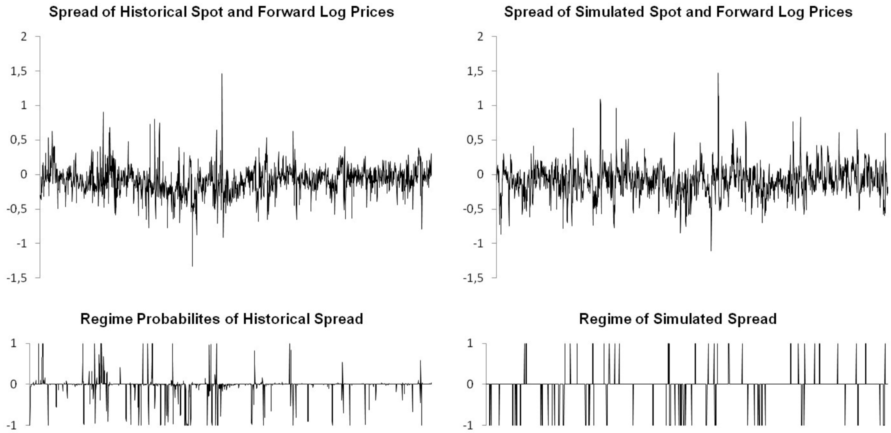

**Figure 4 Historical and Simulated Spread for the UKSPOT Market. The Lower Left Box Presents the Posterior Regime Probabilities and the Lower Right Box the Corresponding Regime Probabilities for the Simulated Data.**




than the corresponding spikes on the EEX, PowerNext, and APX markets, indicating that the system reacts efficiently to supply shortages (starting up a gas-fired power plant is much quicker than starting up a nuclear power plant).

The most appropriate model for the UK market is a three-regime model with Gamma spikes and CEV-dynamics. There is a good match between the historical moments and the moments generated by the model, although this may be misleading. It can be seen by comparing the historical spread and the simulated spread in *Figure 4* that the size and the frequency of the spikes are decreasing over time, whereas

**Figure 5 Historical and Simulated Spread for the APX-NL Market. The Lower Left Box Presents the Posterior Regime Probabilities and the Lower Right Box the Corresponding Regime Probabilities for the Simulated Data.**




the model assumes a fixed jump intensity and jump distributions. The market seems to stabilize during the second part of the sample, and it is possible that this market is one of the most stable markets during the final years of the sample.

### 4.1.4 APX Power NL

The Amsterdam Power Exchange (APX-ENDEX) is the power exchange for the Dutch power grid. The market has the smallest frequency of spot price deviations of all the markets, with only 6% of the observations being classified as spikes or drops. This is most probably a result of the high dependence on gas as fuel (see *Table 2*). Gas power plants can be started up quickly at times of high demand, thereby balancing supply and demand in the power grid.

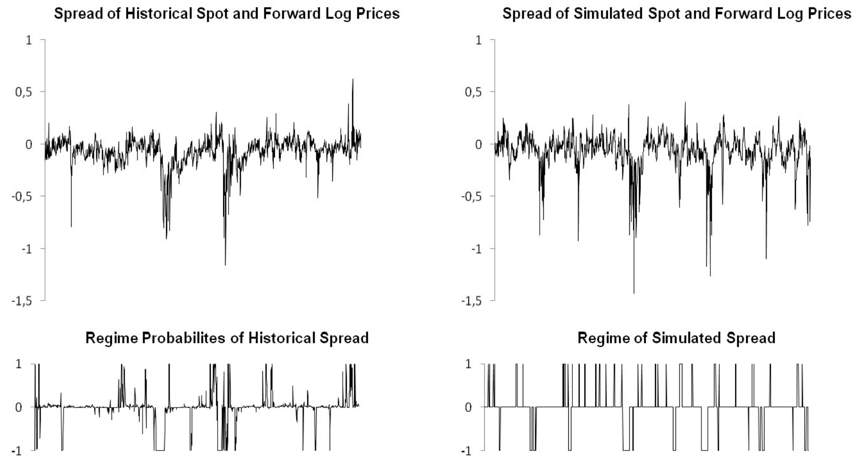
The best-fitting model that was consistent with the model validation criteria was a three-regime model with Gamma spikes and CEV-dynamics, but even a two-state model provided a good fit to the data (see De Jong and Schneider, 2009). In *Figure 5* we see a good similarity between historical and simulated regime switches even though the frequency of the jumps is decreasing (cf. the UKSPOT market).

### 4.1.5 Nord Pool System

Nord Pool ASA was created in Norway in 1991 and has now been extended to all Nordic countries (Sweden joined in 1996, Finland in 1998, Denmark in 2000, and Estonia in 2010), creating an international power market (see Haugom, 2011). The large share of hydro power (52%) in the Nord Pool System gives the dominating market participants the possibility of storing large amounts of power in the form of water reservoirs.

