Price Convergence: What Can the Balassa-Samuelson Model Tell Us?

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1. Introduction

The main purpose of this article is to provide a theoretical foundation to the policy discussions on the adjustment of the price levels and relative prices in Central and Eastern European (CEE) countries to the European Union (EU), which would be consistent with the basic empirical observations – see (Čihák – Holub, 2003). We start with the Balassa-Samuelson model that is a traditional cornerstone of the price convergence theory, and integrate it with the basic neoclassical growth theory for open economies. We show how the model can be extended to the case of more than two goods, and study the implications for relative price structures. By doing this, we develop a consistent – yet easily tractable – framework suitable for analysing many different aspects of the long-run convergence process. We calibrate the model to generate “benchmark convergence” scenarios for the accession economies. Even tough these can not be taken as realistic forecasts of future developments, they can serve as useful inputs into the forecasting process by casting more light on the equilibrium long-run relationships in the economy.

The article is organized as follows. After this introduction, we briefly review the literature on the Balassa-Samuelson model in section 2. We develop our basic model in sections 3 and 4. In particular, section 3 shows the supply side of the model with a three-factor production function, which allows to make the analysis consistent with the open-economy growth theory. Section 4 presents the demand side of the model, providing a deeper look into how the speed of convergence and the allocation of labour across economic sectors are determined by the consumers’ behaviour in interaction with the accumulation of nontradable capital. Section 5 shows

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This article was written as part of the CNB’s research programme. However, it presents authors' own opinions that may not correspond to the views either of the CNB or the IMF. The authors would like to thank to Aleš Bulíř, Vratislav Izák, and Richard Podpiera, participants of two research seminars at the CNB as well as to two anonymous referees for their useful comments, which helped to improve this paper. All remaining errors and omissions are those of the authors.
how the B-S model can be extended for the case when there are many commodity groups with different degrees of tradability. Section 6 concludes.

2. Review of the Literature

The Balassa-Samuelson (B-S) model (Balassa, 1964), (Samuelson, 1964) is one of the cornerstones of the traditional theory of the real equilibrium exchange rate. The key empirical observation underlying the model is that countries with higher productivity in the tradable sector compared with the nontradable sector tend to have higher price levels. The B-S hypothesis states that productivity gains in the tradable sector allow real wages to increase commensurately and, if wages are linked between the tradable and the nontradable sector, prices also increase in the nontradable sector. This leads to an increase in the overall price level in the economy, resulting in a real exchange rate appreciation. The basic version of the B-S model is well described in the literature – e. g. (Mussa, 1984), (Frenkel – Mussa, 1986), (Asea – Corden, 1994), (Samuelson, 1994) –, including by Czech authors – e. g. (Kreidl, 1997), (Čihák – Holub, 2001b).

Until recently, though, the literature did not satisfactorily incorporate both capital and the demand side at once. Obstfeld and Rogoff (1996, pp. 214–216) sketched two examples of possible generalizations of the B-S model, but without formalisation. Asea and Corden (1994) included tradable capital in the model and Asea and Mendoza (1994) presented the model in a general-equilibrium framework; however, neither of the models did include nontradable capital goods and the demand side. Demand side factors have been incorporated in the dependent economy literature, following the pioneering works of Salter (1959), Swan (1960), and other authors. However, as regards capital, most of the dependent economy literature has arbitrarily taken some of the following assumptions: (1) capital is tradable; (2) capital is nontradable; (3) the nontraded sector is capital intensive; or (4) the traded sector is capital intensive. For an example of (1) and (3), see (Bruno, 1982); for an example of (2) and (4), see (Fischer – Frenkel, 1974). These assumptions were criticized as arbitrary or unrealistic (Svensson, 1982), (Fischer – Frenkel, 1974). The first to provide a comprehensive derivation of a dependent-economy model with both traded and nontraded capital were (Brock – Turnovsky, 1994) and (Turnovsky, 1997). However, their model was not fully integrated with the B-S model. Also, due to its high level of generality, it did not allow for closed-form solutions of the key variables.

