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An Endogenous Attitude to Firms' Risk Aversion: A Model*

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Abstract
The paper examines the risk behavior of a competitive firm under price uncertainty. The model developed in the paper departs from Greenwald and Stiglitz (1993a), which singly implies risk-averse behavior. The incorporation of more general assumptions about a firm's financing – access to the equity market, the possibility of a soft budget constraint – allows the identification of a broader range of determinants of a firm's attitude toward risk and, hence, optimal output. The results indicate that price and technology are not the only important factors in a firm's optimal output level, as is the case for the neoclassical firm. The model also demonstrates that a firm's net worth position, managerial sensitivity to bankruptcy, access to capital market, budget constraint softness, and degree of uncertainty about future prices may play important roles toward optimal output considerations.

1. Introduction

In the previous decades there has been a tendency to remove the traditional assumption in the theory of the firm that the demand for products is known with certainty at the time when the output decision is made. The uncertain character of the environment has become a critical component in the theories of the firm. It was shown originally by Sandmo (1971) and Leland (1972) that a firm's attitude to bear the inherent risk of production has important effects on that firm's willingness to produce. Greenwald and Stiglitz (1993a) argue that a firm's risk-averse behavior is a result of financial market imperfections and they demonstrate its link to business cycles. The risk-seeking behavior of firms resulting from too easy access to debt-financing due to soft budget constraint was a central policy issue in the transition economies (Kornai, Maskin, Roland, 2003) and during the collapse of the banking sector in Asia in the 1990s (Krugman, 1998).

However interesting they are, these models of firm behavior are components of the theory with the primary focus on explaining certain phenomena on the macroeconomic level (business cycles, troubles of the banking sector in the transition countries, the nature of the Asian crises). They capture important patterns of a firm's risk behavior, but they focus rather on the specific types of constraints or conditions of a firm's financing. As a result they do not provide a complete picture of the determinants of the firm's attitude to risk. The aim of this paper is to complement this literature and develop a more comprehensive model of a firm, which would encompass a broader scope of important parameters, which may have an effect on how

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the firm behaves – in a risk-averse, risk-neutral or risk-seeking manner. The leading question of the paper is hence the following: What are the major determinants of the firm's attitude to risk and therefore of the firm's willingness to produce under uncertainty?

The interplay between uncertainty related to the productive activity of firms and financing arises from a simple fact: most production takes time and future markets are missing for most of the products. The cost of investment or inputs must be incurred before the revenue is obtained from the sale of its output. The traditional theory of the firm under uncertainty (Sandmo, 1971), (Coes, 1977), (Quiggin, 2001), (Hau, 2003) models a firm as having a von Neumann-Morgenstern utility function, which, besides other properties, is increasing and concave in its profit. The firm is therefore assumed to be risk-averse.[1] The principal result of this approach is the demonstration that (1) optimal output for a risk-averse firm facing an uncertain demand is lower than it would be in the case that the firm faced a certain price of the same expected value and (2) with riskier distribution of prices the risk-averse firm reduces output. However, the attitude to risk is not explained within the model and risk aversion is only assumed and not specifically related to financing decisions.

For Greenwald and Stiglitz (1990, 1993a, 1993b) capital structure is a central issue for the firm's attitude to risk and the following logical framework is utilized for explaining behavior: financing, attitude to risk, optimal level of output. It is argued that as a consequence of severely limited access to the capital market due to information asymmetries and negative signals associated with issuing equity, the reliance on debt-financing is the prevailing phenomenon. Greenwald and Stiglitz (1993a) use a simple model to illustrate how risk considerations affect firms' production decisions. It is assumed that the firm cannot raise equity and it is financed by debt. The debt imposes a risk of bankruptcy on the firm and there are personal costs of bankruptcy for managers. Their value and reputation on the managers' market may be harmed, because it is usually not possible to distinguish whether the bankruptcy is caused by bad management or adverse market conditions. Firms thus take the bankruptcy costs into account in their production decisions, which translates itself into lower output and high sensitivity of firms to any changes in the market prices. Consequently, severe business cycles are more likely to happen.

Although the model of Greenwald and Stiglitz (1993a) lucidly relates the financing and attitude to risk, it is very restrictive about the firm's capital structure possibilities when assuming that bond-financing is the only feasible option for additional financing. Most of the financial theorists with a similar approach to capital structure (e.g. (Myers, Majluf, 1984)) understand the information asymmetries as an explanation for conditions when a firm may refuse to issue equity and prefer debt. As Myers (2001, p. 81) points out: "There is no universal theory of the debt-equity choice, and no reason to expect one." Or agency theory and the incentive argument originally suggested by Jensen and Meckling (1976), Grossmann and Hart (1982) and Fama and Jensen (1983) provide a rationale for a mix between debt and equity as too much debt may imply excessive risk for managers, while too little will encourage

[1] This shift from expected profit to the expected utility of profit in the theory of the firm largely resembles the shift in consumer theory from the expected value to the expected utility originally suggested by Daniel Bernoulli almost three centuries ago via his so-called Petersburg's Paradox.

misuse of funds. Therefore it is more realistic to assume that a firm is financed by both – by debt and by issuing new equity shares.

The soft budget constraint (SBC) literature may add another piece to the picture as it provides a theoretical justification for, among other things, the risk-seeking behavior of a firm.[2] It has been shown in extensive literature that the SBC concept is relevant also in the economic environment based on private ownership (e.g. (Schaffer, 1998), (Djankov, Murrel, 2002)). An important softening instrument in the market economy is some form of credit. Relaxed repayment terms to banks may have the form of governmental guarantees (Krugman, 1998). State-owned banks may apply paternalistic economic policy and give firms easy access to credit even if they are in a troublesome financial situation. Or the banks might not require full repayment and they might provide refinancing for investment projects which in the first period turn out to be unprofitable and rely on the second period's results to be higher than the costs of bankruptcy proceedings (Dewatripont, Maskin, 1995).