The highly flexible hydro power is not enough to stabilize the market, as almost 13% of the observations are classified as spikes or drops, although the size of these unusual events is comparatively small. A plausible explanation is the large

**Figure 6 Historical and Simulated Spread for the Nord Pool System Market. The Lower Left Box Presents the Posterior Regime Probabilities and the Lower Right Box the Corresponding Regime Probabilities for the Simulated Data.**




amount of wind power in the system. Our study also showed that the Nord Pool System is almost decoupled from other European markets, with most spikes occurring independently of when other markets are experiencing spikes.

The best-fitting model is a three-regime model and CEV dynamics. However, the estimate for the $\gamma$ parameter was negative, which is why the preferred model is a three-regime model with Gamma spikes and Vasicek dynamics. The moments are well matched by the model, but the qualitative features are different from the data when prices are low (the area includes Denmark, where negative prices occurred during the time period). The spread is highly left-skewed compared to the other markets, with only a few spikes but many more drops, as seen in *Figure 6*. This effect is captured by the independent spike model, but there may be additional dynamics in the base regime as the spread fluctuates up and down during the year.

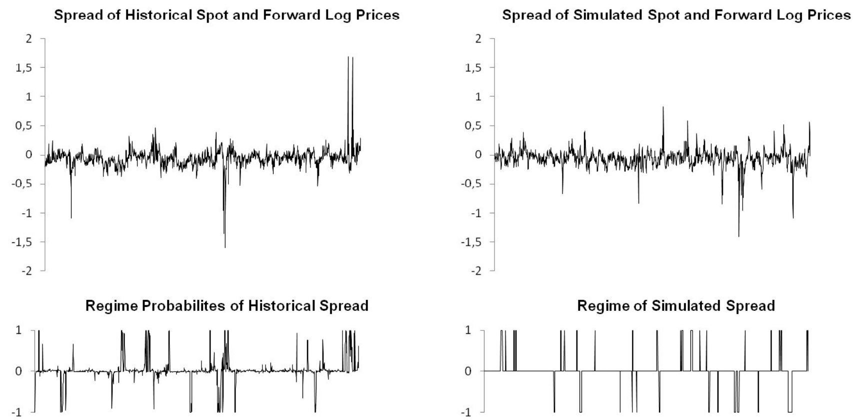
### 4.1.6 Nord Pool Sweden

This regional area in the Nord Pool area has a different spot price than the Nord Pool System price during congestion. The magnitude of the spikes in this area is larger than in the combined (Nord Pool System) market (see Lundgren et al., 2008).

The Schwartz criteria ranks the model with the CEV base regime first, but the negative $\gamma$ parameter in the base regime does not lead to a valid spot price process, as the volatility explodes when prices tend to zero. The model preferred in this paper is again a three-regime model with Gamma spikes and Vasicek dynamics.

The historical spread and the simulated spread are compared in *Figure 7*, where we find similar qualitative features in terms of posterior regime probabilities and the size and frequency of the jumps.

**Figure 7 Historical and Simulated Spread for the Nord Pool Sweden Market. The Lower Left Box Presents the Posterior Regime Probabilities and the Lower Right Box the Corresponding Regime Probabilities for the Simulated Data.**




## 4.2 Gas Markets

The Title Transfer Facility (TTF) and National Balancing Point (NBP) virtual gas hubs have smaller spikes and drops than the power markets. It is also worth noting that the skewness of the spikes and drops is not as prominent as compared with power markets.

It may seem that there are not enough spikes and drops in gas spot prices to motivate an independent spike model. Still, De Jong and Schneider (2009) argue that regime-switching models may improve the fit, so the same methodology is used for gas price spreads.

### 4.2.1 NBP

The NBP is the market for gas in the UK. The qualitative behavior of the spread is similar to the UKSPOT spread. The volatility in the market is decreasing over the time period (the same effect holds for the frequency of the spikes), which indicates that the markets are increasingly efficient at handling unexpected events (cf. the UKSPOT market).

Several two-regime models provided a good fit to the data, but the combined assessment of the validation criteria still selects a three-regime model with Vasicek dynamics and Gamma distributed spikes. The resulting model is compared to the historical spread in *Figure 8*, with satisfactory results.

### 4.2.2 TTF

The TTF is the gas trading hub in the Dutch power grid and is located in Zeebrugge. Gas prices in this hub might be expected to be closely related to electricity prices on the Amsterdam Power Exchange, but this does not seem to be the case in our study in spite of the massive use of gas in the power generation system. Similar results were found in De Jong and Schneider (2009), who only found weak evidence of cointegration between these markets.

