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Estimating the Dynamics of Weak Efficiency on the Prague Stock Exchange Using the Kalman Filter*

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Abstract

The paper builds on the martingale representation of the market efficiency hypothesis and, with the use of an E-GARCH model of the volatility of the PX and PX-GLOBAL daily returns, a state-space model is formulated. Using the Kalman filter, the time-varying dependency of the daily returns on their lagged values is estimated. The estimation of this parameter shows how quickly the Prague Stock Exchange, represented by its PX index and PX-GLOBAL index, has gradually moved toward the condition of weak efficiency.

1. Introduction

The usual approach taken when assessing market efficiency is to examine whether or not a market is efficient with respect to a particular and fixed period of time. In this paper I take a different view of the problem and I attempt to estimate the evolution of market efficiency in the environment of the Prague Stock Exchange.

The theoretical background I make use of is the martingale representation of the weak-efficiency hypothesis. According to this view, a market is weak-efficient when today's returns are not dependent on lagged past returns. Instead of running a time series regression to check the value of the particular coefficients, I will estimate how this parameter of the dependency of present returns on lagged past returns has been changing over time. Thus, one can obtain a good picture of the evolution of the nature of the Czech capital market (the Prague Stock Exchange).

The main idea of the estimation is to formulate a state-space model where the state variable is the estimated time-varying dependency of present returns on lagged past returns. Such a time-varying regression function also has time-varying residuals, whose variance will be modeled by a particular GARCH model. The state variable is then estimated using the Kalman filter. Such an estimation is based on an observable variable – the returns.

Before the state-space model is formulated, I test several versions of the GARCH model to find out which models will be most suitable for use in the state-space model. I will estimate the dynamics for the two Prague Stock Exchange indices: the PX and the PX-GLOBAL.

This approach using Kalman filtering is based on the analyses by Hall (2002), Li (2001, 2003), and Rockinger (2000). Rockinger made a comparison among transition economies including the Czech economy and reported slow convergence of the Czech capital market toward the condition of weak efficiency. With the development of the Czech capital market since that time, especially with respect to in-

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creasing liquidity, it is reasonable to expect that the model estimated in this paper should report an increasing level of convergence toward weak efficiency.

Three papers analyzing the weak-efficiency condition of the Czech capital market have been published recently. In (Pošta, Hackl, 2007) the weak-efficiency condition is tested by comparing Monte Carlo simulations of stock prices with the real behavior of stock prices. The interpretation of such tests is not straightforward, but the results of the tests indicate that the market might be considered to be close to weak efficiency. Tran (2007) and Hájek (2007) do not test the hypothesis within the martingale representation, but rather use random walk as a baseline model. This approach is not usually preferred nowadays; for further discussion see (LeRoy, 1989). Tran finds that the market does not meet the condition of weak efficiency represented by the random walk, especially when non-linear methods are used. Hájek focuses on international comparison and also does not consider the Czech market to be weak-efficient, in contrast to the Hungarian market.

The structure of the paper is as follows: in the next section – methodology – I start with the martingale formulation of the weak-efficiency hypothesis. Then, I proceed with a presentation of the GARCH models I will later use to test and form the state-space models. At the end of section I describe the data used for the analysis.

In the results section, I first present the empirical results of the GARCH testing and then give a general formulation of the state-space model. The empirical estimations of the models are given and the final output – in the form of the estimated time-varying dependency coefficients – is presented.

Finally, in the conclusion I make a few remarks on the evolution of the estimated parameters and thus on the weak-efficiency hypothesis in the Czech environment.

2. Methodology

To test the market efficiency hypothesis, it is necessary to clearly state the model within which the concept is considered. I will assume the martingale exposition of the market efficiency hypothesis. I stress that the random walk representation of weak-efficiency is stricter – a detailed exposition of the problem is given in (LeRoy, 1989).