The model presented in the early parts of this article can be viewed as a modified version of the one by Brock and Turnovsky (1994) and Turnovsky (1997). Compared with their general approach, ours is tailored to addressing monetary policy issues and allows for closed-form solutions for

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1 The basic idea was known already to David Ricardo. Thirty years before Balassa and Samuelson, Harrod (1933) used this observation to explain the international pattern of deviations from purchasing power parity.
the key variables, which can be easily incorporated in other models and simulations. We use the advantages of the closed-form solutions to show how the B-S model can be integrated with the new growth theory and with a theory of the demand side in a comprehensive and consistent way. We illustrate the usefulness of this approach by showing how it can be used in simulations of price level convergence of the CEE countries to the EU.

The literature is also surprisingly thin on generalizing the B-S model to a case with more goods, which would make the B-S model closer to reality. Even though the literature related to the International Comparison Program (ICP) allows for more than two goods – see (Kravis et al., 1982), (Heston – Lipsey, 1999) –, it is focused on measurement issues relating to the ICP rather than on developing the B-S model. Several authors, including Obstfeld and Rogoff (1996, p. 214), suggested that such a generalization of the B-S model is possible, but did not present it (and, to our knowledge, neither did anybody else). In the later parts of this article, therefore, we show how more than two goods can be explicitly incorporated in the B-S model, allowing to study the relative price convergence in a more realistic theoretical setting.

3. Balassa-Samuelson Model with Tradable and Nontradable Capital

The literature usually presents the B-S model with a single-factor production function, in which labour is the only input and its productivity is given by an exogenous productivity coefficient – e. g., (Čihák – Holub, 2001b). Alternatively, the model is presented with a two-factor production function, in which internationally mobile capital is added to the labour input (Obstfeld – Rogoff, 1996) or (Asea – Corden, 1994). However, to gain a better understanding of the relationship between economic growth and price adjustment, it is useful to assume a more elaborate production function, similar to the new literature on economic growth. In particular, the growth literature indicates a key role of accumulation of physical and human capital in economic growth and convergence – e. g., (Barro – Sala-i-Martin, 1995). In this section, therefore, we address the question whether and how much the price adjustment process changes if the various forms of capital are taken into account.

Let us consider an economy with a stock of capital consisting of two parts, namely “tradable capital” and “nontradable capital”, and assume that the production function both in the tradable sector (subscript $T$) and nontradable sector ($N$) has the Cobb-Douglas (C-D) form:

\[ Y_T = A_T L_T^{\alpha - \eta} K_T^\alpha H_T^\beta \]  
\[ Y_N = A_N L_N^{\beta - \phi} K_N^\beta H_N^\phi \]  

Our empirical findings – see, for instance (Čihák – Holub, 2003) – support the view that there are many groups of goods with many different degrees of tradability, rather than two clearly separate groups.
where \( A \) denotes a technological coefficient, \( K \) is tradable capital, \( H \) is nontradable capital, and \( \alpha, \beta, \eta \) and \( \phi \) are tradable-capital and nontradable-capital intensities in the two sectors. We assume that the same technology is applied abroad (denoted by an asterisk throughout the article) as well.

In line with the B-S model, we assume that the law of one price holds for tradable goods, setting (without loss of generality) their world price equal to one. Let us also adopt the standard B-S assumption of perfect labour force mobility among sectors within an individual economy, but zero mobility of labour force among the economies. This assumption implies that the marginal product of labour must be equal to the same wage level in both economic sectors, i.e. that:

\[
(1 - \alpha - \eta) A_T L_T^{\alpha-\eta} K_T^{\alpha} H_T^\beta = (1 - \beta - \phi) p_N A_N L_N^{\beta-\phi} K_N^{\beta}(\tilde{H} - H_T) \phi \tag{3}
\]