As a consequence of all these forms, managers may "discount" the value of debt because they expect that they will not be forced to repay it in the event that the firm gets into financial difficulties. The ability of firms to buy inputs without fully repaying them can significantly alter demand for these inputs and the level of output produced. As pointed out in Kornai, Maskin and Roland (2003) and Krugman (1998) the SBC induces firms towads a higher level of investment and production levels by reducing the downside risk to firms.

In the remaining part of this paper we will broaden the Greenwald and Stiglitz model (1993a) for the usage of equity and the possibility of the SBC and thus we will try to capture other important determinants of the attitude to risk and output behavior under uncertainty. The following model should be a generalization of the preceding ones.

This paper is laid out as follows. In the next section the assumptions will be described and a new model constructed. Section 3 discusses the resulting firm's supply under uncertainty. In Section 4 it will be demonstrated that changes in financing (net worth position, possibility to raise new equity, softness of the budget constraint) and the perception of risk faced by a firm can explain all types of attitudes to risk and can potentially have large effects on the optimal output level. The last section concludes.

2. Model

2.1 Assumptions

A.1. We will assume that a firm can partially finance its expansion of production through new equity shares and the remaining part through debt-financing. Hence $T_t^i = B_t^i + S_t^i$, where T_t^i is the total level of external financing that a firm needs, B_t^i is the level of bond-financing and S_t^i is the total issuing price for new shares at time *t*. At the same time we will assume that the proportion of new equity is given by exogenous parameter $s \in \langle 0;1 \rangle$ so that $S_t^i = sT_t^i$. The parameter *s* represents the level

[2] For a surveying article which defines the term "soft budget constraint" and provides a comprehensive account of contexts in which the SBC concept is relevant, see (Kornai, Maskin, Roland, 2003).

of access to the capital market. This differs from the assumption of Greenwald and Stiglitz (1993a), who consider only a bond-financed firm and thus $s = 0$.

A.2 Firms will be assumed to make decisions at discrete intervals: $t = 1,\ldots, T$. We will assume that future markets for products do not exist and that there is then one period lag between the use of inputs and the availability of output.

A.3 The decision variable is the level of production q_t^i. The output decisions of firms are made by managers who take into account not only profit maximization criteria (relevant for owners), but also the risk connected with financing the production through bonds and limited predictability of future prices. As it is usually impossible to distinguish whether financial bankruptcy was caused by bad luck with investment projects or by bad management, managers inevitably suffer a stigma associated with bankruptcy and assign personal costs to bankruptcy (Eaton, Gersovitz, Stiglitz, 1986), (Greenwald, Stiglitz, 1993a).

A.4 The technology of the firm is represented by the standard real cost function $c = c(q)$ where q is level of production, with $c'(q) > 0$ and $c''(q) \geq 0$. The firm's costs increase with the level of production and technology has increasing marginal costs.

A.5 The firm faces sectoral price level $\tilde{P}_t^i$, which is randomly determined around the overall price level. The uncertain price of output of the firm $\tilde{P}_t^i$ is i.i.d. with a distribution function $F(.)$ and density $f(.)$. The expected price of output is assumed to be equal to the overall price level so that $E(\tilde{P}_t^i) = P$. The relative prices are important for a firm due to the fact that expenditures on inputs are assumed to be dependent on the price level P, whereas the revenues from the output produced depend on specific sector price P_t^i. The firm operates in a competitive market and it is a price--taker.

A.6 It is assumed that if the nominal net worth position of the firm is negative ($A_t < 0$) the firm will go bankrupt and that all the revenues from the sale of q_{t-1}^i will be distributed to creditors.

A.7 All forms of the SBC with respect to a bank result in some form of partial or late repayment of the credit or at least they create the expectation of managers that the firm will have the freedom not to fully repay. In our model we will associate the degree to which the budget constraint related to debt is soft with the parameter I_t^i. The softness of the budget constraint[3] is exogenous in our model and the value of the parameter I_t^i ranges between zero and unity, $I_t^i \in \langle 0;1 \rangle$. The parameter I_t^i can be also understood as a "discount" factor, which managers associate to debt due to the SBC. $I_t^i\ B_t^i$ represents the value of debt B_t^i, which the firm expects that it will be forced to repay.

[3] It is assumed that the degree of softness of the budget constraint is firm specific (e.g. the firm is "too big to fail" or it has strategic importance for the state or financing institution) rather than a feature of the given economic environment. Therefore the notation is and not, although both are possible depending on the source of the SBC.

The decrease in I_t^i will simulate the increase in the degree of softness of the budget constraint. Under standard market conditions I_t^i is equal to one and the more moral hazard there is in the firm's decision-making due to the SBC, the closer the parameter I_t^i will be to zero.

As the net worth position is dependent on the level of debt which the firm has to repay (see the equation (1) below), the lower I_t^i may artificially keep the net worth positive and the company will not go into bankruptcy. The A.5 therefore still holds, even though under standard market conditions ($I_t^i = 1$) the firm would go bankrupt.

A.8 For the sake of simplicity we assume that the overall price level does not change between the periods $P_t = P_{t+1}$ and we will denote the price level P as independent of time. The only source of risk is thus the uncertain price of output $\tilde{P}_t^i$.

A.9 We assume that output q_{t-1}^i has zero supply elasticity and must be sold in its entirety at the beginning of the following period t, when it becomes available.

