

# Cobb-Douglas Production Function: The Case of a Converging Economy

Dana HÁJKOVÁ – Czech National Bank and CERGE-EI (dana.hajkova@cnb.cz)

Jaromír HURNÍK – Czech National Bank, Institute for Economic Studies, Faculty of Social Sciences, Charles University, Prague, and Faculty of Economics and Public Administration, University of Economics, Prague (jaromir.hurnik@cnb.cz).\*

## *Abstract*

*The Cobb-Douglas production function is often used to analyse the supply-side performance and measurement of a country's productive potential. This functional form, however, includes the assumption of a constant share of labor in output, which may be too restrictive for a converging country. For example, labor share in the Czech Republic gradually increased over the last decade. In this paper, we test whether this fact renders the application of the Cobb-Douglas production function unreliable for the Czech economy. We apply a more general form of production function and allow labor share to develop according to the empirical data. For the period 1995–2005, we do not find significant difference between the calculation of the supply side of the Czech economy by the Cobb-Douglas production function and a more general production function.*

## 1. Introduction

The performance of the supply side of an economy is often identified with the growth rate of potential output. Potential output is not observed in reality, however, and has to be approximated. The use of the production function method for the measurement of potential output growth takes into account different sources of an economy's productive capacity, namely the contributions of labour, capital and total factor productivity, the latter containing information about technological and allocative efficiency and hence about the supply-side functioning.<sup>1</sup>

Using the production function, one can discuss changes in the supply-side performance on the basis of the observed simultaneous developments in the quantity of labor, capital and total factor productivity. For instance, an increase in the rate of capital growth accompanied by a rise in trend total factor productivity may signalize some improvement in the supply-side performance. Observing an increase in the rate of the capital growth while trend total factor productivity stagnates, one can, in contrast, deduce that the supply side is functioning ineffectively. The production function thus represents a useful and powerful tool for the macroeconomic analysis and evaluation of the governmental structural policies.

The practical application of the production function method requires making certain assumptions, particularly on the functional form of the production technology, returns to scale, and characteristics of the technological progress, as well as of

\* The views expressed are the views of the authors and may not correspond with the views of the Czech National Bank. The authors would like to thank to anonymous referees for helpful comments

<sup>1</sup> This method is regularly used, for instance, by the European Commission (Denis et al., 2006) and by the OECD (Beffy et al., 2006) to assess the productive potential of countries.

the functioning of markets. The neo-classical two-factor Cobb-Douglas production function with Hicks-neutral technology is frequently used,<sup>2</sup> including the assumptions of positive and diminishing marginal products with respect to inputs of labor and capital, constant returns to scale, no unobserved inputs and perfect competition. These assumptions restrict the elasticity of output with respect to labor and capital to values between zero and one and their sum to being equal to one. Given the assumptions, the theoretical technological coefficients are then in practice approximated with the help of the income share of labor in produced output. Since the technological coefficients are assumed to be stable, the share of factors in production should be stable in time. Furthermore, if there is no presumption that the aggregate technology would differ across countries, the labor shares should also be roughly similar across countries.

The empirical evidence is not fully consistent with this proposition. Labor shares do differ across countries and develop in time. Harrison (2002) shows that labor shares of countries in a panel based on United Nations data are rather volatile over time. Blanchard (1997) finds a declining labor share in four continental European countries (Germany, France, Italy and Spain) between 1970 and 1996. Serres et al. (2001) explain the decline in the labor shares in the United States, Germany, France, Italy, the Netherlands and Belgium between 1975 and 1995 to be mostly the result of the shift of economic activity towards sectors with lower labor intensity.<sup>3</sup> Morel (2006) also finds a considerably falling labor share in Canada between 1998 and 2004.<sup>4</sup> Gollin (2002) argues that proper adjustment for the income of self-employed workers leads to an important convergence of the labor share estimates across countries, but such an adjustment cannot be expected to fully eliminate the trend in labor share for an individual country.