Regarding the other inputs, we assume that \( \alpha > \beta \) and \( \eta > \phi \), i.e. that the production of tradable goods is more intensive in terms of both the tradable and nontradable capital, while the production of nontradable goods is more labour-intensive.\(^3\) The difference between \( K \) and \( H \) is that tradable capital can be moved freely among countries, while nontradable capital can not. The free mobility of tradable capital means also that \( K \) can serve as collateral for foreign borrowing, while \( H \) can not. The notation for tradable and nontradable capital was chosen to be consistent with the growth literature, where \( K \) traditionally stands for physical capital, while \( H \) stands for human capital.\(^4\) However, since the distinction between \( K \) and \( H \) consists in their mobility across country borders rather than their natural characteristics, we refer to them as tradable and nontradable capital goods, following the terminology used in the dependent-economy literature. The mobility of tradable capital implies that its stock adjusts instantaneously to equate its marginal product to the world inter-rate \( r^* \) plus the domestic risk premium \( \sigma \), required by investors, in both economic sectors:

\[
\alpha A_T L_T^{-\alpha-\eta} K_T^{\alpha-1} H_T^\beta = r^* + \sigma = \beta p_N A_N L_N^{-\beta-\phi} K_N^{\beta}(\tilde{H} - H_T) \phi \tag{4}
\]

On the other hand, if the nontradable capital cannot flow between economies, its total amount in the domestic economy (i.e., \( H_T + H_N \)) in a given period is fixed at the level \( \tilde{H} \). The convergence of nontradable capital in per-capita terms to the level of advanced countries, \( h^* \), will thus be only

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\(^3\) Some authors challenged this assumption by pointing out that nontradables include capital-intensive public utilities such as electricity generation or transport (Neary – Purvis, 1982). However, empirical literature tends to support the assumption of higher labour intensity in nontradables. Moreover, for most of our key conclusions, it is sufficient to assume \( \eta/(1-\alpha) > \phi/(1-\beta) \), i.e. that the ratio of nontradable capital intensity with respect to all nontradable factors (labour plus nontradable capital) is greater in the tradable sector than in the nontradable one.

\(^4\) Another reason for this notation is that we want distinguish the abbreviations for capital from those for goods.
gradual. We assume, however, that the nontradable capital can flow freely between the sectors, so that in equilibrium, its marginal return must be the same in both sectors. These assumptions imply that:

\[
\eta A_T L_T^\alpha K_T^\alpha H_T^\beta = \phi p_N A_N L_N^\beta K_N^\beta (\bar{H} - H_T)^\varphi
\]  

(5)

Dividing equations (3) and (5) and rearranging, we get the equilibrium allocation of nontradable capital to the tradable sector as a function of total nontradable capital accumulated in the country and the equilibrium allocation of labour between the two economic sectors:

\[
H_T = \bar{H}Z; \quad Z = \left( \frac{\eta(1 - \beta - \varphi)\gamma}{\phi(1 - \alpha - \eta)(1 - \gamma) + \eta(1 - \beta - \varphi)\gamma} \right)
\]  

(6)

where \(\gamma = L_T / (L_T + L_N)\) is the tradable sector’s share in total employment. This equation shows that the share of total nontradable capital used in the tradable production increases with the share of tradable sector in the overall employment. This is a logical consequence of the assumed equalisation of the nontradable capital’s marginal products between the two economic sectors. If the employment in tradable production goes up, it implies higher marginal product of nontradable capital in this sector and its lower marginal product in the nontradable sector. As a result, human capital starts flowing into the tradable sector from the nontradable one. Equation (6), however, does not mean that the ratio of nontradable capital to labour is the same in both sectors, or that this ration does not depend on the share of each sector in employment.

Combined with (4), equation (6) yields the equilibrium levels of the tradable capital:

\[
K_T = \left( \frac{\alpha A_T L_T^\alpha (\bar{H}Z)^\eta}{r^* + \sigma} \right)^{1-\alpha}
\]  

(7)

\[
K_N = \left( \frac{\beta p_N A_N (L_N)^{1-\varphi} (\bar{H} (1 - Z))^\varphi}{r^* + \sigma} \right)^{1-\beta}
\]  

(8)

Note that a higher price of nontradable goods attracts a higher volume of capital into the nontradable sector. This is an important adjustment mechanism in this model. A poor technology in the tradable sector means that this sector attracts little tradable capital despite the perfect capital