Let's assume a price process $\{P_t\}$ and an information set I_t which consists of all the past realizations of the price process $\{P_t\}$. The price process $\{P_t\}$ is a martingale if the following condition holds:

$$E\left(P_{t+1} / I_t\right) = P_t \tag{1}$$

Expression (1) states that the conditional expectation of the future price based on the given information set is equal to the current price. This can be equivalently expressed by the concept of the fair game:

$$E\left(r_{t+1} / I_t\right) = 0 \tag{2}$$

which states that the conditional expectation of the future return r_{t+1} based on the given information set is equal to zero.

Both expressions (1) and (2) imply that it is impossible to use the past evolution of the price process to make such predictions of the future prices or returns so as to earn systematic extra yields (yields above the market return).

The future return may be expressed as its current conditional expectation based on the information set of the whole evolution of the past returns plus the realized *extra* yield:

$$r_{t+1} = E\left(r_{t+1} / \Phi_t\right) + Y_t \tag{3}$$

where Φ_t is the information set of the past returns and Y_t is the realized *extra* yield.

Equation (3) states that the ex-post future return r_{t+1} consists of its ex-ante expectation, which, of course, usually differs from the real future price. The difference is represented by the realized *extra* yield, which may be positive or negative. Let's express the conditional expectation in (3) with respect to the beginning of the process. Let's assume the process started *n* periods ago (which of course may be an arbitrarily chosen point in time), then the conditional expectation in (3) may be expressed as:

$$E\left(r_{t+1} / \Phi_t\right) = E\left(E\left(r_{t+1} / \Phi_t\right) / \Phi_{t-1}\right) = \ldots = E\left(\ldots\left(E\left(r_{t+1} / \Phi_{t-n+1}\right)\ldots\right) / \Phi_{t-n}\right) \tag{4}$$

while the following holds: $\Phi_{t-n} \subset \Phi_{t-n+1} \subset \ldots \subset \Phi_t$ and $\Phi_{t-n} = r_{t-n} = \alpha$.

In other words, due to the iterated conditioning property of the conditional expectations the current expectation of the future return may be expressed as the starting value of the return, as it is the only member of the first information set Φ_{t-n}:

$$E\left(r_{t+1} / \Phi_t\right) = \alpha \tag{5}$$

Keeping the martingale concept in mind, the realized *extra* yield in (3) must be purely stochastic from the point of view of period *t*:

$$Y_t = \varepsilon_t \tag{6}$$

The martingale model does not require ε_t to be white noise, because the martingale model, as opposed to the random walk model, does not exclude the possibility of predicting the variability of the price process based on its past evolution. Substituting (5) and (6) in (3) the future return may be expressed as:

$$r_{t+1} = \alpha + \varepsilon_t \tag{7}$$

where r_{t+1} is the return on an index. According to such a representation of the weak-efficiency hypothesis, the market is weak-efficient if the beta coefficients in the following regression function are zero or statistically insignificant:

$$r_t = \alpha_t + \sum_{j=0}^{p} \sum_{i=1}^{p} \beta_{t-j} r_{t-i} + \varepsilon_t \tag{8}$$

As already indicated, my attempt is not to run a simple regression such as that of (8) and estimate the betas, but rather to build on equation (8) and, with the help of the state-space representation, to estimate the time-varying betas.

The version of equation (8) I will use in the modeling is as follows:

$$r_t = \alpha \ + \beta \, r_{t-1} + \varepsilon_t \tag{9}$$

It is based on a simple regression which proved that only the one-lagged value of the past returns is significant. The other lags – up to a lag of 20 – were not significant. This applies for both the PX index and the PX-GLOBAL index. The residuals in (9) are assumed to behave as follows:

$$\varepsilon_t \sim N(0, h_t) \tag{10}$$

That is to imply that the variance of the residuals will not be taken as a given number but will be modeled using a GARCH specification. I will test five GARCH representations on the series of past returns on the stock indices. The first two will be general GARCH representations – GARCH (1,1) and GARCH (2,2). Then GARCH-M, E-GARCH, and TARCH will be tested. The general representation of the models consists of equation (9) – except the GARCH-M specification, as will be made clear below – and the particular variance equation according to the GARCH version.