While the assumption of a constant labor share is necessary for the use of the Cobb-Douglas production function method, the abstraction from the labor share movement over time ((and across countries, as in (Denis et al., 2006), for instance)) may be debatable. In particular, it may seem that the assumption of a constant labor share is not fully appropriate for a converging economy which has not yet reached its steady-state and in which GDP per capita is not growing at the rate of technological progress. The argument might be even more important if structural policies with long-lasting effects were introduced during the convergence process. On the other hand, questioning the assumptions of the Cobb-Douglas production function would make this tool unusable for calculating the level of potential product.<sup>5</sup>

Since the labor share has been increasing in the Czech Republic over the past ten years, it may make any supply-side analysis results assuming constant labor share for this country untrustworthy. Dybczak, Flek, Hájková and Hurník (2006) (herein-

<sup>2</sup> I.e., a production function with elasticity of substitution between capital and labor being constant and equal to one.

<sup>3</sup> The last two mentioned works equally include the imputed income of the self-employed in the measure of labor income.

<sup>4</sup> Morel (2006) argues that this partly reflects a sectoral bias which was caused by a boost in commodity prices and a consequent decrease in the weight of labor-intensive industries. An important part of the decrease is, however, explained by the decrease in the labor share in the manufacturing sector linked to increasing labor productivity, decreasing union density and increasing openness to trade.

<sup>5</sup> We would like to thank our anonymous referee for pointing out this fact.

after referred to as “DFHH”) used this assumption of constant labor share and discussed the consequences of raising the labor share on estimation of potential output growth only briefly. We therefore undertake here a more detailed analysis of the impact of the changing labor share of income. In order to do so, we use the DFHH data and analyze the supply-side developments with a general form of production function. Our point is to test the reliability of the use of the Cobb-Douglas production function methodology for the supply-side analysis in general and in a converging economy in particular.

The rest of the paper is organised as follows: Section 2 briefly describes the Cobb-Douglas production function. Section 3 introduces a more general form of production function, computes time-varying factor shares and recalculates the results using the same data set as was used in DFHH. Section 4 makes the relevant comparison and Section 5 concludes.

## 2. The Cobb-Douglas Production Function<sup>6</sup>

DFHH basically follow *Giorno et al. (1995)* and model the potential output using the two factor Cobb-Douglas production function with Hicks-neutral technology:

$$Y_t = A_t \cdot L_t^\alpha \cdot K_t^\beta \quad (2.1)$$

where  $Y$ ,  $L$ ,  $K$  and  $A$  are real GDP, labor input, capital input and the total factor productivity (TFP) level respectively. There are two methodological advances used by DFHH that are worth mentioning. First, they incorporate the concept of a time-varying NAIRU into the production function approach, to derive “potential labor”, and second, they newly introduce the concept of “capital services” to adjust the aggregate capital stock with respect to the real productive impact of this factor input.

The specification of the production function is a special case of the constant-elasticity-of-substitution production function (CES), with the elasticity of substitution equal to one and with the usual theoretical assumptions used in the empirical literature.<sup>7</sup> As mentioned earlier, positive and diminishing marginal products of each input ( $L$ ,  $K$ ) are assumed. This restricts both  $\alpha$  and  $\beta$  to values between 0 and 1. Second, returns to scale are assumed to be constant, i.e.  $\beta = (1 - \alpha)$ .

When applying the Cobb-Douglas production function, DFHH as well as *Giorno et al. (1995)* and others assume the parameter  $\alpha$  (and, hence,  $\beta$ ) to be constant over time.<sup>8</sup> In theory, if the factor markets are competitive, then the marginal product of each input equals its factor price, so  $\partial Y/\partial L = w$  and  $\partial Y/\partial K = r$ , where  $Y$  is output,  $L$  and  $K$  labor and capital inputs,  $w$  and  $r$  are the wage rate and the rental rate of capital respectively. For the Cobb-Douglas production function,  $\partial Y/\partial L = \alpha Y/L = w$ . Under the assumption of constant returns to scale in capital and labor,  $rK + wL = Y$  and

<sup>6</sup> We refer to (*Dybczak, Flek, Hájková, Hurník, 2006*) for additional details on their method.