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5 Note that we make the same assumption about the nontradable capital as about labour, i.e. that it is mobile within country, but immobile internationally. In order to keep the model as simple as possible at this stage, we do not consider the case of sector-specific nontradable (“human”) capital, even though it leads to a fruitful stream of research – e.g., (Dickens – Katz, 1987), (Helwege, 1992), (Neal, 1995), (Strauss, 1998).
mobility among countries. Poor technology and little tradable capital mean low labour productivity, and thus low wages in the tradable sector. These low wages, in turn, imply low prices in the nontradable sector. The low prices, however, mean that the nontradable production attracts little tradable capital, too, even if its technological level is the same as abroad and tradable capital is perfectly mobile. Only as the country converges to the advanced world in terms of technology in the tradable sector, can it converge in the labour productivity in the nontradable sector as well, even assuming that there are no technology differences in this sector among countries. In addition to this mechanism, the stock of tradable capital in both economic sectors also depends positively on the accumulated stock of nontradable capital, which influences the productivity of the tradable capital.

Finally, after combining (5), (6), (7), and (8), and some rearranging, we get the expression for prices in nontradable sector as a function of per capita nontradable capital stock \( h \), the domestic interest rate, sectoral allocation of labour, and technological coefficients:\(^6\)

\[
p_N = \frac{(A_T)^{1-\alpha}}{A_N^{1-\alpha}}(\bar{h})^{\eta(1-\beta)\varphi(1-\alpha)}(r^* + \sigma)^{\beta(1-\alpha)\alpha(1-\beta)}(1-\alpha)X (9)
\]

Let us define the price index as a weighted geometric average of prices of tradable and nontradable goods:\(^7\)

\[
P = (p_T)^\gamma(p_N)^{1-\gamma} (10)
\]

where \( \gamma \) is the share of tradable goods in private consumption. If we assume that this share is the same at home as abroad, the relative price level vis-à-vis the outside world is:

\[
\frac{P}{P^*} = \left( \frac{A_T}{A_T^*} \right)^{\gamma(1-\gamma)(1-\beta)} \left( \frac{A_N}{A_N^*} \right)^{(1-\gamma)(1-\gamma)(1-\beta)} \left( \frac{X}{X^*} \right)^{(1-\gamma)(1-\gamma)(1-\beta)} \left( \frac{r^* + \sigma}{r^*} \right)^{\gamma(1-\gamma)(1-\beta)\varphi(1-\alpha)} \left( \frac{\bar{h}}{\bar{h}^*} \right)^{(1-\gamma)(1-\beta)\varphi(1-\alpha)} (11)
\]

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\(^6\) In which we defined:

\[
X = \frac{\alpha}{\beta} \frac{(1-\beta)}{1-\beta-\varphi} \left[ \frac{1-\alpha-\eta}{\beta} \right]^{1-\beta} \left( \frac{Z}{\gamma_k} \right)^{1-\beta} \left( \frac{1-Z}{1-\gamma_k} \right)^{-\varphi} = \frac{\alpha}{\beta} \frac{(1-\beta)}{1-\beta-\varphi} \left( \frac{\eta}{\varphi} \right)^{\psi} \left( \frac{1-\alpha-\eta}{1-\beta-\varphi} \right)^{1-\beta-\psi} \frac{\gamma_k + \varphi \frac{1-\alpha-\eta}{\eta} \frac{1-\beta-\varphi}{1-\gamma_k}}{1-\alpha}
\]
Equations (9) and (11) lead to many interesting conclusions. Some of them resemble the basic version of the model for the single-factor production function (Čihák – Holub, 2001b). For example, an increase in the technological coefficient in nontradable production leads to a fall in nontradable prices (with a unitary elasticity). An increase in the technological coefficient in the tradable sector has the opposite qualitative impact, even though with a different elasticity (greater than unitary, as per (9)). However, the key mechanism that explains the process of GDP and price convergence in this model is the accumulation of per-capita nontradable capital $\bar{n}$, and perhaps also a gradual reduction in the risk premium. Note also the variable $X$ in equations (9) and (11), which depends not only on the parameters of the model but also on the share of employment in the tradable sector (i.e. $\gamma_k$).