In the case of the general GARCH, the variance equations for GARCH (1,1) and GARCH (2,2) are, respectively:

$$h_t = \gamma + \delta_1 \varepsilon_{t-1}^2 + \delta_2 h_{t-1} \tag{11}$$

$$h_t = \gamma + \delta_1 \varepsilon_{t-1}^2 + \delta_2 \varepsilon_{t-2}^2 + \delta_3 h_{t-1} + \delta_4 h_{t-2} \tag{12}$$

where ε_t are residuals from the mean equation.

The GARCH-M model adds a special parameter into the mean equation so that the mean equation (9) becomes:

$$r_t = \alpha + \beta_1 r_{t-1} + \beta_2 h_t + \varepsilon_t \tag{13}$$

where β_2 measures the sensitivity to the expected risk of the stock index (in this analysis). As the variance equation I use equation (11).

Further, I test the E-GARCH specification, which uses the variance equation in the following form:

$$\ln h_t = \gamma + \delta_1 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \delta_2 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \delta_3 \ln h_{t-1} \tag{14}$$

The E-GARCH specification enables us to test the so-called leverage effect, which states the hypothesis that negative shocks in the form of negative residuals have a larger impact (and also persistence) on the variance (volatility) than positive ones. This effect is embodied in coefficient δ_2. When it is negative and statistically significant, the leverage effect is proved. A big advantage of this model is that, due to its exponential nature, it does not give negative values of volatility.

The last model of volatility tested in this paper is the TARCH (or Threshold GARCH) model. The variance equation is defined as:

$$h_t = \gamma + \delta_1 \varepsilon_{t-1}^2 + \delta_2 \varepsilon_{t-1}^2 \mathit{\Gamma}_{t-1} + \delta_3 h_{t-1} \tag{15}$$

where $\mathit{\Gamma}_{t-1}$ is one when the shock (residuals) is negative and zero otherwise. Therefore, as in the case of the E-GARCH model, the leverage effect is present. If coefficient δ_2 is positive, then negative shocks have bigger impact on volatility than positive ones.

TABLE 1 Descriptive Statistics for the Returns on Indices

Index	Mean	St. dev.	Skewness	Kurtosis	Jarque- -Bera	ADF *t*-statistics
PX	0.0004	0.0119	-0.2661	5.6707	964.7091***	-49.8298***
PX-G	0.0003	0.0100	-0.2809	5.5951	917.1223***	-48.8523***

The GARCH models will be assessed according to the statistical significance of the estimated parameters and also according to the presence of serial correlation in the residuals. The Durbin-Watson statistic and the Ljung-Box Q-statistic will be used:

$$Q = T\left(T+2\right)\sum_{j=1}^{k}\frac{\tau_j^2}{T-J} \tag{16}$$

where τ_j is the j-th autocorrelation, T is the number of observations, and k is the chosen lag.

When the autocorrelation between the residuals is measured, this statistic can be used to test the specification of the mean equation. Of course, no statistically significant serial correlation should be present. When the squared residuals are used in the computation, this statistic can be used to test the specification of the variance equation. Again, no serial correlation should be present. In addition, I use the ARCH LM test to check for the remaining conditional heteroskedasticity in the residuals. No statistically significant heteroskedasticity should be present if the variance equation is correctly specified.

As already stated, the state-space model will be presented in the next section after the GARCH models have been tested. The last part of this section will be dedicated to a description of the data used.

The estimation was carried out with the help of the Prague Stock Exchange indices – the PX and the PX-GLOBAL.