**Figure 8 Historical and Simulated Spread for the NBP gas Market. The Lower Left Box Presents the Posterior Regime Probabilities and the Lower Right Box the Corresponding Regime Probabilities for the Simulated Data.**

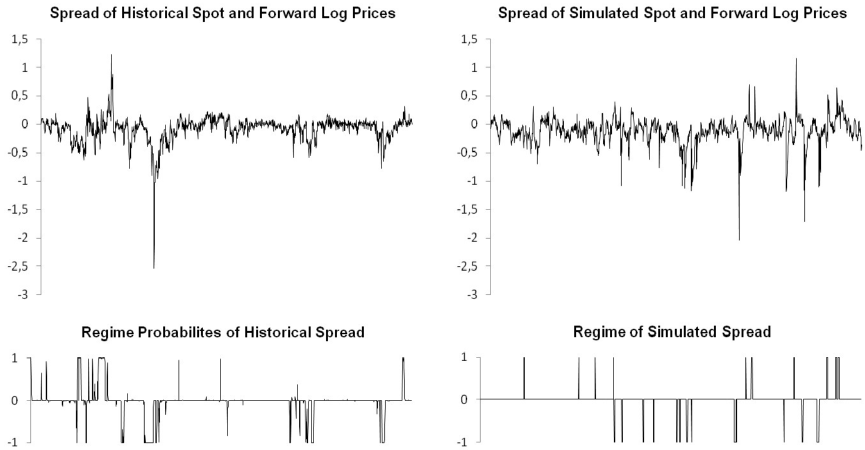
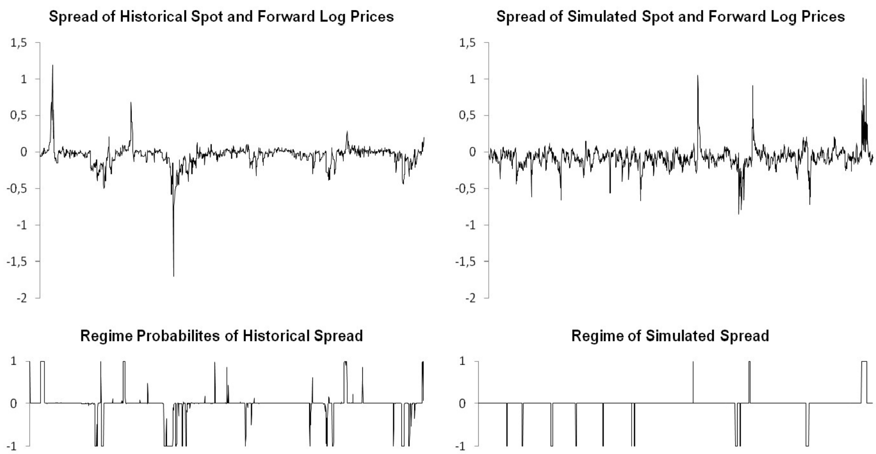



**Figure 9 Historical and Simulated Spread for the TTF Gas Market. The Lower Left Box Presents the Posterior Regime Probabilities and the Lower Right Box the Corresponding Regime Probabilities for the Simulated Data.**




Several two-regime models provide a good fit, indicating that three-regime models may over-fit the data. However, our combined model validation and selection criteria found that a three-regime model with Vasicek dynamics and Gamma distributed spikes was the preferred model. It can be seen in *Figure 9* that the seasonal adjustment was less successful for this market than for most of the electricity markets, indicating that additional modeling may be needed in addition to the independent spike models.

### 4.3 General Remarks

The introduction of Gamma distributed jumps was a success, as not only was it theoretically sound and easy to use within the EM framework, but it was also the statistically preferred jump distribution for six out of the eight markets. The estimated models also demonstrated the need for both state-dependent volatility (CEV dynamics) and jumps (see Escribano et al., 2011).

We applied the models to electricity and gas markets using our framework and found that a three-regime model is needed for the electricity markets (which is consistent with Janczura and Weron, 2011) and for the gas markets, even if a two-regime model may be enough for the gas market once we can find a better model for the seasonal variations (see De Jong and Schneider, 2009). We believe that the difference between the markets is due to the storability of natural gas and the lack thereof for electricity.

We found that the volatility and jump intensity on the UKSPOT and APX-NL markets and on the corresponding gas markets was decreasing over the period. This may be an indication that the European energy markets are converging (see Bunn and Gianfreda, 2011).

## 5. Conclusions

In this paper we applied independent spike models to six electricity and two gas markets. The models were used to describe the price risk by modeling the spread between the spot price and the one-month forward contract used for hedging. Independent spike models are complex models that can be challenging to fit to the data. We compared three different estimation strategies and found, contrary to Ryden (2008), that the EM algorithm (see Janczura and Weron, 2009) was superior in terms of performance and reliability compared to MCMC.

We introduced Gamma distributed spikes and drops in the independent spike model. The Gamma distribution provides the best fit for six out of the eight markets, and does not have the theoretical limitations that log-normal spikes have. The Gamma distribution is also part of the exponential family, making inference using the EM algorithm even more attractive.

We used a mixture of criteria to select the preferred model. The focus was the fit to the market data and the model is well suited for short-term forecasts and spike analysis. The fitted models are able to re-create spikes of the same magnitude and frequency as the historical observations for most markets. There was a large discrepancy between the historical and simulated kurtosis in the EEX market and Nord Pool System. We believe that this was caused by the negative prices that occurred last year. It is obvious that logarithmic spot price models will lead to inaccurate forecasts for small or negative prices. This rather new effect should be included in future models.

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