2.2 Bankruptcy Conditions

The nominal net worth position A_t^i is defined as all assets minus all liabilities. It determines the solvency of a firm and it results from the production and financing decisions made in period *t*–1.

$$A_t^i = P_t^i q_{t-1}^i - (1 + r_{t-1}^i) B_{t-1}^i I_t^i \tag{1}$$

The nominal level of debt B_{t-1}^i was utilized to pay for the inputs that were required for producing q_{t-1}^i . The contractual interest rate of debt is r_{t-1}^i , the contractual repayment owed to debt-holders is $(1 + r_{t-1}^i) B_{t-1}^i$ and the degree to which the budget constraint is soft is I_t^i . The nominal net worth position is also affected by the price P_t^i at which the firm sells the inherited output q_{t-1}^i .

The level of external financing T_t^i needed for production in period *t* depends on the difference between the level of costs and net worth position. Since the cost function is defined in real terms we can write

$$T_t^i = Pc(q_t^i) - A_t^i \tag{2}$$

The firm has two options for supplementing its own resources (net worth) in financing the production: through debt B_t^i or through issuing new equity S_t^i . As noted earlier in A.1, the proportion of new shares in total external financing (and thus of bonds) is given by the parameter *s*.

In the second stage, after the prices in period *t*+1 are revealed, the firm will go bankrupt if it is obliged repay more than its income allows it to repay. Using assumption A.6 and equation (1), we get:

$$P_{t+1}^i q_t^i < (1+r_t^i) B_t^i I_t^i \tag{3}$$

By rewriting and using A.1, A.5 and equation (2) we get the same condition as:

$$\tilde{P}_{t+1}^i q_t^i < (1+r_t^i) I_t^i (Pc(q_t^i) - A_t^i - S_t^i) \tag{4}$$

Dividing the equation by q_t^i we get:

$$\tilde{P}_{t+1}^i < \frac{(1+r_t^i) I_t^i (Pc(q_t^i) - A_t^i - S_t^i)}{q_t^i} = \overline{P}_{t+1}^i \tag{5}$$

where $\overline{P}_{t+1}^i$ is the lowest sectoral price in period t+1 at which the firm is solvent and it shows the extent of changes of relative prices the company is able to absorb without becoming bankrupt. The company is more resistant to bankruptcy risk (i.e. it has lower $\overline{P}_{t+1}^i$) with an increase of its net worth position and the level of new shares issued in period t, and a decrease in I_t^i and decrease in the contractual interest rate.

2.3 Probability of Bankruptcy

A.10 We assume that firms can borrow as much as they wish as long as the lender gets the expected return amounting to $1+r_t^f$, where r_t^f is the risk-free interest rate. The lender takes into account not only the regular repayments amounting to $1+r_t^i$ in the situations when a firm is solvent – if $\tilde{P}_{t+1}^i \geq \overline{P}_{t+1}^i$ –, but also a return $1+r_t^{i,BT}$ when a firm is insolvent – if $\tilde{P}_{t+1}^i < \overline{P}_{t+1}^i$.

Therefore the contractual interest rate is given by the following equation:

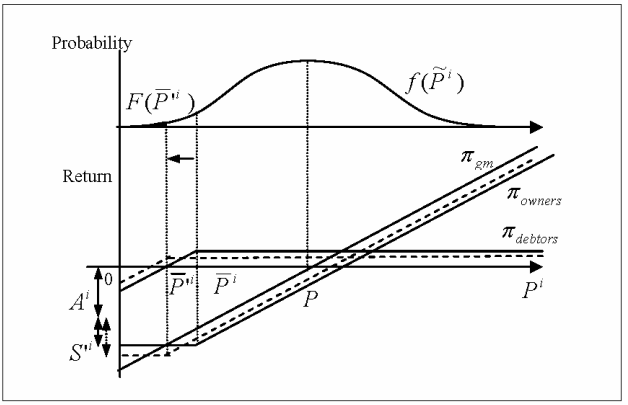
$$1+r_t^f = (1+r_t^i)(1-F(\overline{P}_{t+1}^i)) + F(\overline{P}_{t+1}^i)(1+r_t^{i,BT}) \tag{6}$$

where $F(.)$ is the probability distribution of the sectoral price and $F(\overline{P}_{t+1}^i)$ is defined as the probability of bankruptcy. The individual firm pays via its contractual interest rate not only the opportunity costs (risk-free interest rate), but it also pays to the lender for the risk that the firm will go into bankruptcy and that the bondholder will get only the return $(1+r_t^{i,BT})$, where $r_t^{i,BT} \in <-1; r_t^i>$. As a consequence, the contractual interest rate is higher than, and increases with, the risk-free interest rate, increasing with the probability of bankruptcy and decreasing with the amount the lender can get during the bankruptcy procedure $r_t^{i,BT}$.[4]

The probability of bankruptcy $F(\overline{P}_{t+1}^i) = F(\overline{P}(q_t^i, A_t^i, S_t^i, I_t^i, r_t^f))$ is positively dependent on $\overline{P}_{t+1}^i$, which causes it to increase along with the increase of the interest rate and decrease with the initial net worth position, the amount of new shares and softness of the budget constraint.

[4] The model could be further extended by making the $r_t^{i,BT}$ dependent on other parameters such as random price, level of debt and output level. This extension would not alter the main results of the model and for the sake of simplicity it is thus assumed that $r_t^{i,BT}$ is a constant.