The PX started on April 5, 1994 and is a direct continuation of the former main index the PX-50. This index consists of blue-chip stocks, so its base is narrow. On the other hand, the PX-GLOBAL has a broader base and also includes less liquid stock issues which do not have to meet such strict requirements as the ones included in the PX. Both indices are computed as “price indices” and do not take account of dividend yields. For the analysis I use the history of the indices from January 5, 1995 to July 4, 2007. As is clear from the theoretical presentation of the analysis, daily returns, which are measured as differences in logarithms, are used.

Table 1 reports the basic statistics for the two series of daily returns. In *Table 1*, *,**, and *** show the rejection of the null hypothesis of a normal distribution (Jarque-Bera) or the existence of a unit root (ADF) at significance levels of 10 %, 5 %, and 1 %, respectively.

The Jarque-Bera statistics show that the returns do not follow a normal distribution. This is caused especially by the high values of kurtosis of the distribution. In addition to descriptive statistics, the ADF (augmented Dickey-Fuller) *t*-statistic is reported to show the stationarity of the series. The null hypothesis of a unit root is rejected at the 1% significance level. Thus, the original series of the stock indices are I(1) – integrated of order 1.

TABLE 2 GARCH(1,1)

Index	**PX**	**PX-G**
Coefficient	Value	Value
α	0.000647***	0.000476***
β	0.152282***	0.194306***
γ	3.01E-06***	1.97E-06***
δ_1	0.120591***	0.130987***
δ_2	0.863900***	0.857625***
Statistic	Value	Value
DW	2.08602	2.13651

TABLE 3 GARCH(2,2)

Index	**PX**	**PX-G**
Coefficient	Value	Value
α	0.000635***	0.000465**
β	0.153819***	0.196526***
γ	5.31E-06***	3.44E-06***
δ_1	0.133794***	0.149074***
δ_2	0.081255**	0.083836**
δ_3	0.083177	0.07881
δ_4	0.674801**	0.669219***
Statistic	Value	Value
DW	2.08925	2.14109

3. Results

First. the results for the GARCH models for both the PX and the PX-G (PX--GLOBAL) will be reported. The first two models tested were the standard (1.1) and (2.2) GARCH models. *Tables 2* and *3* report the estimations of the particular coefficients. In all the tables below. *.**. and *** show the rejection of the null hypothesis of insignificance of the particular parameter at significance levels of 10 %. 5 %. and 1 %. respectively. To make the interpretation of the results more convenient I again state the mean and variance equations of the models. GARCH (1.1):

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t$$
$$h_t = \gamma + \delta_1 \varepsilon_{t-1}^2 + \delta_2 h_{t-1}$$

and GARCH (2.2):

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t$$
$$h_t = \gamma + \delta_1 \varepsilon_{t-1}^2 + \delta_2 \varepsilon_{t-2}^2 + \delta_3 h_{t-1} + \delta_4 h_{t-2}$$

In the case of GARCH (1.1) all the coefficients are significant at the 1% level of significance. In the case of GARCH (2.2) one coefficient is not statistically significant. *Table 4* gives the empirical estimation of the GARCH-M model stated as:

$$r_t = \alpha + \beta_1 r_{t-1} + \beta_2 h_t + \varepsilon_t$$
$$h_t = \gamma + \delta_1 \varepsilon_{t-1}^2 + \delta_2 h_{t-1}$$

The only difference between GARCH-M and GARCH (1.1) as I used it is the β_2 coefficient, which measures the sensitivity of the returns to the expected risk. The estimation of this parameter is statistically insignificant.