<sup>7</sup> See, for example (*Grossman, Helpman, 1991*), (*Barro, Sala-i-Martin, 2004*), (*Giorno et al., 1995*), (*Barro, 1998*) or (*Scacciavillani, Swagel, 1999*) for a more detailed discussion as well as criticism of the standard assumptions used in modeling the production function.

<sup>8</sup> *Denis et al. (2006)* calculate the potential output for the EU countries and assume the same specification of the Cobb-Douglas function for all countries with the mean labor share of the EU-15 over 1960–2003 of 0.63 taken as the value of  $\alpha$ .

the capital share  $\beta = \frac{rK}{Y}$  equals the complement to one of the labor share  $\alpha = \frac{wL}{Y}$ .

DFHH calculate the empirical value of parameter  $\alpha$  as the average over the analysed period of the observed ratio of total labor costs and gross value added (i.e. GDP adjusted for net indirect taxes and subsidies),  $\alpha_t$ , as defined in (2.2):

$$\alpha_t = \frac{TLC_t \cdot L_t}{GVA_t} \quad (2.2)$$

where  $TLC_t$  represents total nominal labor cost per employee adjusted for hours worked,  $L_t$  stands for total employment (including self-employment and adjusted for hours worked) and  $GVA_t$  is the economy's gross value added at current prices (i.e. GDP net of indirect taxes and subsidies). Parameter  $\beta$  then simply equals  $(1-\alpha)$ .<sup>9</sup>

The TFP level (corresponding to the parameter  $A$  in equation 2.1) cannot be measured directly. Therefore a so-called *gross* TFP is first estimated. It consists of two parts: (i) the long-run trend of total factor productivity ( $A^*$ ); and (ii) a short-term fluctuation, which is assumed to correspond to the business cycle. Rewriting equation 2.1, the level of gross TFP for each period is given by 2.3 (regardless of the sustainability of its amount in the long term):

$$A_t = \left[ \frac{Y_t}{L_t^\alpha \cdot K_t^{1-\alpha}} \right] \quad (2.3)$$

where  $Y$  is real GDP,  $L$  is the labor input measured by total employment adjusted for hours worked and  $K$  is the capital input measured by capital services. Then, the trend of  $A$  is assumed to represent the long-run trend TFP ( $A^*$ ) and deviations from this trend are assumed to be short-term fluctuations.<sup>10</sup> It is a known fact that the results are sensitive to the smoothing techniques used to find the trend gross total factor productivity. DFHH therefore employ the Band-Pass filter suggested by Baxter and King (1995) and Christiano and Fitzgerald (1999) instead of the commonly used Hodrick-Prescott filter for their trend estimation.

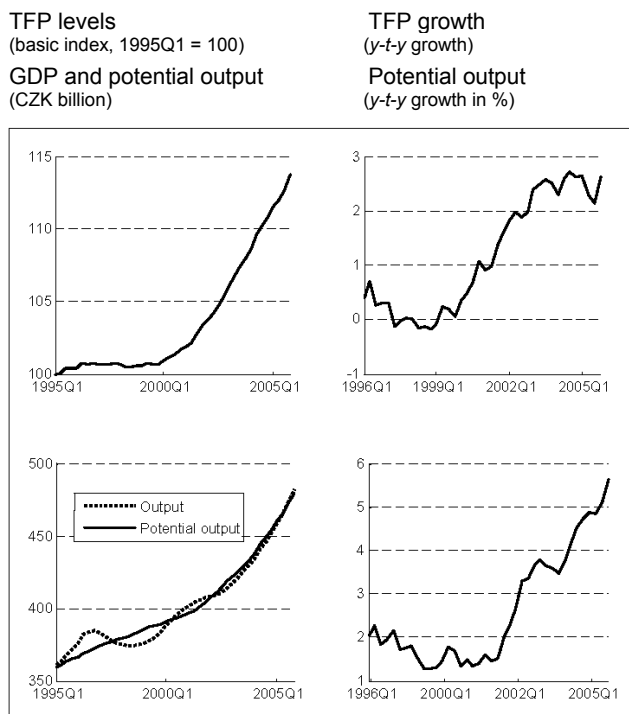
Potential employment ( $L^*$ ) is the level of employment which can be sustained without inducing additional inflationary pressures, i.e.:

$$L^* = \hat{L} \cdot (1 - NAIRU) \quad (2.4)$$

<sup>9</sup> All data come from the national accounts. In theory, the numerator of the share measures the total labor income in the economy. While the income in dependent employment is considered to be well measured, the income of self-employed workers is contained in the mixed income category that includes the income of capital and labor in the unincorporated business sector (for the Czech Republic, the available category comprises both mixed income and gross operational profit). Therefore, one has to assume the "wages" of the self-employed. The method employed here is, as in, e.g., (Serres et al., 2001) and (Gollin, 2002), to impute the average total labor cost per employee to the average self-employed worker.

<sup>10</sup> The variations in the measure of TFP include not only pure changes in technology, but also other factors of improving aggregate productivity, such as improvements in allocative efficiency as well as the existence of mark-ups or technological spillovers (see also Sections 3 and 4 for a discussion).

FIGURE 1 Trend Total Factor Productivity and Potential Output (1995Q1–2005Q4)



Source: (DFHH, 2006)

where  $\hat{L}$  represents the labor force<sup>11</sup> and the term NAIRU stands for the non-accelerating inflation rate of unemployment. To derive the NAIRU, DFHH use the model developed for the Czech economy by Hurník and Navrátil (2005).<sup>12</sup> The measure of capital services is experimentally derived from the measure of existing productive capital stock.<sup>13</sup>

Finally, considering the measure of capital services for  $K$ ,<sup>14</sup> the trend total factor productivity  $A^*$  and potential employment  $L^*$ , they obtain the following equation, which captures the determinants of potential output:

$$Y_t^* = A_t^* \cdot L_t^{*\alpha} \cdot K_t^\beta \quad (2.5)$$

Figure 1 shows the basic results of DFHH that are relevant for our analysis.

The estimated potential output growth is not of primary importance for the economic inference, however. Indeed, given the technique used, the potential output is always a smoothed version of the observed output. This is even more evident when

<sup>11</sup> The labor force is represented by the economically active population, i.e. all persons aged 15 years or over who are classified as employed or unemployed according to the ILO methodology.

<sup>12</sup> See this source for a detailed description of the model applied to estimate the time-varying NAIRU.

<sup>13</sup> See (Hájková, 2006) for a detailed description of the calculation of the measure of capital services.

<sup>14</sup> Since the capital services measure does not account for capacity utilisation, it can be taken as a proxy for the potential productive contribution of capital.

TABLE1 Decomposition of Potential Output Growth (average of  $q$ - $t$ - $q$  annualised growth)

	Potential output (%)	Contribution to growth		
		TFP ( $A^*$ ) (% p.p)	Potential labor ( $L^*$ ) (p.p.)	Capital services ( $K$ ) (p.p.)
1995	2.2	0.6	0.2	1.6
1996	2.0	0.3	-0.1	1.7
1997	1.8	0.0	0.0	1.8
1998	1.3	-0.2	0.0	1.4
1999	1.3	0.0	-0.2	1.4
2000	1.6	1.1	-0.7	1.2
2001	2.2	1.6	-0.5	1.2
2002	3.6	2.0	0.2	1.4
2003	3.4	2.5	-0.4	1.3
2004	4.6	2.6	0.1	1.9
2005	5.1	2.2	0.6	2.2
<b>1995–2005</b>	<b>2.7</b>	<b>1.2</b>	<b>-0.1</b>	<b>1.6</b>

Note: Owing to rounding, the sum of the contributions need not be equal to the total.

Source: (DFHH, 2006).

a statistical smoothing technique for the trend in total factor productivity is applied. What really matters for the analysis of the supply side, in fact, is the decomposition of the potential output into the contribution of its determinants and the consequent interpretation. *Table 1* summarizes the relevant contributions as estimated by DFHH.

