The Perils of Tax Smoothing:
Sustainable Fiscal Policy with Random Shocks
to Permanent Output[*]

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Abstract
If permanent output is uncertain, tax smoothing can be perilous: both debt levels and tax rates are difficult to stabilize and may drift upwards. One practical remedy would be to target the debt. However, our simulations confirm that such a policy would require undesirably volatile fiscal adjustments and may inhibit countercyclical borrowing. An alternative would be to link the primary surplus not only to the debt ratio (like tax smoothing) but also to its volatility, thus preempting further adjustments while gradually reducing the debt.

1. Introduction

According to a recent report by the International Monetary Fund's Independent Evaluation Office (IMF/IEO; International Monetary Fund, 2003), fiscal components of Fund programs are typically plagued by erroneous (often optimistic) output forecasts.[1] In addition, the report suggested that the magnitude of an initial fiscal adjustment was often not clearly justified in terms of long-run fiscal debt sustainability, and the rationale for subsequent program revisions is often unclear.

Most would agree that the goal of a well-intentioned policy adviser (from the Fund or elsewhere) should be to design a fiscal policy that is sustainable in the long run but not excessively harsh during temporary downturns. Regarding the latter criteria, the use of automatic fiscal stabilizers, including borrowing over the business cycle, is generally regarded to be desirable: if a government is permitted to borrow during downturns, it may be able to reduce the volatility of tax rates, expenditures, and aggregate activity.

Moreover, given the uncertainties surrounding output growth, designing such a fiscal policy is a daunting task. To successfully borrow during economic downturns, a government needs to do more than simply adhere to Rogoff's (2003) common-sense admonition to save during upturns. Such a policy requires that the government esti-

[1] On the issue of optimistic output forecasts, see also (Juhn, Loungani, 2002).

mate permanent output – explicitly or implicitly – since the government's spending capacity is determined by the permanent (not current) tax base.

The quality of permanent output forecasts may depend on the technical expertise or the underlying intentions of the policymaker. Alternatively, permanent output may be characterized by unpredictable, persistent shocks. In this case, forecast errors for permanent output are an *inherent* feature of the output process itself, not a reflection the fiscal authority's expertise or intentions.

What are the implications for debt outcomes when it is difficult to forecast permanent output? To address this question, we consider several alternative fiscal rules in an environment where output shocks are partly composed of a permanent (random-walk) component. Importantly, the government is unable to distinguish between permanent and temporary components. However, the government knows that both kinds of shocks are normally distributed, the mean of both kinds of shocks is zero, and it knows the variances of both kinds of shocks (both absolutely and relative to one another). Thus, an informational imperfection arises from the fact that the government cannot distinguish between permanent and temporary shocks. That is, the government faces a "signal extraction" problem similar to one originally proposed by Muth (1960) (see also (Harvey, 1989), and (Miniane, 2004)).

A recurrent theme in this paper is that of a fiscal objective function. McCallum (2000) and others have suggested that a rules-based policy regime reflects the maximization by the authority of an objective function with a time-invariant (not period-by-period) perspective. This echoes Lucas (1976) and Kydland and Prescott (1977), who advocate rules with "good operating characteristics […] (that are) simple and easily understood, so it is obvious when a policy maker deviates from the policy".

Our benchmark regime resembles the *tax smoothing* model proposed by Barro (1979). The essential element of such a policy is that rates are optimally aligned one-to-one with total expenditures (primary plus interest). Such a regime closely resembles the canonical debt sustainability framework widely used by policy makers (see (Blanchard et al., 1990), (Burnside, 2005, Ch. 3)); it is often interpreted to also be a *debt stabilization* framework (International Monetary Fund, 2003b).

This framework uses a common-sense analogy with the permanent income hypothesis of consumer behavior (one that Barro himself emphasized). As in Sargent's (1987, pp. 385–88) extension to Barro's (1979) framework, the deadweight losses of taxation are reflected in a linear quadratic collection cost function.[2] Since primary expenditures are exogenous, tax rates respond to *permanent* changes in the public budgetary burden. If output shocks are temporary, governments should borrow over the business cycle rather than adjust taxes, and government debt optimally fluctuates countercyclically around a deterministic mean.

However, our analysis reveals that it can be difficult to stabilize either the debt ratio or the tax rate if there are permanent shocks to output. Instead, under

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[2] The idea of tax smoothing continues to receive attention in the optimal fiscal policy literature; see, for example (Kingston, 1991), (Lloyd-Ellis, Zhan, Zhu, 2005), and (Fisher, Kingston, 2005). Other recent papers that use a loss function to approximate underlying consumer welfare include Benigno and Woodford (2003) and Angeletos (2003). By contrast, a paper that directly maximizes consumer utility is Chari, Christiano, and Kehoe (1994); they also introduce large capital levies in an initial period to raise consumer welfare (an element absent from our paper).

a tax smoothing regime, fiscal policy takes on the unpredictable flavor described in the IMF/IEO report: both tax rates and debt ratios can be volatile and may even drift upward from their initial values over time. Such unpredictability relates to "debt intolerance" discussed by Reinhart, Rogoff, and Savastano (2003): even at low levels of debt, markets may have difficulties distinguishing permanent from transitory fluctuations in the debt ratio, and they may doubt that a required adjustment will take place.

For a fiscal authority that faces such difficulties, we consider two remedies. First, we consider a debt targeting (DT) regime similar to one found in the European Union and elsewhere: the fiscal authority adjusts the primary surplus (taxes) when there is a *permanent* gap between the desired and actual level of debt. As our simulations confirm, such a regime will stabilize the debt, but at the expense of more tax volatility.

Second, we incorporate a *precautionary* element into fiscal policy: the primary surplus is linked not only to the level of debt (as under tax smoothing), but also to the *variability* of the debt ratio.[3] If the variability of the tax burden rises, the authority optimally increases the primary surplus. Beneath such a rule lies an objective to avoid future fiscal adjustments for all but the most unfavorable circumstances (not unlike a value-at-risk approach). For illustrative purposes, we consider a constant absolute risk aversion (CARA)/hyperbolic objective function, extending the representative consumer analogy, following Caballero's (1990) work on precautionary saving. An objective function of this nature is consistent with recent stochastic approaches to debt sustainability – Monte Carlo simulations and "fan charts" – proposed by Hoffmaister et al. (2001), Garcia and Rigobon (2004), Hostland and Karam (2006), Celasun, Debrun, and Ostry (2006), and Tanner and Samake (2008).

Under the simulated precautionary regime (PR), the initial primary surplus is modestly higher than under the alternatives, both tax rates and the debt ratio fall over time. For cases where the permanent component to output shocks is important, the precautionary regime provides a smoother path of taxes than under debt targeting.

The remainder of the paper is organized as follows. Section 2 presents the economic environment. Section 3 discusses the loss function associated with tax smoothing. Section 4 discusses some additional aspects of tax smoothing. Section 5 presents a debt targeting regime. Section 6 presents a precautionary regime. Section 7 presents simulations. Section 8 concludes.

2. The Economic Environment

In this paper, we will compare several alternative fiscal regimes. All share a common economic environment with three key elements: the output process, the government's intertemporal budget constraint, and the sequencing of fiscal policy decisions in any period.

The output process. Output Y, is the sum of permanent and transitory components. It follows a "random-walk-plus-noise" model (see (Harvey, 1989; p. 102)):

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[3] Other recent work on fiscal policy that features a precautionary element includes the International Monetary Fund's (2003) stress test framework, Value-at-Risk (VaR) methodologies like that of Kopits and Barnhill (2003), and the discussion of the fiscal authority's role as a "tormented insurer" found in (Mendoza, Oviedo, 2004).

$$Y_t = Y_t^P + \varepsilon_t \tag{1a}$$

$$Y_t^P = Y_{t\text{-}1}^P + \eta_t \tag{1b}$$

where Y_t^P is permanent output. The permanent component is a random walk with shock η_t. The transitory component ε_t is pure noise. The shocks to the permanent and transitory component are uncorrelated at all leads and lags. The separate components are never observed. The "signal-to-noise" ratio $q = \text{var}(\eta_t)/\text{var}(\varepsilon_t)$ is assumed to be known. Only the *sum* of three elements $(Y_{t\text{-}1}^P + \varepsilon_t + \eta_t)$ is observed – not the individual elements.