The last two tested models were E-GARCH and TARCH. *Tables 5* and *6* report the results for E-GARCH:

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t$$
$$\ln h_t = \gamma + \delta_1 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \delta_2 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \delta_3 \ln h_{t-1}$$

TABLE 4 GARCH-M

Index	PX	PX-G
Coefficient	Value	Value
α	0.00086	0.000544
β_1	0.151885***	0.194079***
β_2	-0.022815	-0.008655
γ	2.98E-06***	1.97E-06***
δ_1	0.119999***	0.130700***
δ_2	0.864673***	0.857913***
Statistic	Value	Value
DW	2.09358	2.13646

TABLE 5 E-GARCH

Index	PX	PX-G
Coefficient	Value	Value
α	0.000550**	0.000511***
β	0.146855***	0.193149***
γ	-0.536634***	-0.602910***
δ_1	0.231787***	0.247583***
δ_2	-0.035849***	-0.031248***
δ_3	0.960421***	0.956179***
Statistic	Value	Value
DW	2.07573	2.13382

TABLE 6 TARCH

Index	PX	PX-G
Coefficient	Value	Value
α	0.000547**	0.000416**
β	0.155212***	0.195456***
γ	3.57E-06***	2.25E-06***
δ_1	0.093367***	0.108097***
δ_2	0.056037***	0.047467***
δ_3	0.857311***	0.852836***
Statistic	Value	Value
DW	2.09281	2.13931

and TARCH:

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t$$
$$h_t = \gamma + \delta_1 \varepsilon_{t-1}^2 + \delta_2 \varepsilon_{t-1}^2 \Gamma_{t-1} + \delta_3 h_{t-1}$$

As is clear from the results, all the estimated parameters are statistically significant. Also, one can see that the leverage effect was proved. In the case of E-GARCH, it is coefficient δ_2. As it was estimated to be negative, the impact of negative shocks on volatility is higher than that of positive shocks. In the case of the TARCH model it is again coefficient δ_2, which here must be positive if the leverage effect is present.

All tables on the GARCH models report the Durbin-Watson statistic to check the serial correlation of the residuals. It is approximately 2.1 in all cases, which is acceptable.

Based on the significance of the estimated coefficients, only GARCH (1,1), E-GARCH, and TARCH are suitable for the state-space models. All three models also show no serial correlation in the squared residuals as measured by the Ljung-Box Q-statistic, and the ARCH LM test shows no remaining heteroskedasticity (I do not present the results of the tests here).

TABLE 7 Model – PX

Coefficient	Value
α	0.000526**
γ	-0.787559***
δ_1	0.243039***
δ_2	-0.009054
δ_3	0.934919***
η	-10.42898**
Statistic	Value
Log Likelihood	9 774.098
AIC	-6.261601

TABLE 8 Model – PX-Glob

Coefficient	Value
α	0.000428**
γ	-0.706962***
δ_1	0.247314***
δ_2	-0.006010
δ_3	0.947533***
η	-9.906154***
Statistic	Value
Log Likelihood	10 321.81
AIC	-6.612700

Based on these results, the state-space models are formulated. I present here only the state-space model with the E-GARCH specification, first as an example on which it is easy to visualize the other representations and, second as the model which gave the best results in both cases and which will be discussed further below.

The state-space formulation is as follows:

$$\begin{aligned} r_t &= \alpha + \beta_t r_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N\left(0, h_t\right) \\ \ln h_t &= \gamma + \delta_1 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \delta_2 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \delta_3 \ln h_{t-1} \\ \beta_t &= \beta_{t-1} + \eta_t,\ \eta_t \sim N\left(0, \sigma^2\right) \\ \sigma^2 &= e^{\eta} \end{aligned} \tag{17}$$

The first equation is the mean equation (and also the regression function (9)) described above. The time subscript, t, behind β indicates that this coefficient is not to be estimated as a single value but as a time-varying parameter, forming a series. The second equation is the variance equation of the E-GARCH model, which describes the behavior of the variance of the residuals in the first equation. The third equation describes the behavior of the beta coefficient. It is supposed to follow a random walk whose variance is defined by the exponential function. Parameter η is also to be estimated. The covariance of the residuals is set to zero. The Kalman filter is a good way of estimating an unobserved (state) variable from the observed variable(s). The idea behind the Kalman filter is presented in *Appendix 1*.