DFHH<sup>15</sup> interpret the results in the following way: “Given the roughly zero contribution of potential labor in the second half of the 1990s and, after 1995, also a similar contribution of total factor productivity, the main driver of the relatively sluggish potential output growth in that period was the flow of capital services. Although the latter was growing, the zero TFP growth at the same time signals that the investment activity was probably far from effective. As the potential output growth was fully dependent on the growth of just one input, i.e. capital services, this can hardly be viewed as consistent with an efficiently functioning supply side.” And, “After 2001, the annual average contribution of TFP to potential output growth exceeded 2 %. This, along with accelerating capital services and a positive contribution of labor in the last two years, gradually raised the annual growth of potential output to 5 % in 2005. Such an increase in potential output growth, together with the growth in both capital services and TFP, signals more efficient use of resources, i.e. improvements in supply-side performance.”

It is evident that the development of the trend total factor productivity is viewed as critical for the evaluation of the supply-side performance.<sup>16</sup> In a nutshell, whereas the second half of the 1990s is evaluated as a period of an inefficiently working supply side, given the stagnation of the trend total factor productivity, the pe-

<sup>15</sup> (DFHH, 2006, pp. 15–16)

<sup>16</sup> It is worth recalling that in a growth accounting exercise, the contributions of factors do not identify the ultimate sources of growth. Since, in theory, changes of capital are endogenous to technological change, the contribution of technology in growth accounting underestimates the full effect of technological change on output. See (Barro, Sala-i-Martin, 2004, pp. 457–460) for a detailed description.

riod from 2001 onwards is evaluated as a period of more efficient use of resources simply because of the positive growth of the trend total factor productivity. In Sections 3 and 4 we evaluate whether this conclusion holds even if we apply a more general form of the production function.

It is worth mentioning that, similarly to the results of DFHH, Hájek (2006) comes to the conclusion that the growth of the trend total factor productivity reached 0.7 % in the period 1992–1998 and 2.2 % in the period 1999–2004.<sup>17</sup> Though quantitatively similar, the results of Hájek (2006) are, however, not directly comparable, since Hájek (2006) employs the growth accounting approach that differs in several aspects from the methodology of DFHH. First, it does not work explicitly with the concept of potential labor and NAIRU. Second, it does not split the gross total factor productivity into the trend total factor productivity and the short-term fluctuations. It therefore seems that the results of Hájek (2006) may be significantly influenced by business cycle fluctuations. In contrast, given the methodology of growth accounting, the results of Hájek (2006) are not based on the assumption of a constant labor share and the Thörnqvist index is used instead. Finally, Hájek (2006) employs the measure of capital stock to account for the productive contribution of capital. Taking all these into account, one should be aware that the results of Hájek (2006) and DFHH may be similar rather accidentally.

### 3. Labor Share: A Case of a Converging Economy

The usual assumption of a constant labor share may not be fully convenient for an economy that has not yet reached its steady state and is still in the progress of economic convergence, which we believe characterizes the Czech economy. The question we therefore ask is to which extent the assumption of a constant labor share influences the results.

In our alternative setting, we abandon the assumption of unit elasticity of substitution between labor and capital and adopt a more general form of production function that would allow  $\alpha$  and  $\beta$  to change over time, i.e. assuming that they asymptotically converge to their steady state values. The form of production function is described by (3.1):

$$Y_t = F(\tilde{A}_t, K_t, L_t) \quad (3.1)$$

where  $Y_t$ ,  $K_t$  and  $L_t$  are output, and the inputs of capital and labor are as before. In contrast,  $\tilde{A}$  is not identical to  $A_t$  from equations (2.1) and (2.3). This is because the total factor productivity, which is *de facto* residual, depends on the form of the production function that differs in these two cases. It is clear that the functional form (3.1) does not allow for computing potential output in levels (or, consequently, for identifying the output gap) and therefore could not be consistently used in the DFHH exercise. It is, however, adequate for analysing the functioning of the supply side of the economy and also for computing the growth in potential output.