FIGURE 1 The Effect of New Shares on the Probability of Bankruptcy



The shape of the distribution function $F(.)$ expresses the degree of uncertainty concerning future prices. The relationship between these parameters and probability of bankruptcy can be easily graphically illustrated. On the horizontal axis is the uncertain sectoral price $\tilde{P}^i_{t+1}$ and on the vertical axis we plot:

– the firm's gross margin defined as $\pi_{gm} = P^i_{t+1} q^i_t - Pc(q^i_t)$,

– the return of owners defined as $\pi_{owners} = P^i_{t+1} q^i_t - Pc(q^i_t) - r^i_t B^i_t$, if $\tilde{P}^i_{t+1} \geq \overline{P}^i_{t+1}$ (the firm is solvent) or as $\pi_{owners} = -A^i_t - S^i_t$, if $\tilde{P}^i_{t+1} < \overline{P}^i_{t+1}$ (the firm is bankrupt),

– the return to bondholders defined as $\pi_{debtor} = r^i_t B^i_t$, if $\tilde{P}^i_{t+1} \geq \overline{P}^i_{t+1}$ or as $\pi_{debtor} = P^i_{t+1} q^i_t - Pc(q^i_t) + A^i_t + S^i_t$, if $\tilde{P}^i_{t+1} < \overline{P}^i_{t+1}$.

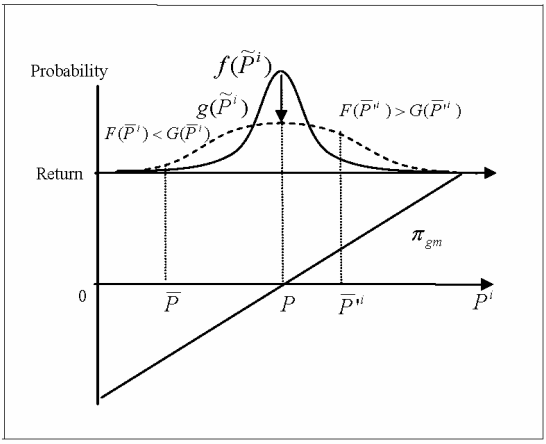
In the upper part of *Figure 1* we have plotted the probability density *f*(.) of the sectoral price, which illustrates the uncertainty.[5] The area under the density function up to the solvency price $\overline{P}^i_{t+1}$ measures the probability of bankruptcy $F(\overline{P}^i_{t+1})$. For illustrative purposes, let's take a look at the effect of new shares and increased uncertainty for a given q^i_t.

An increase of new shares S^i_t has two effects. It increases the owner's liabilities and it works as a substitute for bond-financing, which is reduced. Consequently, the contractual interest rate and thus repayments to debtors decrease for all levels of relative prices – π_{debtor} shifts down, π_{owners} shifts to the left. Both effects lead to a decrease of the probability of bankruptcy.

The shape of density function *f*(.) also influences the size of the area illustrating the probability of bankruptcy $F(\overline{P}^i_{t+1})$. Let us define an increase in price uncertainty as a "stretching" of the probability distribution around a constant mean (Sand-

[5] The distribution function of output prices is assumed to have a shape similar to normal distribution with the mean equal to the overall price level, the lower boundary equal to zero and the upper boundary equal to double the price level.

FUGURE 2 Impact of Increased Price Uncertainty on the Probability of Bankruptcy



mo, 1971). In *Figure 2* the probability function $g(.)$ corresponds to the riskier distribution of prices. We can observe opposite effects of uncertainty depending on the firm's financial situation. If the solvency level of the output price is lower than the overall price level, then riskier distribution increases the probability of bankruptcy – $F(\bar{P}) < G(\bar{P})$. In other words, if it is more probable that the firm will survive than that it will go bankrupt, then price uncertainty is a bad thing from the perspective of the firm's survival. In the opposite situation, when $\bar{P}'$ is very high and the firm has a poor chance for survival, the increase in price uncertainty may help the firm out of trouble via the increased probability of extremely high prices. The probability of bankruptcy thus decreases for riskier distribution of prices – $F(\bar{P}) > G(\bar{P})$.

2.4 The Firm's Objective Function and First Order Conditions

We focus on a firm with divided ownership and control and therefore consider managers as the primary decision-makers in the firm. The managers select the output level so as to maximize expected profit of the operational firm (i.e., total sales revenues minus repayments to lenders and opportunity costs of new equity capital) minus their personal costs of possible bankruptcy.

The objective function with these features can be expressed in the following way:

$$\underset{q\geq 0}{MAX}[P_{t+1}^{i,E} q_t^i - (1+r_t^i) I_t^i B_t^i - (1+r_t^f) S_t^i - K_t^i F(\bar{P}_{t+1}^i)] \tag{7}$$

In compliance with Greenwald and Stiglitz (1993a) the bankruptcy costs are defined as the costs of bankruptcy K_t^i multiplied by the probability of bankruptcy. Bankruptcy costs represent the negative attitude of managers towards bankruptcy. They are subject to an agency agreement that rewards them with a share of profits, but they have to bear a large penalty in the event of bankruptcy (stigma of an unsuccessful manager).

A.11 We assume that the bankruptcy costs increase with the size of the firm. The bigger the firm is, the more managers (and employees) are involved, whose loss of position and income would increase with the firm's size. We will use quantity as the proxy variable for the firm's size as it is the only scale variable in the model and thus the bankruptcy costs are defined as follows:

$$K_t^i = Kq_t^i \tag{8}$$

The bankruptcy costs are linearly related to the firm's level of production, where K is the sensitivity of managers to bankruptcy.[6]

We can rewrite the objective function as:

$$\underset{q\geq 0}{MAX}[P_{t+1}^{i,E} q_t^i - (1+r_t^i) I_t^i (Pc(q_t^i) - A_t^i - S_t^i) - (1+r_t^f) S_t^i - Kq_t^i F(\overline{P}_{t+1}^i)] \tag{9}$$

If the above-stated assumptions are fulfilled, then the firm's behavior in terms of its real output is determined by interest rates, softness of the budget constraint, technology, the firm's ability to issue new shares, equity, real bankruptcy costs and relative price uncertainty.