In the well-known signal extraction problem, an economic agent attempts to distinguish permanent and temporary components. The recursive estimate of permanent output at time t conditional on information available at time t-1, $\hat{Y}^P{}_{t|t\text{-}1}$, is:

$$\hat{Y}^P{}_{t|t\text{-}1} = (1-\alpha)\ \hat{Y}^P{}_{t\text{-}1|t\text{-}2} + \alpha Y_{t\text{-}1} \tag{1c}$$

where α, derived from a least-squares optimization problem, is a function of the signal-to-noise ratio, q: $\alpha = q + [q^2+4q]^{0.5}/[2+q+[q^2+4q]^{0.5}]$ (see (Muth, 1960), (Harvey, 1989), (Miniane, 2003)). Notice that information acquired in a period is only useful for forecasting in the following period.

The government budget constraint. In any period, the government's constraint in constant prices is:

$$B_{t\text{-}1}(1+r) - PS_t = B_t \tag{2}$$

where B is real debt and PS is the real primary surplus (tax revenue T minus primary government expenditures G). As a ratio to output, the ex post budget constraint is:

$$b_{t\text{-}1}\theta_t - ps_t = b_t \tag{3}$$

where b is the ratio of government debt to output (Y), θ is the *ex post* growth adjusted discount factor $(1+r)/(1+\lambda)$, $\lambda_t = (Y_t/Y_{t\text{-}1} - 1)$ is the *realized* growth rate of real output, and ps_t is the primary surplus ratio to output. The intertemporal budget constraint is obtained by successive substitution of (1) over an infinite horizon:

$$b_{\text{-}1} - \sum_{t=0}^{\infty} ps_t\ /\ \prod_{t=0}^{\infty}\theta_t = \lim_{t\to\infty} b_t\ /\ \prod_{t=0}^{\infty}\theta^{\text{-}1} \tag{4a}$$

The transversality (or "no-Ponzi game") condition is:

$$\lim_{t\to\infty} b_t/(1+r)^{t\text{-}1} = 0 \tag{4b}$$

Equation (4b) says that the limiting discounted value of government liabilities cannot differ from zero.[4]

The sequence of fiscal policy decisions. Common to all regimes is an assumed sequence that reflects some important aspects of fiscal policymaking. In the initial

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[4] Note that trend growth of output is assumed to be zero. The model can be modified to include a drift term in permanent output.

period ($t = 1$) the government inherits debt level B_0. Likewise, the debt level B_{t-1} is known at the beginning of every subsequent period.

The government must make certain decisions at the beginning of each period ($t = 1, 2, \ldots$) – before the output shocks are realized. We assume that governments control the level of spending. They make commitments at the beginning of the period, and they fulfill their commitments through the end of the period. We also assume that our government wishes to smooth its expenditures in the sense that it programs a constant proportion γ of forecasted permanent output in period t *conditional on information in time t-1* (that is, before output is realized $\hat{Y}^P_{t|t-1}$). That is,

$$G_t = \gamma \hat{Y}^P_{t|t-1} \tag{5}$$

And, at the beginning of each period the government also chooses the tax *rate τ* which is levied on realized output Y_t.[5] As discussed below, depending on which regime is chosen, τ^i is chosen, where i designates the chosen regime. Typically, a regime reflects some relationship between tax rate τ^i_t, the fixed primary expenditure ratio γ and interest payments rb_{t-1}.

3. Tax Smoothing (TS) under a Linear Quadratic Loss Function

Ideally, fiscal regimes should reflect an underlying objective function. In this paper, the government's objective is similar to one proposed by Barro (1979): the government is assumed to minimize tax collection costs. Assuming exogenous values for the primary expenditure ratio γ, the initial debt ratio b_{-1}, and the real interest rate r, and the current period estimate of permanent output $\hat{Y}^P_{t|t-1}$, the government chooses a path of tax rates τ_t that minimizes the discounted expected loss function:

$$\sum_{t=0}^{\infty} E_t \Phi(\tau_t)/(1+\varphi)^t \tag{6}$$

where E_t is the conditional expectations operator and φ is the government's rate of time preference, subject to constraints (4a) and (4b) and the framework for expenditure decisions (5). In this section, we consider a specific functional form, namely the linear quadratic form, similar to one found in Sargent's (1987, pp. 385–88) extension of Barro's model:

$$\Phi(\tau_t) = \phi \tau_t^2 \tag{7}$$

In this case, for $\varphi = r$, the first order condition in any period t is $2\phi\tau_t = \varpi, \forall t$ (ϖ = Lagrange multiplier), and thus the Euler equation implies that $\tau_t = E_t(\tau_{t+1})$, $\forall t$. In our context, the government seeks to maintain a constant expenditure share of permanent output. Thus, substituting the Euler equation into ex ante budget constraint (4a) yields the following policy rule:

$$\tau_t^{TS} = \gamma + rb_{t-1}^{TS} \tag{8}$$

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[5] More precisely, since there is no time inconsistency in this model, optimization is assumed to take place at $t = 0$; thereafter, the government simply executes decisions based on policy functions calculated at $t = 0$ and realizations of the random variables.

or all t, where $b_{t-1}^{TS} \equiv rB_{t-1}^{TS}/\hat{Y}^P_{t|t-1}$ is the debt level in the previous period scaled by *expected* permanent output. (Equivalently, the rule says that the expected primary surplus should be $ps_t^{TS} = rB_{t-1}^{TS}/\hat{Y}^P_{t|t-1}$). Ex post, the debt *level* B_t is:

$$B_t^{TS} = B_{t-1}^{TS}(1+r-r\,\Psi_t) + \gamma[\hat{Y}^P_{t|t-1}-Y_t] \qquad (9)$$

where $\Psi_t \equiv Y_t/\hat{Y}^P_{t|t-1}$. And, the ex post debt *ratio* b_t ($\equiv B_t/Y_t$) evolves according to:

$$b_t^{TS} = b_{t-1}^{TS}(Y_{t-1}/Y_t)(1+r-r\,\Psi_t) + \gamma[\Psi_t^{-1} - 1] \qquad (10)$$

Thus, the tax smoothing framework reflects a standard logic: temporary changes in government spending are financed by temporary debt issue, while permanent spending changes are matched one to one with tax rate changes. Importantly, equations (9) and (10) reveal that if there are no errors in forecasting permanent output ($\Psi = 1$) the debt ratio (relative to permanent output) is stabilized; this feature is widely recognized (see, for example (International Monetary Fund, 2003b)). The tax smoothing regime closely resembles the sustainability frameworks of Blanchard et al. (1990), Talvi and Végh (2000), and Burnside (2005).

4. Does a Tax Smoothing Regime Stabilize the Debt?

Equations (9) and (10) also reveal how debt fluctuations stem from errors in forecasting permanent output, in both ratio form Ψ_t and difference form $[\hat{Y}^P_{t|t-1}-Y_t]$. In this section, we discuss why it may be difficult to stabilize the debt under a tax smoothing regime.

It has been well known since Barro (1979) and Sargent (1987) that both debt and taxes can follow random walks. However, in their models, the random walk stems from the assumption that there are random shocks to expenditures (that is, wars). By contrast, we assume that the authority aims to keep the expenditure ratio constant.