As I have already stated, the state-space models with other variance specifications gave poor results. In the *Table 7* and *8* I present the results for the model defined by (17) for the PX and PX-GLOBAL indices.

All the parameters except δ_2 are statistically significant. I just recall that this parameter points to whether or not negative shocks have a larger impact on the volatility of the returns than positive shocks. The parameter was estimated as negative, which would indicate the leverage effect. However, it was not significant, even at the 10% level.

FIGURE 1 Beta Coefficient Series for PX

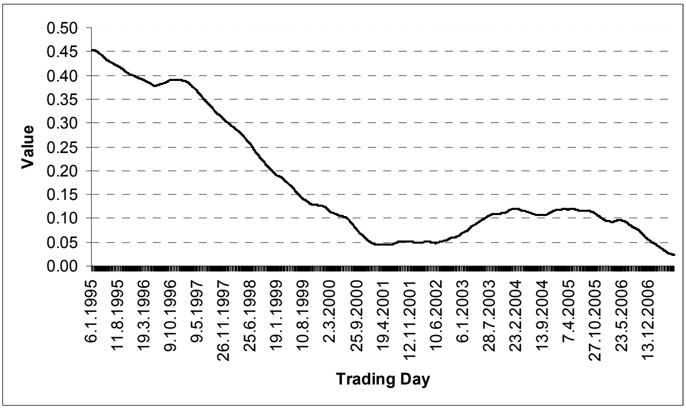
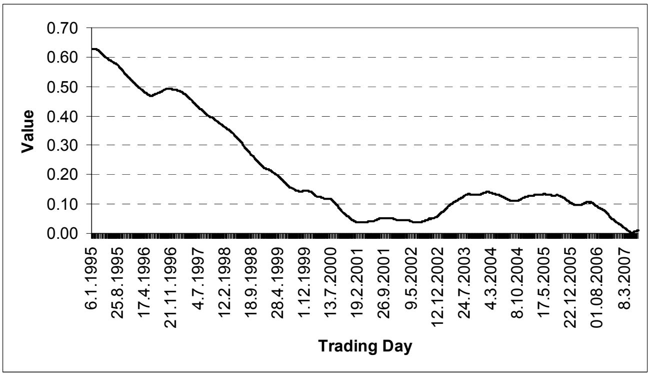


FIGURE 2 Beta Coefficient Series for PX-Glob



Higher values of the log likelihood function are preferred. The value of the log likelihood function enters the Akaike information criterion, of which lower values are preferred. It is defined as:

$$AIC = -2\frac{l}{T} + 2\frac{k}{T} \tag{13}$$

where l is the value of the log likelihood function, T is the number of observations, and k is the number of parameters estimated. Of course, there are no precise values of these parameters which should be reached. As compared with the other estimated models, the value of the log likelihood function is the highest and the AIC is sufficiently low, although the main deficiency of the other models consisted in the low statistical significance of more than one of the estimated parameters.

No starting values for the parameters or for the covariance matrix were used. The estimated state variable in the form of the time-varying β parameter for the cases of the PX index and PX-GLOBAL index is depicted in *Figures 1* and *2*.

The initial estimated value of the beta coefficient was 0.45 in the case of the PX index and 0.63 in the case of the PX-GLOBAL, which indicates clear weak-

TABLE 9 (Year over Year) Indices of Traded Volumes

Year/Year	Main Market	Secondary Market	Free Market
2001/2000	0.51	0.31	1.87
2002/2001	1.59	0.46	3.42
2003/2002	1.33	0.93	0.58
2004/2003	1.90	1.58	3.14
2005/2004	2.28	0.09	1.22
2006/2005	0.83	0.56	0.52

-inefficiency. The value of beta then kept falling almost continuously to the level of 0.04 as of April 6, 2001. The evolution of the beta for the PX-GLOBAL is almost identical, except it reached a slightly lower value at that time. The beta for the PX started rising again in 2002, reaching 0.12 in March 2004. Approximately in the third quarter of 2006 it again started falling significantly, reaching 0.02 on June 29, 2007. The beta for the PX-GLOBAL followed a similar pattern and started rising sharply in July 2002, reaching 0.14 in February 2004. In May 2006 it started falling significantly and reached 0.01 on June 29, 2007.