First, a decomposition of actual economic growth into contributions of labor, capital and total factor productivity via growth accounting is to be carried out. In a standard way ((see, e.g., (Barro, Sala-i-Martin, 2004)), after taking logarithms and derivatives with respect to time of equation (3.1), one gets:

<sup>17</sup> (Hájek, 2006, p. 179)

$$\frac{\dot{Y}}{Y} = \frac{F_A \tilde{A}}{Y} \frac{\dot{\tilde{A}}}{\tilde{A}} + \frac{F_K K}{Y} \frac{\dot{K}}{K} + \frac{F_L L}{Y} \frac{\dot{L}}{L} \quad (3.2)$$

where  $F_K$  and  $F_L$  are the social marginal products of capital and labor, respectively.

The part of growth ascribed directly to the change in technology is  $g = \frac{F_A \tilde{A}}{Y} \frac{\dot{\tilde{A}}}{\tilde{A}}$ . For

the following analysis we assume that the technology is Hicks-neutral, which implies  $g = \frac{\dot{\tilde{A}}}{\tilde{A}}$ . If the markets are competitive, i.e. the factors are paid their social marginal

products,  $F_K$  is equivalent to the rental price of capital,  $r$ , and  $F_L$  is equivalent to the wage rate,  $w$ , which should be observable. Then the contribution of total factor productivity can be measured as

$$g = \frac{\dot{Y}}{Y} - \frac{rK}{Y} \frac{\dot{K}}{K} - \frac{wL}{Y} \frac{\dot{L}}{L} \quad (3.3)$$

This is the well-known Solow residual that is interpreted as the contribution of total factor productivity to economic growth. We maintain the neo-classical assumption of constant returns to scale in capital and labor. This implies that the capital share is  $\beta = \frac{rK}{Y}$  and equals the complement to one of the labor share  $\alpha = \frac{wL}{Y}$ .<sup>18</sup>

Since equation (3.3) describes the economy in continuous time, one needs to use the discrete time approximation to implement available statistical data. The index proposed by Thörnqvist (1936) can be used. According to that approach, we measure the direct contribution of total factor productivity to economic growth as:

$$\hat{g} = \ln \frac{Y_t}{Y_{t-1}} - (1 - \hat{\alpha}_t) \ln \frac{K_t}{K_{t-1}} - \hat{\alpha}_t \ln \frac{L_t}{L_{t-1}} \quad (3.4)$$

where  $\hat{g} = \ln \frac{\tilde{A}_t}{\tilde{A}_{t-1}}$ ,  $\hat{\alpha}_t = \frac{1}{2}(\alpha_{t-1} + \alpha_t)$  and  $\alpha_t$  is defined by (2.2).

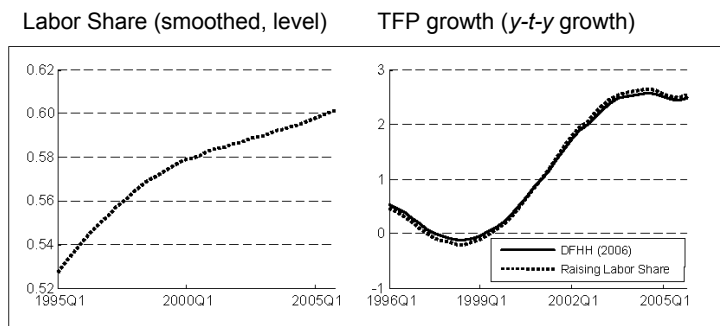
We construct an index of gross TFP ( $\tilde{A}$ ) with 1995Q1 = 1 and smooth it using the Band-Pass filter as in DFHH to obtain the long-term trend TFP ( $\tilde{A}^*$ ). Similarly, to exclude the influence of the business cycle on the labor share series we also apply the Band-Pass filter to smooth out the fluctuations with the business cycle periodicity and obtain  $\alpha^*$ . In practice we exclude all the fluctuations with periodicity higher than six and lower than thirty-two quarters.