However, if there is a substantial permanent component to output shocks (var(η) large relative to var(ε)), debt and taxes may also drift substantially from their origin. Moreover, the random walk in the debt ratio (and implicitly, the tax rate) may contain an *upward drift*. This point may be illustrated by a simple example. Suppose that output follows a random walk ($q = \infty$), and that $Y_0 = 100$. Assume also that both the tax rate and ex ante expenditure ratio are set at 20 % ($\tau = \gamma = 0.2$), with no initial debt. At the beginning of period 1 – before that period's shock – expected permanent output $\hat{Y}^P_{1|0} = 100$. If expectations are correct, ex post $Y^P_1 = 100$ and the deficit equals zero – both as a level and as a ratio to permanent output Y^P. Alternatively, consider symmetric shocks of ±5 units. For the pessimistic case $Y^P_1 = 105$, the surplus as a ratio to output is 0.952 % ([21–20]/105). For the optimistic case $Y^P_1 = 95$, that (*deficit*) ratio is -1.053 % of output ([19–20]/95). Thus, the mean surplus ratio-to-output is -0.033 % (*deficit*); the shocks do not exactly cancel out but instead exhibit a deficit bias. Alternatively, we may verify more formally that there is an upward drift to debt. First, rewrite (10) as:

$$b_t^{TS} = b_{t-1}^{TS} + b_{t-1}^{TS}r(1-\Psi_t) + \gamma[\Psi_t^{-1} - 1] \qquad (11)$$

Next, by appealing to Jensen's Inequality, it can be shown that the two error terms $(1 - \Psi_t)$ and $[\Psi_t^{-1} - 1]$ may have positive (rather than zero) means.[6]

Seen either way, this result helps illustrate how the debt ratio might drift upwards under a TS regime – even if the policymaker does not intend it to do so. Simulations (discussed below) will reveal that such an upward drift is more modest in cases where permanent shocks are relatively unimportant. But, such a drift is more apparent as permanent shocks become more important relative to temporary shocks ($\mathrm{var}(\eta)$ grows relative to $\mathrm{var}(\varepsilon)$).

Even for the extreme case with only temporary shocks ($\mathrm{var}(\eta) = 0$, $\mathrm{var}(\varepsilon) > 0$), there will be ex ante permanent changes to the debt under this regime. To see this, note that an adverse shock in the initial period will cause a *one-period* deficit – and an increase in the debt. Since the *expected value* of all future shocks is zero, the debt has risen permanently ex ante. (Ex post, however, simulations of such a case yield debt outcomes that are distributed around the initial mean.)

Also, under intermediate cases of (1a)–(1b) wherein both $\mathrm{var}(\varepsilon)$ and $\mathrm{var}(\eta)$ are nonzero, under a TS regime, *any* shock – permanent or temporary – will cause persistent build-ups / drawdowns of debt. Indeed, as simulations later in the paper show, debt *levels* at the end of J periods are more uncertain if there are both temporary and permanent shocks to output $\mathrm{var}(\eta)$, $\mathrm{var}(\varepsilon)$ both nonzero than at either of the two polar extremes ($\mathrm{var}(\eta) = 0$, or $\mathrm{var}(\varepsilon) = 0$). This happens because agents do not know whether a given shock is temporary or permanent until some time after the shock has occurred.

To illustrate, assume that in period 1 the inherited debt stock is zero ($b_0 = 0$) and permanent output, known before the first period shock, is Y^P_0. Assume also that the first period shock (the sum of the unobserved components) is adverse: $error_1 = \varepsilon_1 + \eta_1 < 0$. Finally, assume that after period 1 there are no additional shocks ($error_t = 0$, $t = 2,3,4...$).

Consider now two polar cases of shocks that are fully permanent ($error_1 \equiv \eta_1$) and fully temporary ($error_1 \equiv \varepsilon_1$). In *either* case, the government runs a period 1 deficit equal to $\gamma[Y^P_0 - Y_1]$. However, if the adverse shock is fully permanent ($error_1 \equiv \eta_1$), permanent output will be overestimated in period 1 by the amount $\hat{Y}^P_{1|1} - Y^P_1 = \eta_1(1-\alpha) > 0$. Since the estimate of permanent output in successive periods is a recursion, the error will persist and die out gradually. Accordingly, in subsequent periods, the government will "overspend" by the amount $\gamma[\hat{Y}^P_t - Y^P_t]$ – a persistent deficit and a permanent accumulation of debt.

By contrast, if the adverse shock is fully temporary ($error_1 \equiv \varepsilon_1$), permanent output will be underestimated in period 1 by the amount $\hat{Y}^P_{1|1} - Y^P_1 = -\alpha\varepsilon_1 < 0$. Again, noting the recursive nature of permanent output forecasts, the underestimate will die out only gradually. Accordingly, in the subsequent periods, the government will "underspend" by the amount $-\gamma[\hat{Y}^P_t - Y^P_t]$ – a persistent surplus/debt drawdown.

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[6] Specifically, note first that $(1 - \Psi_t) = [\hat{Y}^P_{t|t-1} - Y_t]/\hat{Y}^P_{t|t-1}$. Next, we define $Z_t \equiv [\hat{Y}^P_{t|t-1} - Y_t]$, where $E(Z) = 0$, and $f(Z) = Z/\hat{Y}^P$. Since the partial derivatives are $\partial Z/\partial \hat{Y}^P = -(\hat{Y}^P)^{-2}$, $\partial^2 Z/\partial \hat{Y}^{P2} = 2(\hat{Y}^P)^{-3} > 0$, $f(Z)$ is a convex function. Consistent with Jensen's Inequality, if $f(Z)$ is convex, $E(f(Z)) \geq f(E(Z))$ and, since $f(E(Z)) = 0$, $E(f(Z)) \geq 0$. A symmetric proof holds for $[\Psi_t^{-1} - 1]$.

The potential for large and permanent shifts in debt levels, even when policy makers are well-intentioned, brings to mind the "debt intolerance" suggested by Reinhart, Rogoff, and Savastano (2003). Large and permanent shifts to debt levels will require large adjustments. Markets may have difficulties distinguishing permanent from temporary shocks to government debt. Moreover, markets may doubt that the required adjustment will take place. In these cases, a crisis of "debt intolerance" may take place – even at relatively low levels of debt.

The question arises as to whether the government could have other mechanisms besides distortionary taxation for dealing with uncertainty. In the optimal fiscal policy literature, some (Lucas, Stokey, 1983) have suggested that the public sector could issue state-contingent assets. However, such contingencies on debt need not be explicitly specified. Instead, according to the fiscal theory of the price level, unanticipated inflations will do the job under a "non-Ricardian" regime – one in which the government does not explicitly adjust the primary surplus (Cochrane, 1998), (Woodford, 2001), (Chari, Christiano, Kehoe, 1994), and (Hall, Krieger, 2002).

Real-world examples of explicit state-contingent government debt are rarely found. And, neither explicit default nor unanticipated inflation appears to be widely embraced as optimal policy in the real world. Rather, most governments would like to *avoid* such events.

Several kinds of market imperfections may help to explain this disconnect between theory and reality. First, there may be important informational asymmetries (including moral hazard) in the conduct of fiscal policy: the government itself has control over spending levels. Second, in a model with predetermined prices, state-contingent inflationary pressures will cause misallocations of resources whose welfare costs may exceed the benefits arising from an inflationary shock absorber (Siu, 2004), (Schmitt-Grohe, Uribe, 2002), (Angeletos, 2003). Third, certain features of financial markets may make state-contingent inflation an undesirable way to balance the budget. For example, some have argued that safe public debt serves an important comparator *benchmark* role. For such debt to be marketable, its return should be safe from credit risk. (For further discussion of these issues, see (Kumhof, Tanner, 2005.))

5. A Debt Targeting (DT) Regime

To address the potentially large and unpredictable fluctuations in debt under the TS regime, we next consider an (ad hoc) debt targeting regime (DT). Under this regime, taxes are determined by:

$$\tau_t^{DT} = \gamma + rb_{t-1}^{DT} + \alpha [b_{t-1}^{DT} - b^*] \tag{12}$$

where $b_{t-1}^{TS} \equiv B_{t-1}^{DT}/\hat{Y}_{t|t-1}^P$ and b^* is the (ad hoc) target debt (assumed to be equal to the initial debt level b_0) and α is the adjustment coefficient implied by the signal extraction model: the authority only responds to *permanent* deviations from the debt target.

This regime resembles some fiscal rules currently in place. An example would be the European Union's Stability and Growth Pact (SGP). The SGP calls for a long-run debt ceiling to output ratio in member countries not to exceed 60 percent of output (as well as a flow deficit target that is not always binding).