We will now discuss the impact of the following three changes on the results of the model: (i) an increase in the world interest rate or the risk premium, (ii) an increase in the nontradable capital, and (iii) an increase in the share of nontradable sector in total employment.

First, an increase in the world interest rate or the risk premium, which together determine the equilibrium domestic interest rate for tradable capital, reduces the relative price of nontradable goods (and the overall price level) under the plausible assumption that $\alpha > \beta$. This is due to the fact that a higher domestic interest rate reduces the tradable capital stock in both sectors, but its impact on labour productivity is more pronounced in the tradable sector which is more tradable-capital intensive. The marginal product of labour thus goes down more in the tradable sector than in the nontradable one, implying reduced unit labour costs in the production of nontradables, given the fact that the wage rate is determined in the tradable sector. If there is no risk premium, however, a change in the world interest rate has no effect on the relative price level, as it affects all countries symmetrically (provided that $(1-\gamma)$ is the same at home and abroad). This is not true for changes in the risk premium, which can be country-specific. A higher/lower risk premium leads to a lower/higher price of nontradables and thus also to lower/higher relative price level. This is important, as a gradual decline in the risk premium has been (and is likely to remain) an integral part of the convergence process in transition countries. The decline in the risk premium not only decreases the equilibrium real interest rates in these economies, but also increases the equilibrium speed of price and GDP convergence process.

Second, a relationship exists between the per-capita nontradable capital on the one hand, and the price of nontradables and the overall price level on the other one. As shown in (9), the elasticity of nontradables’ prices with respect to per-capita stock of nontradable capital is $[\eta(1-\beta)-\varphi(1-\alpha)]/(1-\alpha)$. If we recall the assumptions that $\alpha > \beta$ and $\eta > \varphi$ (or at

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7 The geometric average is an optimal price index, provided that we assume a C-D utility function with unitary elasticity of substitution between tradable and nontradable goods; the traditional arithmetic average can be thought of as a log-linear approximation of this optimal price index (see, for instance, (Obstfeld – Rogoff, 1996)).
least that $\eta/(1-\alpha) > \varphi/(1-\beta)$, the elasticity is unambiguously positive, which means that a higher per-capita stock of nontradable capital increases the price of nontradables (and price level), and vice versa. The exact magnitude of this effect depends on the values of parameters $\alpha$, $\beta$, $\eta$, and $\varphi$.

Third, an increase in the share of nontradable sector in total employment has a positive impact on the price of nontradable goods and the overall price level. The explanation can be derived from (6). If the country starts allocating more labour to the nontradable sector, which is less capital intensive, it means that the constraint stemming from the limited stock of nontradable capital becomes less “binding”. In response, some parts of the country’s nontradable capital stock will shift from tradable to nontradable sector, eventually increasing the nontradable capital per employee in the production of nontradables; however, this cannot be enough to prevent the nontradable capital per employee from rising in the tradable sector, too. The latter factor dominates in equilibrium, being magnified by the responses of tradable capital investments, implying a bigger increase of labour productivity in the tradable than in the nontradable sector. This implies a higher price of nontradables.

As a next step, we can use the results so far to derive an expression for nominal GDP per employee:\footnote{Where we define:}

$$GDP_{nom} = (A_T)^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{r^* + \sigma} \right)^{\frac{\alpha}{1-\alpha}} (\bar{h})^{\frac{\eta}{1-\alpha}} R$$

Four factors determine the nominal GDP in (12). First, a positive relationship exists between the nominal GDP and the tradable productivity parameter (which is its only determinant in the single-factor production function case – see (Čihák – Holub, 2001b)). The exponent reflects the impact of technology on the tradable capital stock invested into the economy. Second, a crucial role is played by the nontradable capital, which influences the labour productivity in a similar way as technological coefficients. Note that the exponent of the nontradable capital reflects the relative importance of nontradable and tradable capital in the tradable sector. Third, an important factor is the risk premium, which has a negative impact, since an increase in the risk premium reduces the capital stock and thus production per employee in both economic sectors for the given level of technology. Fourth, the nominal GDP is also influenced by the allocation of labour between the two sectors.