Then, we compute the (logarithmic approximation of) potential output growth, using the trend TFP,  $\tilde{A}^*$ , smoothed series of labor share,  $\alpha^*$ , potential employment,  $L^*$ , and capital services:

<sup>18</sup> It is worth mentioning that even the use of (3.2) is not fully consistent for estimating TFP growth rates under changing labor and capital shares. It is implied by the fact that equation (3.2) is strictly valid only if  $F_K$  and  $F_L$  are time independent. This complication is, however, usually omitted. We would like to thank our anonymous referee for pointing out this fact.



FIGURE 2 Labor Share and Trend Total Factor Productivity



Source: (DFHH, 2006); the authors' own computation

$$\ln \frac{Y_t^*}{Y_{t-1}^*} = \ln \frac{\tilde{A}_t^*}{\tilde{A}_{t-1}^*} + (1 - \alpha_t^*) \ln \frac{K_t}{K_{t-1}} + \alpha_t^* \ln \frac{L_t^*}{L_{t-1}^*} \quad (3.5)$$

Whereas the constant value of the  $\alpha$  parameter used by DFHH equals approximately 0.57, *Figure 2* shows the smoothed series for the  $\alpha_t^*$  parameter as well as the resulting growth of the trend total factor productivity,  $\tilde{A}^*$ .<sup>19</sup>

*Figure 2* clearly shows a gradual increase in the labor share over the whole sample. This indicates that the assumption about its ongoing convergence towards a steady state may be relevant. In addition, it is important to recognize that  $\alpha_t^*$  ignores temporary movements in the labor share that do not reflect trend developments on the supply side and that could obscure the picture. The result for TFP, however, shows a limited impact of this change for the story about the supply side.

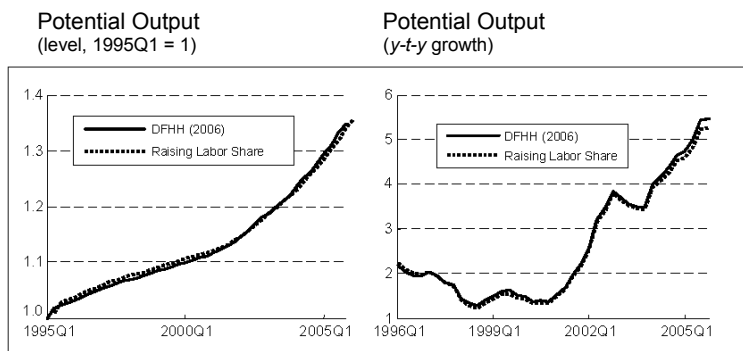
#### 4. Basic Comparisons

The move from a constant share of labor  $\alpha$  to time-varying  $\alpha_t^*$  does not have a significant impact on the estimated trend total factor productivity. For the constant labor share, total factor productivity had a positive contribution to potential output dynamics only after the year 2000 and the same holds for the gradually growing labor share. To put it strictly technically, the observed difference vis-à-vis the case with constant  $\alpha$  is caused by the fact that, given the growth of capital input being higher than that of labor input, allowing the  $\alpha$  parameter to increase makes the contribution of capital to the potential output growth higher in the beginning of the sample (i.e. where values of  $\alpha^*$  are lower than the average value used in the case with constant  $\alpha$ ) and lower thereafter. Given the same values of observed output, the residual total factor productivity will adjust.

Using the equation (3.5), the path of the potential output is easy to calculate for the dynamic development of the labor share. In the left graph of *Figure 3*, there is

<sup>19</sup> The data we use in our analysis are those used by DFHH (2006). See (DFHH, 2006, pp. 10–11, for their detailed description.