After derivation of equation (9) according to q we get the first order equilibrium condition

$$P_{t+1}^{i,E} - (1+r_t^i) I_t^i \left(Pc'(q_t^{i*}) - s\frac{\partial T_t^i}{\partial q_t^i} \right) - (1+r_t^f) s \frac{\partial T_t^i}{\partial q_t^i} - \eta_t^i = 0 \tag{10}$$

where η_t^i is the marginal bankruptcy costs of firm i in period t:

$$\eta_t^i = KF(\overline{P}_{t+1}^i) + Kq_t^i f(\overline{P}_{t+1}^i) \frac{\partial \overline{P}_{t+1}^i}{\partial q_t^i} \tag{11}$$

$\eta_t^i = \eta(K, q_t^i, A_t^i, S_t^i, I_t^i, r_t^i)$ can be understood as risk premium for additional production.

3. The Firm's Supply under Uncertainty

The equilibrium condition (12) below is the central outcome of our model. At first we will try to show that the resulting behavior is a generalization of the existing concepts of a firm under uncertainty. Consequently we will perform a sensitivity analysis of the optimal output on the major parameters. This exercise will allow us to devise nine propositions about the determinants of the firm's attitude to risk and the impact on the firm's behavior.

Using derivation of equation (2) $\frac{\partial T_t^i}{\partial q_t^i} = Pc'(q_t^i)$ we can rewrite the equilibrium condition (11) so that:

$$P_{t+1}^{i,E} = Pc'(q_t^{i*})[(1-s) I_t^i (1+r_t^i) + s(1+r_t^f)] + \eta_t^i \tag{12}$$

The output is selected so as to equalize price on one hand and on the other

[6] Besides this reasoning, the dependence of bankruptcy costs on the size of a firm is important from the technical point of view of this model as with bankruptcy costs having been fixed, they could be easily mitigated through expansion of production.

hand (1) marginal costs that include the costs of financing weighted for the relative proportion of new shares and debt and (2) the personal bankruptcy costs.

The first question which is naturally raised by the introduction of price uncertainty and financing issues is how the optimal output compares with the well-known neoclassical solution. The traditional neoclassical analysis says that a firm maximizes its profit and therefore that firm produces up to the point when the market price is equal to the present value of the marginal costs. In terms of our model it is the situation when:

– the firm faces a hard budget constraint ($I_t^i = 1$) and

– the costs of production expansion can be fully financed through new shares ($s = 1$).

As the event of bankruptcy is possible only when the company is at least partially financed by debt, the risk premium η_t^i is equal to zero. We can then write:

$$P_{t+1}^{i,E} = Pc'(q_t^{i*})(1 + r_t^f) \tag{13}$$

This outcome is the traditional neoclassical result taking into account the fact that the costs are paid one period before the output is received and opportunity costs must be included.

The second situation of interest is when a firm faces a hard budget constraint (HBC, $I_t^i = 1$) and it is partly financed through debt. The managers choose the level of output according to the following equation:

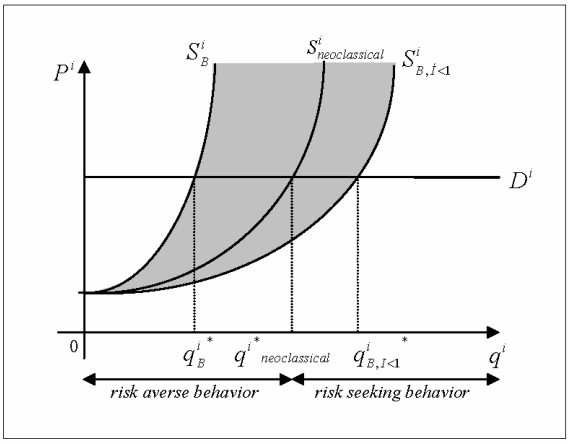
$$P_{t+1}^{i,E} = Pc'(q_t^{i*})[(1-s)(1+r_t^i) + s(1+r_t^f)] + \eta_t^i \tag{14}$$

The costs of debt r_t^i and the opportunity costs of new shares r_t^f are weighted according to their relative proportion to give the total costs of financing. As the contractual interest rate related to debt is higher than the risk-free interest rate (equation 6), better access to the capital market approximated by the exogenous parameter *s* reduces the costs of financing. It is interesting to notice that to the extent a firm can issue new shares on the capital market ($s \rightarrow 1$) in order to finance its additional production, the marginal costs of production and financing move towards the level of marginal costs related to a traditional risk-neutral firm as we have shown above.

The item η_t^i is another departure from the traditional risk-neutral neoclassical equilibrium. The bond-financed or co-financed firm has to face the risk of bankruptcy. As a consequence, the marginal bankruptcy costs η_t^i (risk premium) are positive. The impact of bankruptcy risk is thus a restriction of the optimal output level.

Greenwald and Stiglitz (1993a) model a situation when a firm is solely bond-financed ($s = 0$), faces an HBC and pays a higher contractual interest rate for the overall financial needs $T_t^i = B_t^i$. At the same time the risk premium with solely bond-financing is the maximum of the risk premiums associated with co-financing (bonds and new shares). Both of these effects lead to a firm having the highest possible risk aversion and lowest optimal level of output (supply curve S_B^i in *Figure 3*) compared to our model. Here, new shares work as a buffer to absorb risks. The more the firm

FIGURE 3 Firm's Supply under Uncertainty



has access to equity financing, the less risk-averse it will be and the supply curve will shift to the right towards the supply curve associated with a risk-neutral firm.

Let's look at the behavior of a firm which faces an SBC, in which case the probability of fully repaying its debt obligations is lower than unity ($I^i_t < 1$). So far we have assumed it is desirable for managers to search for funding on the capital market relative to loans and that they are constrained by the parameter *s*. If, however, $I^i_t < \frac{1+r^f_t}{1+r^i_t}$, it is preferable for managers to finance production completely via loans, because doing so reduces the costs of financing. Managers also feel less threatened by the firm's debt obligations and thus the bankruptcy risk premium associated with bonds decreases with the softness of the budget constraint.