The key advantage of this regime is that it helps prevent the wide swings in debt that might occur over time under a tax-smoothing regime. That is, looking J periods ahead, the authority will have a better idea what the end-period debt ratio b_J will be under the DT regime than under the TS regime.

However, this regime also has a clear drawback: it implies more volatile tax rates – fiscal adjustments that are more frequent and deeper than under TS. Such a policy inhibits countercyclical borrowing insofar as it may require adjustments during a recession. Also, spreading out the adjustment over time more gradually – setting the adjustment coefficient in (12) to some number less than α – would simply increase the permanent accumulation of debt.

6. A Precautionary (PR) Fiscal Regime

In this section, we consider a third regime that is designed to guard against future tax and debt increases. In doing so, we rephrase the government's objective function: the fiscal authority targets a primary surplus today with the aim of precluding future fiscal adjustments for all but the most unfavorable z-percent of macroeconomic circumstances (where z is some "small" number – presumably less than 50 percent).

In recent years, policy has been increasingly cast in such terms. For example, the IMF (2003) has recently developed a "stress test" approach under which the authority aims for a primary surplus that is expected to stabilize (or reduce) the debt. Under this approach, scenarios are presented in which key variables (including growth and interest rates) are assumed to deviate from their baseline value by two standard deviations.

In a similar vein, Kopits and Barnhill (2003) suggest using a value-at-risk (VaR) methodology to fiscal sustainability. In such an approach, the government estimates both the mean and the variance of relevant fiscal variables. Likewise, Mendoza and Oviedo (2004) emphasize the role of the fiscal authority as a "tormented insurer." Under their approach, as the variability of fiscal shocks rises, the country's maximum sustainable debt ratio falls.

Such an objective function relates to recent stochastic simulations of debt sustainability – including "fan charts" – proposed by Hoffmaister et al. (2001), Garcia and Rigobon (2004), Hostland and Karam (2006), Celasun, Debrun, and Ostry (2006), and Tanner and Samake (2008). In their frameworks, substantial increases in public debt become increasingly likely over time, even when the fiscal stance remains unchanged. This must be so: the forecast variance of debt increases with horizon length. These approaches plainly reveal the probability of undesirable outcomes – essentially the upper tails of the "fan chart." Our objective function helps recast such simulations in a normative light: it summarizes the maximum fiscal adjustment that a country is willing to suffer in order to avoid further increases in debt (in probability terms, over a given horizon).

If the government aims to avoid further adjustment with a probability of less than 50 percent, it will also, on average, reduce the debt. This is because, under such a regime, the primary surplus must exceed that prescribed by the TS regime, by some factor that is linked to the variability of the tax burden. (Formally, the tax burden is all expenditures $rb + \gamma$. However, in our model, the major source of tax burden va-

riability comes from the debt b, since the primary expenditure ratio γ remains constant.)

We assume a linear relationship between the debt/output ratio and the standard deviation of the tax burden by the coefficient ω. That is, the variability of the tax burden is ωb. A priori, such an assumption makes sense, since the only other source of variance would be the primary expenditure ratio γ – a variable that we have assumed that the government maintains ex ante constant. However, this assumption is also borne out by simulations (discussed below).

Thus, under a precautionary regime, tax rates are determined according to:

$$\tau_t^{PR} = \gamma + (r + \kappa\omega)b_{t-1}^{PR} \tag{13}$$

where $b_{t-1}^{PR} \equiv B_{t-1}^{PR}/\hat{Y}_{t|t-1}^{P}$. As a formal matter, the objective function underlying (13) *cannot* be linear quadratic. As the literature on precautionary savings (Kimball, 1990), (Carroll, Kimball, 1996), (Caballero, 1990) emphasizes, such a precautionary element like that found in (13) requires that the government's objective function also has a nonzero third moment.

At the same time, it is difficult to link regime (13) to a *specific* functional form in a way that is consistent with the linear, least-squares forecasting problem implied by equations (1a)–(1b). As Sargent (1987, p. 396) explains, under quadratic objective functions, linear forecasts can be derived separately from the optimization problem. *This property – widely known as "certainty equivalence" – does not generalize to objective functions with higher-level moments.* That is, while the first order condition implied by quadratic function (7) will include linear forecasts of future marginal tax collection costs; the first order condition implied by objective functions with a third moment will include nonlinear marginal collection costs.

Most noncertainty equivalent models are technically complex. We confine ourselves here to a special case that can motivate policy rules with a precautionary element. Thus, consider the constant absolute risk aversion (CARA)/exponential loss function:

$$\Phi(\tau_t) = -1/\phi \exp(-\phi\tau_t) \tag{14}$$

By analogy with Caballero's (1990) work on household consumption, this functional form has an important implication: it yields an expression for the optimal tax rate that is contingent on expenditures, debt, and volatility.[7] Such an analogy is straightforward if there is no signal extraction problem (the polar cases $q = \infty$ and $q = 0$ in (1a)–(1b) that are equivalent to ones explicitly considered by Caballero). In other cases, such a functional form is better thought of as a heuristic device.[8]

Note that, so long as ϕ is nonzero, the function has a nonzero third derivative, which gives rise to a precautionary motive for household saving – a positive relationship between saving and the variance of the driving process. Moreover, the ratio

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[7] This functional form is not necessary to obtain precautionary behavior, but it does yield an exact solution. Talmain (1998) provides approximations for other functional forms.

[8] Caballero considered several cases of a *single* shock to labor output, including random walk ($q = \infty$ in our model) and white noise ($q = 0$). However, the nonpolar cases of (1a)–(1b) involve two *separate* shocks – ones that agents are unable to contemporaneously distinguish.

Φ'''/Φ'' is said to be a measure of "prudence" (Kimball, 1990).[9] Note next that the Euler equation for any two periods t (τ known) and $t+1$ (τ unknown) is:

$$\exp(-\phi\tau_t)/E[\exp(-\phi\tau_{t+1})] = (1+\varphi)/(1+r) \qquad (15)$$

Then, following Caballero (1990), it can be shown that if output variance is normally distributed, Euler equation (15) implies that the *nonstochastic* component to tax rates is:

$$\tau_t = \tau_{t-1} + \phi\sigma_t^2/2 \qquad (16)$$

where σ^2 is the variance of the total tax burden (discussed below).[10] Thus, the case of $\phi = 0$ is equivalent to the tax smoothing regime discussed in the previous section. By contrast, if $\phi < 0$, fiscal policy may be said to have a *precautionary* element, insofar as the authority attempts to avoid future adjustments by raising today's primary surplus beyond the minimum required to keep the (expected) debt ratio constant.

Note also that substituting (16) into the budget constraint (4) and the transversality condition (5) yields a long-run (expected) relationship between tax rates, expenditure, and debt:

$$\tau_t^* = \kappa_t^* + \gamma_t + rb_{t-1} \qquad (17)$$

where $\kappa_t^* \equiv \kappa(\sigma_t^2)^* = -\phi\sigma_t^2/2$. By assumption, κ_t^* is time-varying: the variance of the tax burden ($\gamma + rb$) falls as the debt ratio itself falls.

The prudence parameter $-\phi$ has an alternative, common-sense interpretation, consistent with the value-at-risk approach: it summarizes the willingness to pay for a precautionary cushion. If $-\phi$ equals or exceeds some critical value $-\phi(z)$, $0 < z < 1$, the government will be willing to levy taxes today sufficient to "cover itself" with pro-

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[9] Alternatively, the statistical term for the third moment, namely *skewness*, helps illustrate the nature of the preference function: the authority places an extra weight on the unattractive prospect of raising taxes.