FIGURE 3 Potential Output



Source: (DFHH, 2006); the authors' own computation

TABLE 2 Potential Output Growth and Contribution of Total Factor Productivity (average of  $q$ - $t$ - $q$  annualised growth)

	Constant Labor Share (DFHH 2006)		Raising Labor Share (Smoothed for Business Cycle Volatility)	
	Potential Output (%)	TFP (A*) (%, p.p)	Potential Output (%)	TFP (A*) (%, p.p)
1995	2.2	0.6	2.3	0.5
1996	2.0	0.3	2.0	0.2
1997	1.8	0.0	1.7	-0.1
1998	1.3	-0.2	1.3	-0.2
1999	1.3	0.0	1.4	0.2
2000	1.6	1.1	1.3	0.8
2001	2.2	1.6	2.2	1.6
2002	3.6	2.0	3.8	2.2
2003	3.4	2.5	3.4	2.6
2003	4.6	2.6	4.5	2.6
2005	5.5	2.6	5.3	2.5
<b>1995–2005</b>	<b>2.7</b>	<b>1.2</b>	<b>2.7</b>	<b>1.2</b>

the level (index) of potential output as it comes from DFHH (2006) and our analysis. Similarly, the right graph of *Figure 3* shows the growth rate of both versions of the potential output estimations.

The general observation is that the estimates of potential output using a constant labor share and a rising labor share smoothed for the business cycle volatility seem to be almost identical. The right graph of *Figure 3* shows, however, that from 2003 onwards the estimated growth of potential output is slightly lower when the rising labor share is applied. *Table 2* shows in detail that the application of the general production function with an increasing labor share does not change the overall picture of the total factor productivity contribution to the potential output growth. The sizeable change in the relative weights of labor and capital in the exercise thus had a fairly limited impact on measured TFP. Its growth, which as a residual was expected to change most, remains roughly the same as described by DFHH. Our results thus sup-

port the interpretation in DFHH that after a period of an inefficiently functioning supply side in the second half of the 1990s, it improved after 2001.

In general, a stagnant labor force and its increasing share will both slow the potential output growth, given the growth of capital services and estimation of the total factor productivity.<sup>20</sup> If the growth of total factor productivity and capital input are known, the effect of employing a time-variant labor share instead of a constant one on the estimated potential product will be higher the farther the actual value of the labor share is from that constant. The fact that TFP is not observed and is endogenous to the chosen methodology, however, dampens this effect. Still, we have found a small risk that the use of the Cobb-Douglas production function may lead to some overestimation of the potential output growth, though not a dramatic one.

## 5. Conclusions

There is little doubt that the Czech economy has experienced relatively slow annual average growth in potential output of around 2.5–3.0 % over the last ten years with a tendency to accelerate from 2001 onwards. A common story about the low level of economic growth in the second half of 1990s seems to be that of slowly growing or even stagnating trend total factor productivity during that period. This may consequently be interpreted as evidence of clear inefficiency in the supply-side functioning that only from 2001 onward indicates a certain degree of improvement.

A conclusion like the one above is based on the application of standard production function methodology to the Czech economy. At the same time, however, there may be doubts about the robustness of those results, given the shortcomings of the methods used in recent studies, e.g. the applicability of all used assumptions for a converging economy. A particular question we raise in this study is the frequently used assumption of stability of factor shares in total output.

For our exercise, we use the results of (Dybczak et al., 2006), who evaluate the supply-side functioning and calculate the potential product using the Cobb-Douglas production function and thus the assumption of constant factor shares. Applying the general form of the production function to DFHH data, we relax the assumption of a constant labor share and show the impact on the quantitative results and their interpretation. We obtain roughly the same growth of total factor productivity and thus confirm the robustness of the economic story. Our estimated growth of potential output is slightly lower at the end of the sample. Because nothing else changes between the two exercises, the difference in results is caused by raising the share of labor in output while the labor force is growing only slightly. The results show, however, that the sensitivity of results to the changes in the labor share is fairly low.

The purpose of this paper is not to literally challenge the results of recent studies. It aims rather to evaluate the effects of a particular assumption about the production function. Although we find limited influence of this particular assumption, our main message is still that the results of a supply-side analysis based on the production function methodology should be taken with a certain degree of caution.

<sup>20</sup> See *Table 1* for the development of Czech labor force.

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