We can rewrite the equation (12) so that:

$$P^{i,E}_{t+1} = Pc'(q^{i*}_t) I^i_t (1 + r^i_t) + \eta^i_t \tag{15}$$

The softer the budget constraint is, the more the costs of production are "discounted" and the closer the bankruptcy premium is to zero. In an extreme case, when the firm has certainty that it will not have to repay its debt ($I^i_t = 0$), the managers do not include the costs into their considerations about optimal output. Consequently, the supply curve of a firm facing an SBC ($S^i_{B,I<1}$) is shifted to the right and the optimal level of production ($q^{i\,*}_{B,I<1}$) may become even higher than for a neoclassical risk-neutral firm. *Figure 3* graphically illustrates the three situations discussed above.

S^i_B is the supply curve of a firm as understood by Greenwald and Stiglitz (1993a). It is only bond-financed, very risk-averse and produces low levels of output. The inclusion of the possibility of equity financing creates a continuous interval for the firm's supply curve with the upper extreme being a neoclassical risk-neutral firm. The introduction of the SBC has further broadened the model for risk-seeking be-

havior. The managers may put greater weight on the favorable development of prices because they are not so motivated to reduce the chance of failure and, as a result, the firm produces more even relative to the risk-neutral producer. When the degree of softness I_t^i is switched for unity due to, for example, privatization of state banks, the optimization issue changes rapidly. The firm starts to face an HBC and it may cause substantial shrinkage in the firm's supply.

4. The Determinants of Firm's Attitude to Risk and Optimal Output

In this section we will elaborate algebraically how the major parameters influence the firm's attitude to risk and its optimal output.

4.1 Net Worth Position, the Interest Rate and Sensitivity to Bankruptcy

By differentiating the optimality condition (12) according to the net worth position we get:

$$\left[(1-s)I_t^i(1+r_t^i)+s(1+r_t^f)\right] Pc''(q_t^{i*})\frac{\partial q_t^{i*}}{\partial A_t^i} = -Pc'(q_t^{i*})(1-s)I_t^i\frac{\partial r_t^i}{\partial A_t^i} - \frac{\partial \eta_t^i}{\partial A_t^i} \qquad (16)$$

The first item on the right-hand side represents the change in costs of financing due to the change in net worth. The contractual interest rate required by banks is negatively dependent on the net worth position and the proportion of debt on the value of marginal costs is non-negative, and thus the whole first item is positive. The second item on the right-hand side is also positive as the change in the risk premium associated with bankruptcy is negatively dependent on the net worth position, because a stronger net worth position decreases the probability of bankruptcy. Since we assume that marginal costs increase with quantity, we can conclude that the optimal output has to increase with the net worth position ($\frac{\partial q_t^{i*}}{\partial A_t^i} \geq 0$).

Result 1: *Under price uncertainty stronger net worth position decreases the costs of production financing, decreases risk premium associated with bankruptcy and therefore increases the optimal level of production.*

The lower boundary of the contractual interest rate is the risk-free interest rate and the minimum risk premium is zero. There is thus a level of A_t^i above which the decrease of the interest rate and bankruptcy premium due to the increase in A_t^i will be close to zero.

Result 2: *There is certain level of net worth above which the impact of increased net worth has an insignificant positive impact on the optimal level of production.*

The positive relationship between net worth and the level of production is determined also in (Greenwald, Stiglitz, 1990, 1993a). Using the differentiating exercise with respect to the risk-free interest rate and the sensitivity of managers to bankruptcy *K* we can obtain two more results which are also very similar to those of the Greenwald and Stiglitz model.

Result 3: *Under price uncertainty the increase of the risk-free interest rate increases the costs of financing and increases the risk premium associated with bankruptcy and*

therefore decreases the optimal level of production. Moreover, the optimal output decreases faster than that of a risk-neutral firm.

Result 4: *Under price uncertainty the increase of the sensitivity of managers to bankruptcy increases the risk premium associated with bankruptcy and therefore decreases the optimal level of production.*

The results that follow are products of additional features of our model.

4.2 New Shares and Optimal Output

The effect of new shares resembles the situation for the net worth position with one exception. The equation (17) shows that the negative relationship between new shares and costs of financing is conditional on the existence of and HBC. In the case that an SBC is present, the effect of increased *s* is ambiguous. On one hand, the interest rate required from lenders decreases up to risk-free interest rate. On the other hand, if a firm faces an SBC it may be cheaper to rely on loans, because they may not be fully repaid. In such a situation we cannot determine a sign of a change in the costs of financing. In any case, the risk premium decreases with the level of new shares. Thus, for a firm that faces an HBC the optimal level of production is positively dependent on the proportion of new shares in its financing $\frac{\partial q_t^{i*}}{\partial s} \geq 0$.

$$\left[(1-s)I_t^i(1+r_t^i)+s(1+r_t^f)\right] Pc''(q_t^{i*})\frac{\partial q_t^{i*}}{\partial s} = -Pc'(q_t^{i*})\left[(1-s)I_t^i\frac{\partial r_t^i}{\partial s}-(1+r_t^i)I_t^i+(1+r_t^f)\right]-\frac{\partial \eta_t^i}{\partial s} \tag{17}$$

Result 5: *Under price uncertainty and the existence of a hard budget constraint, the increase in financing through new shares decreases the costs of financing and decreases the risk premium associated with bankruptcy, and therefore increases the optimal level of production.*

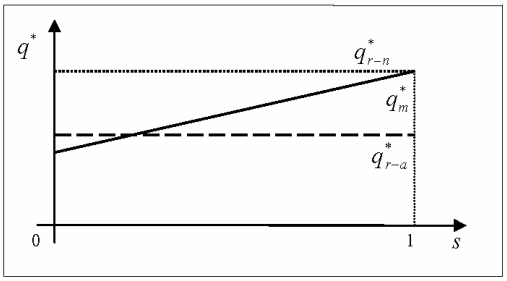
Figure 4 illustrates these relationships and the independence of output from financing in the theory of a risk-neutral firm (q^*_{r-n}) and the traditional theory of a firm under uncertainty (q^*_{r-a} (Sandmo, 1971)). In our model the optimal output q^*_m increases up to the level of production associated with a risk-neutral firm for $s = 1$. We can easily observe that if we take the extreme case used in the models of Greenwald and Stiglitz, where $s = 0$, the level of production under uncertainty will be the lowest possible.