[10] The following informal argument will show that, if $\phi < 0$, there will be a positive relationship between volatility and the optimal tax rate. First, repeating for convenience expression (15), the Euler equation for any two periods t (τ known) and $t+1$ (τ unknown) is:

$$\exp(-\phi\tau_t)/E[\exp(-\phi\tau_{t+1})] = (1+\varphi)/(1+r)$$

In steady state ($r = \varphi$), taking the natural logarithm of (15) yields:

$$\ln\{\exp(-\phi\tau_t)\} = \ln\{E[\exp(-\phi\tau_{t+1})]\}$$

Following Caballero (1990), we conjecture that the future tax rate τ_{t+1} contains two components: a constant one (τ_{t+1}^*) and a variable one (v_{t+1}). Then, note that $E(v_{t+1}) = 0$. The corresponding marginal loss associated with $E(v_{t+1})$ is:

$$\phi'(0) = \exp(-\phi*0) = 1$$

The natural logarithm of the above term is $\ln(\phi'(0)) = \ln(\exp(-\phi*0)) = 0$. By Jensen's inequality, since $E(\ln(X)) < \ln(E(X))$, $E(\ln(\exp(-\phi v_{t+1}))$ must be less than zero. If Euler equation (15) is re-written as:

$$\phi'(\tau_t) = E(\phi'(\tau_{t+1}^*)) + E(\phi'(v_{t+1}))$$

it is easily seen that the *marginal losses* behave according to $\phi'(\tau_t) < E(\phi'(\tau_{t+1}^*))$ (since $E(\phi'(v_{t+1})) < 0$). Therefore, for $\phi < 0$, $\tau_{t+1} < \tau_t$.

TABLE 1 Variance Parameters for Simulations

Var(ε) (permanent)	Var(η) (temporary)	q	α
6.25	0.00	∞	1.00
5.47	0.78	7.00	0.88
4.69	1.56	3.00	0.79
3.13	3.13	1.00	0.62
1.56	4.69	0.33	0.43
0.78	5.47	0.14	0.31
0.00	6.25	0.00	0.00

bability $(1-z)$.[11] Thus, the critical value of $-\phi(z)$ tells us the minimum "prudence" required for a government to choose to run a primary surplus sufficient to avoid further adjustment for all but the worst z percent of state contingencies.

7. Simulation Results

This section presents simulations of regimes TS, DT, and PR for alternative parameter values.[12] Repeated J period histories are drawn, for both short and long horizons, $J = 20$ and 100. In all cases, 1000 simulations are computed.

Key parameters were chosen to conform broadly to emerging markets. However, alternative assumptions for variances and interest rates, not presented here, yield qualitatively similar results. In all cases, the standard deviation of total output (permanent and temporary components combined, var($\varepsilon+\eta$)) is assumed to be 2.5. For each simulation, several alternative values for the signal-to-noise ratio (q) are presented – see *Table 1*.[13]

Periods are thought of as "quarters". Output in the initial quarter is 100, initial debt is 200 (50 percent of *yearly* output); the primary expenditure ratio $\gamma = 20$ percent. The interest rate is assumed to be 2 percent (about 8 percent yearly). Thus, the mean tax rates under TS and DT $\tau_0 = 24$ percent; under perfect certainty, the debt stabilizing primary surplus is 4 percent of *yearly* output.

Under the precautionary regime *PR* the initial tax rate τ_0 is assumed to be 24.5 percent; this is ½ percent of output greater than τ_0 under TS/DT. This "front--loaded" adjustment keeps the debt from rising in all but the worst 15–20 percent of cases, depending on the value of q.

7.1 The Behavior of Debt across Regimes

While the debt *ratio b* is typically most interesting for policymakers, looking at the debt level B_T reveals some effects of the chosen fiscal policy itself – as opposed

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[11] Thus, assuming that shocks are distributed normally, choice of z % corresponds to $k(z)$ times σ ($k(0.5) = 1.64$, $k(0.1) = 1.3$, $k(0.15) = 1.04$, etc.). And, $\phi = -2/\sigma[k(z)]$.

[12] All simulations are performed in RATS.

[13] A referee correctly points out that, in the strict sense, the variance pairs in *Table 1* do not keep total variance constant. To see this, suppose (hypothetically) that $Y^P_t = \rho Y^P_{t-1} + \eta_t$, where $\rho < 1$. Thus, var(Y^P) $= $ var(η)/(1-ρ^2): the variance of Y^P does not move one-to-one with the variance of η. As a remedy, we performed two supplemental sets of simulations. One set of simulations considers alternative values of var(η) for a fixed value var(ε) = 3.13. The other set considers alternative values of var(ε) for a fixed value of var(η) = 3.13. Results, available from the authors, largely confirm the analysis in the body of the paper.

TABLE 2 Selected Statistics, Simulated Debt Ratios (b_J), and Levels (B_J) after 20 Periods (J=20); (SD = standard deviation)

		Debt Ratio (b_{20})				Debt Level (B_{20})			
		Mean	SD	Percentiles		Mean	SD	Percentiles	
				75th	90th			75th	90th
$q \to \infty$, $\alpha = 1$	TS	0.51	0.07	0.55	0.59	200.0	2.7	201.9	203.5
	DT	0.50	0.00	0.50	0.50	200.0	22.6	215.4	228.9
	TP	0.48	0.06	0.52	0.57	190.3	2.7	192.1	193.7
$q = 8$, $\alpha = 0.88$	TS	0.50	0.06	0.54	0.59	199.9	2.8	201.7	203.5
	DT	0.50	0.00	0.50	0.51	200.7	20.7	215.3	227.0
	TP	0.48	0.06	0.51	0.56	190.2	2.8	192.0	193.8
$q = 3$, $\alpha = 0.79$	TS	0.51	0.06	0.54	0.59	200.0	2.9	202.0	204.0
	DT	0.50	0.01	0.50	0.51	199.8	18.9	212.4	223.7
	TP	0.48	0.06	0.51	0.56	190.3	2.9	192.3	194.3
$q = 1$, $\alpha = 0.62$	TS	0.50	0.05	0.53	0.57	200.0	3.0	202.0	204.0
	DT	0.50	0.01	0.51	0.51	199.8	14.9	210.0	218.4
	TP	0.48	0.05	0.51	0.54	190.3	3.0	192.3	194.2
$q = 0.33$, $\alpha = 0.43$	TS	0.50	0.04	0.53	0.55	200.0	3.0	202.0	204.1
	DT	0.50	0.01	0.51	0.52	200.0	9.8	207.1	212.5
	TP	0.48	0.04	0.50	0.53	190.3	2.9	192.3	194.3
$q = 0.14$, $\alpha = 0.31$	TS	0.50	0.03	0.52	0.54	199.9	2.9	201.9	203.6
	DT	0.50	0.01	0.51	0.52	200.1	6.4	204.7	208.3
	TP	0.48	0.03	0.49	0.51	190.2	2.9	192.2	193.9
$q = 0$, $\alpha = 0$	TS	0.50	0.02	0.51	0.52	199.9	2.7	201.6	203.3
	DT	0.50	0.02	0.51	0.52	199.9	2.7	201.6	203.3
	TP	0.48	0.02	0.49	0.50	190.2	2.6	191.9	193.5

to effects due to changes in the scale variable Y. Thus, *Tables 2* and *3* summarize selected properties of both the debt ratio (b) and the debt level (B) at the end of J periods ($J = 20$, 100), for 1000 simulations of regimes TS, DS, and PR. The table reports the mean, the standard deviation, and values for the 75th and 90th percentiles. These latter statistics are consistent with value at risk (VaR) approaches (for example (Kopits, Barnhill, 2003)): they tell us that the debt (ratio or level) exceeds the critical value in 25 or 10 percent of the simulations.

Results reveal that, under a tax smoothing (TS) regime, for the cases where permanent shocks are more important (higher q) it is more difficult to stabilize the debt ratio. For both horizons $J = 20$ and $J = 100$, the increase in the *mean* debt ratio (previously discussed) is evident. As q increases, so does the increase in mean debt b_J. And, the standard deviation of the terminal debt *ratio* b_J increases monotonically with q. Likewise, as q rises so do the 75th and 90th percentile statistics.

Hence, for more permanent shocks (higher q), debt increases are more severe in the worst 25 and 20 percent of cases. As *Table 2* shows, for a twenty-period horizon ($J = 20$, about five years), if all shocks are permanent (infinite q), the terminal value of the debt ratio b_{20} equals or exceeds 55 percent of output in 25 percent (75th percentile statistic) and 59 percent of output in 10 percent (90th percentile statistic).