To close the model, (11) and (12) can be used to derive an expression for the relative GDP in PPP:

\footnote{Where we define:}

$$R = \left( \frac{Z}{\gamma_k} \right)^{\frac{\eta}{1-\alpha}} \left[ \gamma_k + (1-\gamma_k) \frac{1-\alpha-\eta}{1-\beta-\varphi} \right]$$
In line with our previous results, GDP in PPP now depends not only on productivities in the two sectors, but also on the nontradable capital stock, risk premium, and sectoral allocation of labour.

We can now use (11), (12), and (13) to illustrate the results of the model in simulations. We provide two examples, one focusing on the role of nontradable capital, the other on the impact of the risk premium. To make the simulations easier, we make two assumptions for both examples. First, we treat the sectoral distribution of labour as constant. Second, we assume the speed of convergence towards the steady state to be constant, too. We calibrate the examples consistently with the empirical estimates of the convergence speed from cross-country regressions – e.g. (Barro, 1991). In particular, we assume that the country tends to close about 2.7 percent of the GDP gap relative to its steady state, implying a 2.7 percent speed of convergence also for the nontradable capital. Also, to derive a “benchmark” convergence scenario, let us set \( \alpha = 0.4 \), \( \beta = 0.1 \), and \( (1-\gamma) = 0.60 \). These values are broadly consistent with the cross-country empirical estimates in (Cihák – Holub, 2003).

In the first example, shown in Figure 1, we focus on the role of nontradable capital. In particular, we consider a country that starts with a nontradable capital stock equal to 15 percent of the steady state value, which is assumed to be represented by the EU average in our policy discussions, and no risk premium. These values imply starting nominal GDP per em-

\[
\frac{GDP_{\text{PPP}}}{GDP^*_{\text{PPP}}} = \left( \frac{A^*_T}{A_T} \right)^{(1-(1-\gamma)(1-\beta))} \left( \frac{A^*_N}{A^*_N} \right)^{(1-\gamma)} \left( \frac{r^*}{r^* - \sigma} \right)^{(1-\gamma) \beta + \frac{\alpha [1-(1-\gamma)(1-\beta)]}{1-\alpha}}
\]

(13)

In the end, we thus decided to do with the constant convergence speed assumption. This choice can be supported by two additional arguments. First, the empirical estimates implicitly assume a constant convergence speed, which means that a realistic calibration of the model would be indirectly influenced by this assumption in any case. Second, practical policy discussions – including those at the CNB – are usually based on the simple “beta-convergence” scenarios rather than on numerical solutions of the Ramsey model.

\footnote{In section 4 below, we discuss under what special circumstances one can achieve the stable allocation of labor between sectors. We admit, though, that this assumption is not fully realistic (see footnote 16).}

\footnote{From a theoretical point of view, it would be more appropriate to base the convergence scenarios on an exact numerical solution of our model, using the conditions of consumer maximization derived in section 4. Strictly speaking, the constant convergence speed we employ here applies around the steady state only, which is not the situation of accession countries (we thank for this remark to an anonymous referee). It might be realistic to assume that for less advanced countries the convergence would be faster than around the steady state – see e.g. (Barro – Sala-i-Martin, 1995). This would imply that our simulations underestimate the starting growth rates of all key variables. Nonetheless, if one takes the broad concept of capital – which we do in this article assuming that the total capital share in the production of tradable goods reaches 80 percent, this bias is likely to be relatively small. In the end, we thus decided to do with the constant convergence speed assumption. This choice can be supported by two additional arguments. First, the empirical estimates implicitly assume a constant convergence speed, which means that a realistic calibration of the model would be indirectly influenced by this assumption in any case. Second, practical policy discussions – including those at the CNB – are usually based on the simple “beta-convergence” scenarios rather than on numerical solutions of the Ramsey model.}
ployee equal to 28 percent of the steady state (i.e. the EU), GDP in PPP equal to 50 percent of steady state, and price level equal to 57 percent of the steady state. All the variables are assumed to smoothly converge to their steady-state levels in the limit. Figure 1 shows that such a country (which is close to the Czech Republic in terms of GDP in PPP per employee) should achieve a growth rate of nontradable capital starting from above 5 percent a year and gradually declining below 3 percent over 25 years. As a result, the country's real growth differential vis-à-vis a steady-state economy should be roughly 2 percent a year initially, and decline gradually to 1 percent over next 25 years. The implied equilibrium real exchange rate appreciation is slightly lower than that (starting from about 1.5 percent a year and declining below 1 percent). The B-S effect is thus important in this “benchmark” convergence scenario, but its impact is smaller than the allowed inflation differential under the Maastricht criteria, and much smaller than the nominal exchange rate appreciation allowed under the ERM II regime, if the membership in it is minimised to 2 years and the ±15 % fluctuation band is considered.