4.3 Price Uncertainty and Optimal Output

The determinants we have analyzed so far influence first the minimal sectoral price level when a firm is solvent $\overline{P}_t^i$ and then through the probability of bankruptcy $F(\overline{P}_t^i)$ they change the contractual interest rate and the risk premium. This time we keep the solvency level of the sectoral price constant and we model the impact of the changes in the variance of the distribution function *F*(.).

It is not straightforward how the effect of increased variance can be formalized in a manner similar to that by which we demonstrated the changes in a firm's

FIGURE 4 New Shares and Optimal Output



risk behavior for other parameters. Let's define the subsidiary function $X(\bar{P}_t^i, \sigma)$, which represents the distribution function for different levels of $\bar{P}_t^i$ and different levels of uncertainty represented by σ. We can imagine the departure from $F(\bar{P}_t^i)$ to $X(\bar{P}_t^i, \sigma)$ as adding another dimension σ, where the probability of bankruptcy depends not only on the solvency level of price $\bar{P}_t^i$ as is the case for $F(\bar{P}_t^i)$, but also on its variance. Therefore, $X(\bar{P}_t^i, \sigma) = F(\bar{P}_t^i)$, if the "variable" variance in the subsidiary probability of bankruptcy is the same as the given variance behind our original probability of bankruptcy.

Let's substitute the originally utilized probability of bankruptcy with the subsidiary one and let's differentiate the optimality condition according to variance. We get:

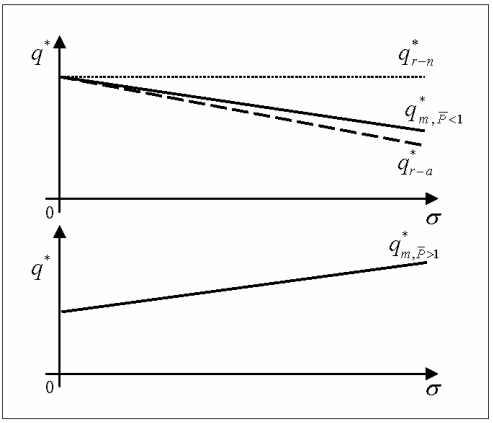
$$\begin{aligned} &\left[(1-s)I_t^i(1+r_t^i)+s(1+r_t^f)\right]Pc''(q_t^{i*})\frac{\partial q_t^{i*}}{\partial \sigma} = \\ &= -Pc'(q_t^{i*})(1-s)I_t^i \frac{\partial r_t^i}{\partial X(\bar{P}_t^i,\sigma)}\frac{\partial X(\bar{P}_t^i,\sigma)}{\partial \sigma} - \frac{\partial \eta_t^i}{\partial X(\bar{P}_t^i,\sigma)}\frac{\partial X(\bar{P}_t^i,\sigma)}{\partial \sigma} \end{aligned} \tag{18}$$

We have already analyzed the relationship between increased uncertainty and the probability of bankruptcy in Section 3 (*Figure 2*). There is a positive relationship between the probability of bankruptcy and uncertainty ($\frac{\partial X(\bar{P}_t^i,\sigma)}{\partial \sigma} > 0$) if the firm's solvency price is lower than the overall price level ($\bar{P}_t^i < P$). A negative relationship is present if $\bar{P}_t^i > P$, because its probability of survival[7] is low and a big variance of prices may help the firm out of trouble.

The increased probability of bankruptcy increases both the interest rate $\frac{\partial r_t^i}{\partial X(\bar{P}_t^i,\sigma)} > 0$ and the risk premium associated with bankruptcy $\frac{\partial \eta_t^i}{\partial X(\bar{P}_t^i,\sigma)} > 0$.

[7] Probability of survival is defined as one minus the probability of bankruptcy.

FIGURE 5 Price Uncertainty and Optimal Output



Therefore the overall impact of uncertainty on the optimal output $\frac{\partial q_t^{i*}}{\partial \sigma}$ depends on the financial situation of the firm.

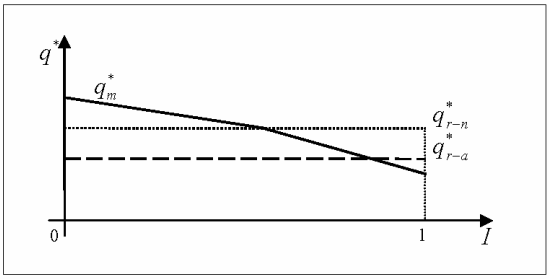
Result 7*: If the probability of bankruptcy is lower than the probability of the firm's survival, then the increase in the price uncertainty increases the costs of financing and the risk premium associated with bankruptcy and therefore decreases the optimal level of production.*

Result 8*: If the probability of bankruptcy is higher than the probability of the firm's survival, then the increase in the price uncertainty increases the optimal level of production.*

In other words, under normal circumstances, if a firm may go bankrupt only in the case of extremely adverse market conditions (low prices), the increased price uncertainty induces the firm to reduce its output level. On the other hand, if the firm has financial troubles, then the managers may welcome the increased uncertainty of its output prices and increase the optimal output in order to increase the chance that the firm will get of trouble. In such a situation the managers have an incentive to gamble their way out of the coming insolvency through risky investments and risky levels of production. (*Figure 5*)