TABLE 3 Selected Statistics, Simulated Debt Ratios (b_J), and Levels (B_J)
after 100 Periods (J=100) (SD = standard deviation)

		Debt Ratio (b_{100})				Debt Level (B_{100})			
		Mean	SD	Percentiles		Mean	SD	Percentiles	
				75th	90th			75th	90th
$q\rightarrow\infty$, $\alpha = 1$	TS	0.55	0.24	0.62	0.76	200.0	6.0	204.2	207.7
	DT	0.50	0.00	0.50	0.50	200.0	50.0	234.6	264.9
	TP	0.43	0.19	0.49	0.60	155.9	5.4	159.7	162.6
$q = 8$, $\alpha = 0.88$	TS	0.55	0.20	0.62	0.77	200.4	6.5	204.2	207.7
	DT	0.50	0.01	0.50	0.51	197.2	47.2	229.8	258.3
	TP	0.43	0.16	0.49	0.61	156.2	5.7	160.0	163.8
$q = 3$, $\alpha = 0.79$	TS	0.52	0.14	0.58	0.70	199.4	6.5	203.7	207.7
	DT	0.50	0.01	0.50	0.51	203.9	43.0	233.2	260.5
	TP	0.40	0.11	0.46	0.54	155.4	5.8	159.1	162.8
$q = 1$, $\alpha = 0.62$	TS	0.52	0.12	0.58	0.66	199.8	6.7	204.6	208.2
	DT	0.50	0.01	0.51	0.52	201.2	34.1	224.6	245.0
	TP	0.40	0.09	0.45	0.52	155.6	5.9	160.0	163.1
$q = 0.33$, $\alpha = 0.43$	TS	0.51	0.09	0.56	0.62	199.8	6.8	204.3	208.3
	DT	0.50	0.01	0.51	0.52	200.8	24.3	216.7	231.9
	TP	0.40	0.07	0.44	0.48	155.7	6.0	159.7	163.3
$q = 0.14$, $\alpha = 0.31$	TS	0.51	0.07	0.55	0.59	200.3	6.8	204.8	209.0
	DT	0.50	0.02	0.51	0.52	199.3	17.3	211.4	221.1
	TP	0.40	0.05	0.43	0.47	156.1	6.0	160.1	163.8
$q = 0$, $\alpha = 0$	TS	0.50	0.02	0.51	0.53	200.3	6.1	204.3	207.6
	DT	0.50	0.02	0.51	0.53	200.3	6.1	204.3	207.6
	TP	0.39	0.02	0.40	0.41	156.1	5.4	159.6	162.6

Likewise, as *Table 3* shows, the corresponding statistic for a 100 period horizon ($J = 100$, about 25 years), the terminal value of the debt ratio b_{100} equals or exceeds 62 percent of output in 25 percent of cases (75th percentile); b_{100} equals or exceeds 76 percent of output in 10 percent of cases (90th percentile.)

Interestingly, the unscaled debt levels B_J reveal a *nonmonotonic* relationship between q and the volatility of B_J. The relationship has an "inverse *U*" shape: the standard deviation of end-period debt B_J and its 75th and 90th percentiles are highest at *intermediate* values of q (specifically $q \approx 0.33$) – cases with both temporary and permanent shocks. This result reflects the fact that (as discussed previously) persistent shocks to debt take place in a TS regime precisely *because* agents do not initially know whether a given shock is temporary or permanent.

Simulations also reveal that the debt targeting (DT) regime achieves its goal. In all cases, the standard deviations of debt ratios b_J (*but not levels B_J*) fall dramatically, as do the 75th and 90th percentiles. (As previously mentioned, for $q = 0$, TS and DT are equivalent.)

And, the precautionary regime (PR) reduces the mean debt level and ratios relative to both TS and DT as expected. As *Table 2* shows, after 20 periods, the mean debt ratio b_{20} drops from its initial value of 50 percent of output to about 48 percent.

Moreover, as *Table 3* shows, after one hundred periods, the mean debt ratio b_{100} drops further still to about 43 percent.

And, PR provides less debt volatility relative to TS (but not DT).

However, for cases where permanent shocks are more important (higher *q*), the precautionary regime PR implies a relatively larger "cushion." Hence, for the 20-period simulation (*Table 2*), under the PR regime, the 75th and 90th percentiles for b_J are 52 percent and 57 percent of output respectively (as opposed to 55 percent and 59 percent of output under the TS regime). Likewise, for the 100-period simulation (*Table 3*), under the PR regime, the 75th and 90th percentiles for b_y are 49 percent and 60 percent respectively (as opposed to 62 percent and 76 percent under the TS regime).

7.2 The Behavior of Tax Rates across Regimes

As discussed above, tax rates, rather than debt ratios, directly impact the citizens of a country. Collection cost functions (7) and (14) are meant to reflect the distortionary impacts of taxes. Thus, we also examine the behavior of the tax rate τ. How do tax rates behave under the alternative regimes? Does the debt targeting (DT) regime represent an attractive policy option? Accordingly, simulated data on tax rates are presented both graphically and in tables.

As mentioned previously, the initial tax rate τ_0 is about 24 percent under TS and DT but 24.5 percent under PR. Note that under PR (and unlike TS or DT) taxes fall over time. As *Table 4* shows, after 20 periods, under regime PR, the tax rate τ_J falls to about 24.3 percent; after 100 periods, τ_{100} falls slightly below the tax rates implied by TS or DT.

Average simulated tax rates for regimes TS, DT, and PR (taken over 1000 draws) are graphed in *Figures 1a–1g* (at the end of this section) for alternative values of *q*. Casual observation reveals that for high values of *q* (for example $q=\infty$, *Figure 1a*) the TS regime yields a tax rate that rises gradually over time. Under the DT regime, taxes are more volatile but do not appear to drift upwards. And, under the PR regime, tax rates are initially higher than under DT or PR but fall over time. However, as *q* falls, the upward drift under TS becomes less pronounced; for the case of $q = 0$ (*Figure 1g*), the tax rate under TS appears to be roughly constant over time. Also, as *q* falls, the behavior of DT gradually becomes more similar to that of TS; for the case of $q = 0$ (*Figure 1g*), TS and DT are identical.

To supplement the graphs, *Tables 4* and *5* present the mean, standard deviation, 75th, and 90th percentiles of within-period and end-period tax rates (τ_j, and τ_J respectively) for 20 and 100 period horizons, respectively.

According to these tables, for cases where permanent shocks are more important (high *q*), the TS regime is a relatively less effective policy for stabilizing tax rates. Instead, tax rate variability and *q* are positively related. Thus, at the end of a 20-year horizon ($J = 20$), if all shocks are permanent ($q = \infty$), there is a 25 percent chance (75th percentile statistic) that the tax rate τ_{20} will be at least 24.4 percent; for the worst 10 percent of cases (90th percentile statistic), the tax rate τ_{20} will be at least 24.7 percent. Likewise, after 100 periods ($J = 100$), if all shocks are permanent ($q = \infty$), there is a 25 percent chance (75th percentile statistic) that the tax rate τ_{100} will be