In the second example, shown in Figure 2, we focus on the impact of the risk premium. In particular, we consider a country that starts with a nontradable capital stock equal to 23 percent of the steady state, and we assume that the country faces a risk premium of 3 percentage points compared with the world interest rate of 6 percentage points (i.e., the domestic real interest rate is 1.5-times the world level). The risk premium implies a lower equilibrium stock of tradable capital, which offsets the fact that compared to the benchmark case, the economy starts with a higher nontradable capital stock. In terms of the “observable variables”, the economy thus starts at values similar to the first example: the nominal GDP

\[\text{GDP}_{\text{nom}}/\text{GDP}_{\text{nom}}^*\text{, }\text{GDP}_{\text{ppp}}/\text{GDP}_{\text{ppp}}^*\text{ and }\text{P}/\text{P}^*\text{ are the same differentials for the nominal GDP, the GDP in PPP, and price level, respectively.}\]

11 Note that the model is purely deterministic, which implies the smooth convergence towards the steady state. A more realistic setting would need to take stochastic shocks into account.
per employee reaches roughly 28 percent of the steady state, GDP in PPP 50 percent of steady state (close to the Czech starting point) and price level 57 percent of the steady state. After the initial period, the risk premium starts declining by 15 percent each year, converging gradually towards zero. Figure 2 shows that very little convergence takes place in the starting period. The economy’s output is far from the EU levels; however, given the risk premium, it is not very far from its conditional steady state. Once the risk premium starts to decline, through, the GDP growth both in nominal terms and in the PPP, as well as the price level convergence speed up immediately, in line with (11), (12), and (23). Moreover, this primary effect is further magnified by the fact that the reduced equilibrium real interest rate creates an additional motivation for nontradable capital accumulation, which gradually starts to gain momentum. As a result, the real GDP growth differential increases above 2 percentage points, and only gradually declines towards 1 percentage point. The real exchange rate appreciation goes up to 2 percentage points initially, and gradually decreases below 1 percentage point. The results are thus similar to the “benchmark” scenario of Figure 1, even though the main driving force is different. The contribution of this alternative simulation consists in illustrating the links between the risk premium, GDP growth, and real exchange rate appreciation, which are usually the key equilibrium variables entering the forecasting process at the central bank – see e.g. (Beneš et al., 2002).

4. Demand Side of the Economy and the Speed of Convergence

In section 3, we focused on the supply side of the economy only. In particular, we assumed that the speed of convergence was constant (which is broadly consistent with the results of cross-country regressions by Barro
(1991) and others), and the allocation of labour across economic sectors was fixed for each economy. In this section, we provide a deeper look into how the above two factors are determined by the consumers’ behaviour in interaction with the production and accumulation of nontradable capital. The discussion in this section is based on the Ramsey model in an open economy with nontradable ("human") capital (Cohen – Sachs, 1986), (Barro – Sala-i-Martin, 1995), extended for the distinction between tradable and nontradable goods – e.g. (Obstfeld – Rogoff, 1996).

Let us adopt several assumptions to make the analysis easier to present. First, we assume that preferences between tradable and nontradable goods have a Cobb-Douglas (C-D) form, implying unitary elasticity of substitution between them – see e.g. (Obstfeld – Rogoff, 1996, pp. 222). Second, we assume that while one unit of tradable capital is equivalent to one unit of tradable good, in order to produce a unit of nontradable capital, one needs to give up one unit of the consumption basket $C_t$ (see the definition in (14)).