The evolution of the dependency of the returns on one-lagged past returns clearly shows how the Czech capital market, as approximated by the Prague Stock Exchange, has become weak-efficient. Even though the betas are not strictly zero, the dependency of current returns on past returns is insignificant. The analysis also shows that the evolution is not smooth. Indeed, there is an apparent reversal approximately from the last quarter of 2002 to the first half of 2006.

The possible explanation of this shift lies in the economic development of the Czech economy together with the liquidity of the market. Of course, these factors are partly interlinked. The market probably reacted to the economic slowdown that began at the end of 2000 and continued till the end of 2002. We might have expected the market to react much sooner, but this later reaction just points to its semi-strong inefficiency. This apparent economic slowdown was partly accompanied by a decline in the liquidity of the market, which is reported in *Table 9*.[1] In *Table 9* there is an apparent decrease in the volume traded on the market between 2003 and 2002. The lower liquidity supports the possibility of the exploitation of past prices to predict future prices, because the prices are more stable. From *Figures 1* and *2* one can see that most of the increase in the betas occurred in this period.

In *Figures 1* and *2* we can also see that there was not such an abrupt reaction of the betas to the economic recession in 1997 and 1998, which might cast a little doubt on the presented explanation. However, one must take account of the absolute values of the coefficients, which were much higher at that time, pointing to apparent inefficiency of the market.

To assess the possible impact of economic development and liquidity on the betas more rigorously, I formulate two autoregressive models. The data used for

[1] The issues traded in the main, secondary, and free market differ in terms of the requirements the issue must meet and the disclosure duties. On July 1, the main market and secondary market merged and were named the main market. The indices are calculated as year-over-year indices of the volume traded on the particular market.

TABLE 10

MODEL - Beta-PX	
Coefficient	Value
Beta-PX (-1)	1.267914***
Beta-PX (-2)	-0.353889***
HDP (-2)	-6.582250*
Statistic	Value
LM test statistic	1.11575

MODEL - Beta-PX-GLOB	
Coefficient	Value
Beta-PX-GLOB (-3)	0.298262*
HDP (-2)	-14.44444*
Statistic	Value
LM test statistic	1.18145

the models include the estimated betas, GDP in levels, and the traded volume of stocks and units.[2] While the betas and traded volumes are on a monthly basis, GDP is on a quarterly basis. To exploit the longer monthly series, I disaggregate the GDP series to a monthly basis (using the second-order polynomial so that the sum of three months amounts to the value of the respective quarter). All series were seasonally adjusted and entered in logs and first-differenced. *Table 10* presents the results for the autoregressive models which include GDP as a regressor (the Breusch-Godfrey LM statistic is presented as an indicator for serial correlation; the null hypothesis of no serial correlation was not rejected).

The GDP parameter, which accounts for economic development, enters with lag (-2) and its coefficient is negative and significant at the 10% level. This means that a decline in economic activity leads to a rise in the betas, which supports the hypothesis formulated above. It is noteworthy that the GDP parameter enters with lag (-2) to (-4) with statistical significance up to 10 %, with the best result presented in the table. I report this fact because of the frequency transformation.

Regression with the liquidity parameter did not prove the parameter of interest to be statistically significant at reasonable lags, so I do not present the results. However, it is important to make two remarks. First, the coefficient signs were negative at lags (-1) to (-3), which is in line with the above reasoning. And second, it is important to bear in mind that it was not possible to extract the traded volume of units from the liquidity parameter (the volume traded).