TABLE 4 Selected Statistics, Simulated Tax Rates, Within Period (τ_j, j = 1 to J), and End-Period (τ_J), 20 Periods (J = 20) (SD = standard deviation)

| | | Within-Period Tax Rate τ_j, j = 1 to 20 | | | | End-Period Tax Rate τ_{20} | | | |
| | | Mean | SD | Percentiles | | Mean | SD | Percentiles | |
				75th	90th			75th	90th
$q\rightarrow\infty$, $\alpha = 1$	TS	0.240	0.002	0.242	0.243	0.241	0.005	0.244	0.247
	DT	0.241	0.050	0.273	0.300	0.243	0.052	0.276	0.307
	TP	0.244	0.002	0.246	0.247	0.244	0.006	0.247	0.251
$q = 8$, $\alpha = 0.88$	TS	0.240	0.002	0.241	0.242	0.240	0.005	0.243	0.247
	DT	0.241	0.041	0.267	0.290	0.242	0.044	0.270	0.300
	TP	0.244	0.002	0.246	0.247	0.243	0.005	0.246	0.250
$q = 3$, $\alpha = 0.79$	TS	0.240	0.002	0.241	0.242	0.240	0.005	0.243	0.247
	DT	0.241	0.035	0.264	0.283	0.243	0.037	0.267	0.291
	TP	0.244	0.002	0.246	0.247	0.243	0.005	0.246	0.250
$q = 1$, $\alpha = 0.62$	TS	0.240	0.001	0.241	0.242	0.240	0.004	0.243	0.245
	DT	0.241	0.024	0.256	0.270	0.239	0.025	0.256	0.271
	TP	0.244	0.002	0.245	0.246	0.243	0.004	0.246	0.249
$q = 0.33$, $\alpha = 0.43$	TS	0.240	0.001	0.241	0.241	0.240	0.003	0.242	0.244
	DT	0.241	0.014	0.249	0.258	0.241	0.016	0.251	0.263
	TP	0.244	0.001	0.245	0.246	0.243	0.003	0.245	0.247
$q = 0.14$, $\alpha = 0.31$	TS	0.240	0.001	0.241	0.241	0.240	0.002	0.241	0.243
	DT	0.240	0.008	0.246	0.250	0.240	0.011	0.247	0.254
	TP	0.244	0.001	0.245	0.245	0.243	0.002	0.244	0.246
$q = 0$, $\alpha = 0$	TS	0.240	0.000	0.240	0.240	0.240	0.001	0.240	0.241
	DT	0.240	0.000	0.240	0.240	0.240	0.001	0.240	0.241
	TP	0.244	0.001	0.244	0.245	0.243	0.001	0.243	0.244

at least 24.9 percent; for the worst 10 percent of cases (90th percentile statistic), tax rates τ_{100} will be at least 26.1 percent.

   Tables 4 and *5* thus confirm an unattractive aspect of a debt targeting DT regime: taxes are, by definition, made more volatile – perhaps unacceptably so. For $q = \infty$, under a DT regime, the standard deviations for end-period taxes τ_{20} and τ_{100} are substantially higher than under a TS regime. At the end of a 20-period horizon ($J = 20$), if all shocks are permanent ($q = \infty$), there is a 25percent chance (75th percentile statistic) that the tax rate τ_{20} will be at least 27.6 percent; for the worst 10 percent of cases (90th percentile statistic), the tax rate τ_{20} will be at least 30.0 percent. A 100-period horizon ($J = 100$) yields similar results. Thus, a DT regime does stabilize the debt, but with substantially more tax rate variability.

   By contrast, under the precautionary (PR) regime, simulations reveal that its initially higher tax rates yield some advantages. First, over the short term ($J = 20$), these advantages are more evident for higher values of q. For example, if all shocks are permanent ($q = \infty$), during periods $t = 1$ through J, the standard deviations of tax rates under TS and PR are about the same (0.002) – and substantially lower than under DT (0.05). Since the mean tax rate under PR is higher than under TS, so too are the 75th and 90th percentile statistics. Thus, the PR regime might be thought of as

TABLE 5 Selected Statistic, Simulated Tax Rates, Within Period (τ_i, j = 1 to J), and End-Period (τ_J), 100 Periods (J = 100) (SD=standard deviation)
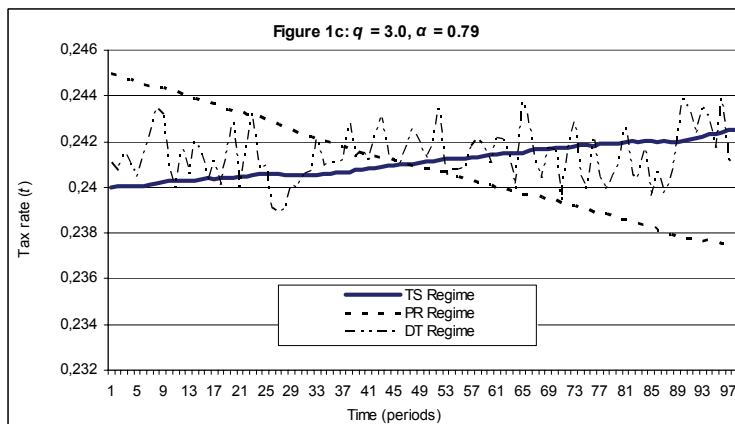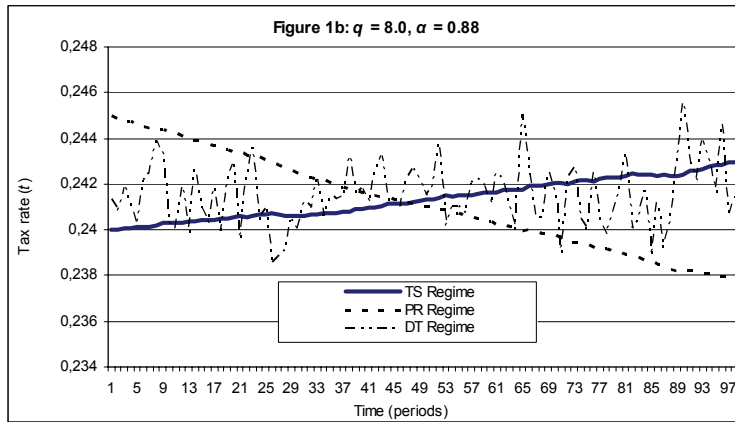
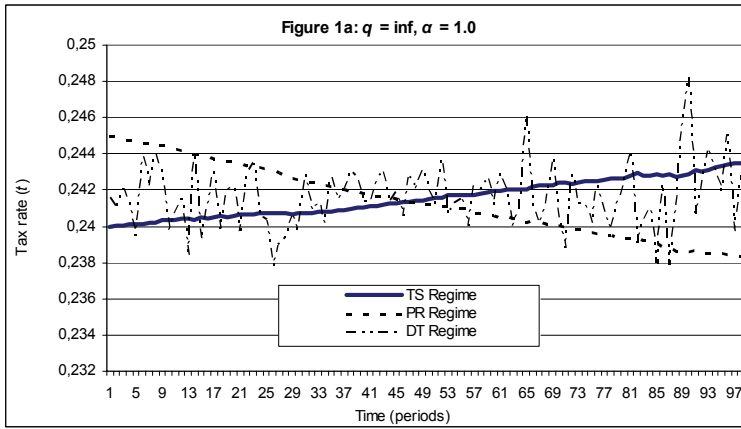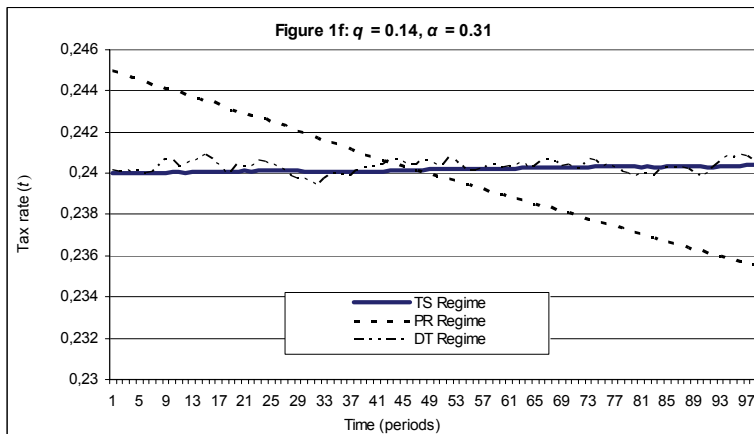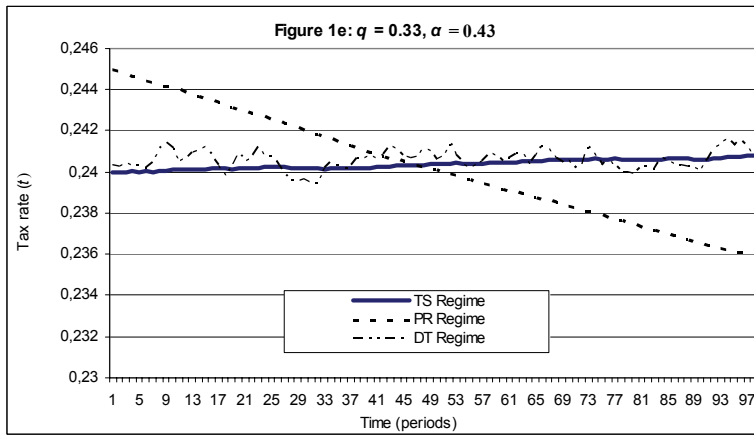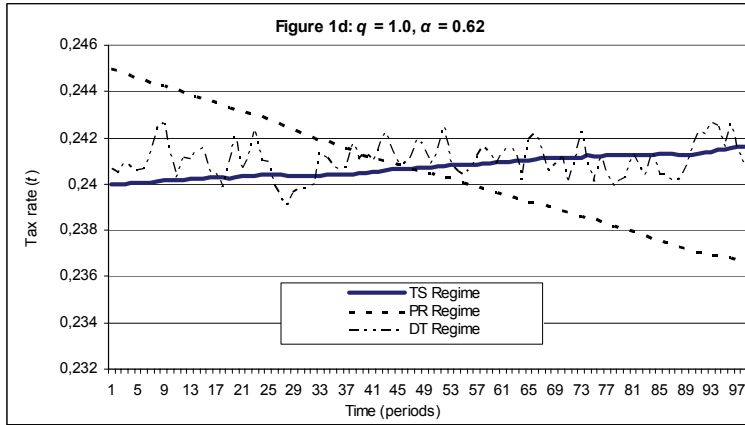|		Within-Period Tax Rate τ_i, j =1 to 20				End-Period Tax Rate τ_{20}			
		Mean	SD	Percentiles		Mean	SD	Percentiles	
				75th	90th			75th	90th
$q \to \infty$, $\alpha = 1$	TS	0.240	0.002	0.242	0.243	0.241	0.005	0.244	0.247
	DT	0.241	0.050	0.273	0.300	0.243	0.052	0.276	0.307
	TP	0.244	0.002	0.246	0.247	0.244	0.006	0.247	0.251
$q = 8$, $\alpha = 0.88$	TS	0.240	0.002	0.241	0.242	0.240	0.005	0.243	0.247
	DT	0.241	0.041	0.267	0.290	0.242	0.044	0.270	0.300
	TP	0.244	0.002	0.246	0.247	0.243	0.005	0.246	0.250
$q = 3$, $\alpha = 0.79$	TS	0.240	0.002	0.241	0.242	0.240	0.005	0.243	0.247
	DT	0.241	0.035	0.264	0.283	0.243	0.037	0.267	0.291
	TP	0.244	0.002	0.246	0.247	0.243	0.005	0.246	0.250
$q = 1$, $\alpha = 0.62$	TS	0.240	0.001	0.241	0.242	0.240	0.004	0.243	0.245
	DT	0.241	0.024	0.256	0.270	0.239	0.025	0.256	0.271
	TP	0.244	0.002	0.245	0.246	0.243	0.004	0.246	0.249
$q = 0.33$, $\alpha = 0.43$	TS	0.240	0.001	0.241	0.241	0.240	0.003	0.242	0.244
	DT	0.241	0.014	0.249	0.258	0.241	0.016	0.251	0.263
	TP	0.244	0.001	0.245	0.246	0.243	0.003	0.245	0.247
$q = 0.14$, $\alpha = 0.31$	TS	0.240	0.001	0.241	0.241	0.240	0.002	0.241	0.243
	DT	0.240	0.008	0.246	0.250	0.240	0.011	0.247	0.254
	TP	0.244	0.001	0.245	0.245	0.243	0.002	0.244	0.246
$q = 0$, $\alpha = 0$	TS	0.240	0.000	0.240	0.240	0.240	0.001	0.240	0.241
	DT	0.240	0.000	0.240	0.240	0.240	0.001	0.240	0.241
	TP	0.244	0.001	0.244	0.245	0.243	0.001	0.243	0.244