Traditional theories of the firm under price uncertainty demonstrate a decline in production q^*_{r-a} due to uncertainty. The Greenwald and Stiglitz model (1993a) determines a negative relationship between the degree of uncertainty and the optimal output as well. However, in other places Stiglitz (1994) highlights the incentives of insolvent banks to make high-risk loans in order to avoid insolvency and uses the similar logic we explained for the case of our firm. This phenomenon is theoretically explained also in a number of other places. For example, Hlaváček (1999, pp. 106–110) uses his model of a firm which maximizes the probability of its survival instead of profit, and he argues that there are situations which force the firm to behave in a risk-seeking manner even though normally the firm would behave in a risk-averse manner. The immediate threat of a firm's liquidation is taken as an example of such a situation.

FIGURE 6 SBC and Optimal Output



4.4 SBC and Output Behavior

As the final parameter we will analyze the impact of an SBC. There are two effects similar to the preceding variables: the change in financing costs and the change in the risk premium. These two effects will push a firm to the level of production of a risk-neutral firm (similarly as new shares and net worth position). In addition, in an SBC environment managers "discount" the value of costs financed by debt. This effect is represented by the third item on the right-hand side, which is constantly negative. This third effect can push a firm into production levels even higher than those of a risk-neutral firm and ex-post this production expansion can be considered as excessively risky. All three effects lead to a negative relationship between the hardness of the budget constraint and the optimal level of production ($\frac{\partial q_t^{i*}}{\partial I_t^i} \leq 0$).

$$\begin{aligned} &\left[(1-s)I_t^i(1+r_t^i)+s(1+r_t^f)\right] Pc''(q_t^{i*})\frac{\partial q_t^{i*}}{\partial I_t^i} = \\ &= -Pc'(q_t^{i*})(1-s)I_t^i\frac{\partial r_t^i}{\partial I_t^i} - \frac{\partial \eta_t^i}{\partial I_t^i} - Pc'(q_t^{i*})(1-s)(1+r_t^i) \end{aligned} \tag{19}$$

Result 9*: Under price uncertainty, an increase in the softness of the budget constraint decreases the costs of financing, decreases the risk premium associated with bankruptcy and discounts the value of debt perceived by managers, and therefore increases the optimal level of production.*

The increase in optimal output q_m^* with the softness of the budget constraint is faster below the level of production of a risk-neutral firm, because all three effects are present. Above q_{r-n}^* the output q_m^* grows at a slower pace as only the third effect (managers discounting production costs financed by debt) is at work.

5. Conclusions

The attitude to risk directly influences a firm's willingness to produce in the environment of uncertainty. The presence of uncertainty thus changes many of the predictions of the neoclassical theory of the firm under certainty, where the only important determinants of the firm's optimal output are technology and relative prices. This paper aimed to answer the following question: What are the major determinants

TABLE 1 Summary of the Theories of Firm under Uncertainty

	Financing	Attitude to risk	Output behavior
Neoclassical firm	Ø	A: certainty A': risk neutral	P: $p = MC(q^*_{r-n})$
Traditional theory of firm under uncertainty	Ø	A: risk averse	P: $q^*_{r-a} < q^*_{r-n}$
Greenwald and Stiglitz (1993a)	A: B>0 S=0	P: risk averse	P: $q^*_B < q^*_{r-n}$
Soft budget constraint concepts	A: B>0 ∃ SBC	P: risk seeking	P: $q^*_{r-n} < q^*_{SBC}$
This model	A: B>0 S>0 ∃ SBC	P: risk averse risk neutral risk seeking	P: $q^*_m \in < q^*_B ; q^*_{SBC} >$

Note: We denote the assumptions as "A" and the results of the models as "P" (propositions).

of the firm's attitude to risk and therefore of the firm's willingness to produce under uncertainty? In doing so, we have used the following logical framework: financing, attitude to risk and optimal output. A model of a firm has been developed, where the managers are the primary decision-makers and include personal costs of bankruptcy into the profit maximization as originally suggested by Greenwald and Stiglitz (1993a).

Unique to our model is more general approach to the firm's financing possibilities and conditions. Our firm is not restricted to only debt-financing as is assumed in the otherwise-very-inspiring model of Greenwald and Stiglitz (1993a), which implies risk-averse behavior of a firm and a lower optimal output level. We have incorporated other plausible assumptions about a firm's financing, most importantly partial access to the equity market and the possible existence of the soft budget constraint related to debt-financing. The model thus provides comprehensive endogeneisation of the firm's attitude to risk, which ranges from strongly risk--averse behavior to risk-seeking behavior and which to my knowledge is not present in the existing literature. It has been shown that from the perspective of our core question the benchmark models can be understood as special cases in relation to this model (see *Table 1*).

Nine propositions about the determinants of the firm's behavior under uncertainty were devised. Of these, the following considerations are perhaps the most interesting ones. Firstly, firms with a stronger net worth position and with more ready access to the equity market will tend to produce more, other things being equal. The impact of these parameters decreases with their absolute value with the maximum being at the level associated with the traditional risk-neutral firm. Secondly, the price uncertainty has a dual effect depending on the financial condition of the given firm. Under normal circumstances, when the probability of bankruptcy is relatively low, an increase in the price uncertainty will lead to a reduction of output. On the other hand, a firm teetering on the edge of bankruptcy will be tempted by the increased uncertainty towards higher levels of production. Thirdly, the soft budget constraint leads to excessively risky behavior and to the expansion of output. The optimal output of the firm facing a soft budget constraint may even go beyond the neoclassical optimal output level.

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