4. Conclusion

In the paper I presented a state-space model and the Kalman filtering technique to estimate the evolution of the dependency of the current returns on the PX and PX-GLOBAL indices on their one-lagged past values. The sample starts in 1995 and the results of the analysis show that the market was clearly weak-inefficient at that time. The results also show how the market has neared weak-efficiency since that period. The analysis also shows the speed at which this nature of the capital market has changed.

As a preliminary analysis, several models of volatility were tested. The results support the leverage effect hypothesis, indicating that negative shocks may have a larger impact on the volatility of the returns than positive shocks. However, this result was not supported within the state-space models.

[2] It was impossible to exclude units from the figures on the traded volume.

The development of the sensitivity of the current returns on the past values was discussed in the broader context of the economy. This discussion pointed out the relationship between the development of weak-efficiency and economic development together with the liquidity of the capital market. This hypothesis was tested within an autoregressive model. The liquidity parameter was not found to be statistically significant, which may be due to methodological problems. The role of economic development as measured by GDP proved to be a significant factor of the evolution of market efficiency.

Based on the results of this analysis and my other recent paper, I conclude that the Czech capital market approximated by the Prague Stock Exchange may be considered weak-efficient. Another important note is that the condition of weak-efficiency should not be considered to be static. Even a once weak-efficient market may go through periods characterized by weaker fulfillment of the condition. This may be caused by the macroeconomic development of the economy, which results in temporary microeconomic changes in the framework within which the market operates.

APPENDIX

Kalman Filter

Appendix provides a basic representation of the Kalman Filter method.

The Kalman Filter consists of two stages: filtering and smoothing. The first equation below describes the observed variable and the second describes the unobserved (state) variable:

$$Y_t = ZX_t + RD_t + \nu_t$$
$$X_t = TX_{t-1} + \eta_t$$

where Z, R, and T are the coefficient matrices, Y and X are vectors of the observed and unobserved variables, respectively, D is a vector of exogenous variables, and v and η are stochastic variables with variance/covariance matrices H and Q, respectively:

$$H_t = \sigma_\nu^2$$
$$Q_t = \sigma_\eta^2 r$$

r is called the signal-to-noise ratio.

As new information on the observed variables is released, the filtering procedure creates estimates of the unobserved variables. Let's assume A_t is an optimal estimate of the vector X_t and P_t is the corresponding variance/covariance matrix. Then, knowing A_{t-1} and P_{t-1}, the optimal estimate may be expressed as:

$$A_{t+1/t} = \left(T - K_t Z\right) A_{t/t-1} + K_t \left(Y_t - D_t\right)$$
$$\text{kde: } K_t = TP_{t/t-1}Z'F_t^{-1} \quad \text{a} \quad F_t = ZP_{t/t-1}Z + H_t$$
$$P_{t+1/t} = T\left(P_{t/t-1} - P_{t/t-1}Z'F_t^{-1}ZP_{t/t-1}\right)T' + Q_t$$

Based on these estimates the estimation error is computed:

$$\upsilon_t = Y_t - ZA_{t/t-1} - RD_t$$

The estimation error enters the log likelihood function, which is to be maximized. This is the criterion for the optimal estimate:

$$l = -\frac{1}{2}\log 2\pi - \frac{1}{2}\log\left|F_t\right| - \frac{1}{2}\upsilon_t F_t^{-1}\upsilon_t$$

The second stage uses all the available information. It is a backward recursive computation which starts with the last estimate of the filtering procedure and goes back to the beginning of the sample:

$$A_{t/T} = A_t + P_t^*\left(A_{t+1/T} - T_{t+1}A_t\right)$$

$$P_{t/T} = P_t + P_t^*\left(P_{t+1/T} - P_{t+1/t}\right)P_t^{*\prime}$$

$$P_t^* = P_t T_{t+1}{}' P_{t+1/t}^{-1}$$

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