a compromise between the TS and DT regimes: it helps attenuate debt volatility while avoiding additional tax rate volatility.
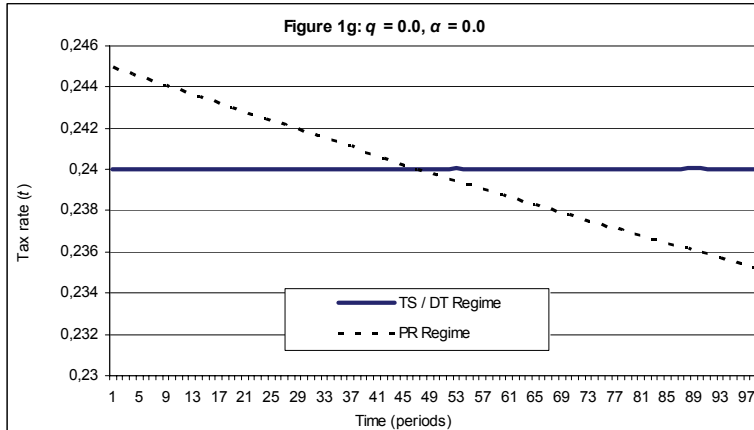
Second, if the authority keeps with a PR regime, both the mean and the variability of tax rates will fall. For a 100-period horizon ($J = 100$), the within period mean tax rate for PR is about the same or slightly less than under TS/DT; for cases in which the permanent shock is important ($q \geq 8$), the standard deviation, the 75th, and the 90th percentile statistics are about the same under PR and TS. Thus, as before, PR appears to be even more appealing relative to TS when the shocks are mainly permanent: it is more urgent for policymakers to insure against permanent adverse shocks than temporary adverse shocks.

Finally, the simulations reveal the benefits of debt reduction over the long term – *for any value of q*. At the end of a 100-period horizon ($J = 100$) the mean end-period tax rate τ_J under *PR* is less than under TS/DT; as well, the standard deviation, 75th, and 90th percentile statistics are all less under PR than either TS or DT for *all* values of *q*.

FIGURES 1a–g Simulated Tax Rates, Regimes TS, DT, and PR

Figure 1a: q = inf, α = 1.0

Figure 1b: q = 8.0, α = 0.88

Figure 1c: q = 3.0, α = 0.79

Figure 1d: q = 1.0, α = 0.62

Figure 1e: q = 0.33, α = 0.43

Figure 1f: q = 0.14, α = 0.31

Figure 1g: q = 0.0, α = 0.0

8. Conclusions

Tax smoothing is a canonical framework for well-intentioned fiscal policy. Under certainty, tax smoothing implies a constant debt-to-output ratio. Accordingly, such a policy does not favor the present over the future – or vice versa.

However, policymakers typically face uncertainty and they are often unsure about the true value of permanent output. Our analysis has illustrated the ways that tax smoothing can be perilous in such an environment: both debt levels and tax rates are difficult to stabilize and may drift upwards.

One practical remedy would be to target the debt. However, our simulations confirmed the undesirable nature of such a policy: it increases tax rate volatility over time and may inhibit countercyclical borrowing.

As an alternative we consider a precautionary regime. Unlike tax smoothing, which links the primary surplus to the debt ratio, a precautionary regime also links the optimal primary surplus to the *volatility* of the debt ratio.

Over time, a precautionary regime will reduce the debt. Plans to reduce government debt are not novel. However, the choice of any particular debt target is arbitrary and may be difficult to defend. In this sense, the credibility of government policy may be bolstered by an explicitly articulated objective function. Accordingly, under a precautionary regime, the objective is to guard against future tax and debt increases for all but the worst macroeconomic circumstances. By contrast, tax smoothing provides no such safeguard. In this sense, the ultimate goal of this paper is to foster good, clear, and understandable fiscal policies – echoing Lucas (1976) and Kydland and Prescott (1977).

In this spirit, Tanner and Samake (2008) incorporate an objective function similar to the one considered in this paper into a stochastic simulation of debt outcomes in three emerging market economies, namely Brazil, Mexico, and Turkey.

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