Third, we assume that the borrowing constraint is binding for a converging economy, implying that the sum of tradable capital and foreign borrowing is zero, and the net assets of the economy are thus equal to the nontradable capital only. This allows us to formulate the optimisation problem of a representative consumer in time $t$ as follows:

$$\max_{c_t} U_s = \sum_{t=s}^{\infty} \left( \frac{1}{1 + \rho} \right)^{t-s} u(C_t), \quad u(c_t) = \frac{(C_t)^{1-\gamma} - 1}{1- \theta}, \quad C_t = (c_{T,t})^{\gamma} (c_{N,t})^{1-\gamma}, \quad \theta > 0$$

s.t. $P_t (H_{t+1} - H_t) = w_t - (c_{T,t} + p_{N,t} c_{N,t}) + \tilde{r}_t P_t H_t$

where $c_{T,t}$ is the consumption of tradable goods in time $t$, $c_{N,t}$ is consumption of nontradable goods in the same period, $\theta$ is the coefficient of relative risk aversion, $\rho$ is the subjective discount rate, and $\tilde{r}_t$ is the market return on renting nontradable capital in time $t$ (which is in equilibrium equal to its marginal product). $P_t$, defined in (10), is the optimal price index for the C-D utility function. Similarly to section 3, $H_t$, and $w_t$ denote the level of the nontradable capital and the wage level, respectively.

The above specification of the utility function implies that the demand functions for tradable and nontradable goods are given by – see e.g. (Obstfeld – Rogoff, 1996):

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12 The assumption that the production of the nontradable capital requires nontradable goods fits quite well with the assumption that the nontradable capital cannot move across borders. The assumption is also convenient, for two reasons. First, the price of the nontradable capital develops in the same way as the price of the consumer basket, which means that we do not have to take into account relative price changes of the nontradable capital in the Euler equation (17). Second, the assumption means that the employment distribution between the tradable and nontradable sectors is constant (see below). A more general assumption would be that the nontradable capital is produced as a C-D combination of the tradable and nontradable good, with weights $\gamma_t$ and $(1-\gamma_t)$, respectively. Here we restrict our attention to the case of $\gamma_t = \gamma$. Note that in this respect we differ from the earlier dependent-economy literature, which assumed that the nontradable capital is identical with the nontradable good (Brock – Turnovsky, 1994), (Turnovsky, 1997), i.e. another special case with $\gamma_t = 0$. 

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\[ c_{T,t} = \gamma P_t C_t; \quad c_{N,t} = (1-\gamma) \frac{P_t}{p_{N,t}} C_t \] (15)

Note that the C-D utility function leads to constant nominal shares of tradable and nontradable goods in consumer expenditures in equilibrium. This allows us to rewrite the flow budget constraint of equation (14) as:

\[ (H_{t+1} - H_t) = \frac{w_t}{P_t} - C_t + \tilde{r}_t H_t \] (16)

which is similar to the budget constraint of a representative agent in an economy without the distinction between tradable and nontradable goods.

When we maximise the utility function of equation (14) subject to (16), we get a standard optimality condition (“Euler equation”):

\[ \frac{C_{t+1}}{C_t} = \left(1+\tilde{r}_t\right)^{\frac{1}{\alpha}} \] (17)

If we assume away depreciation of the nontradable capital, the return on the nontradable capital should be equal to its marginal product (denoted \( \dot{f}_{h,t} \)) in equilibrium. Thanks to the assumption of perfect mobility of the nontradable capital – and thus equality of its marginal product – between sectors, we can concentrate on the marginal product of the nontradable capital in the tradable sector only. Using our previous results from equations (5), (6), and (7), this marginal product can be expressed as:\(^13\)

\[ \dot{f}_{h,t} = \eta B_{T_t} \left( \frac{\bar{h} Z_t}{\gamma E_t} \right)^{\epsilon-1} = \epsilon (1-\alpha) B_{T_t} \left( \frac{\bar{h} Z_t}{\gamma E_t} \right)^{\epsilon-1} \]

\[ B_{T,t} \equiv (A_{T,t})^{-\frac{1}{1-\alpha}} \left( \frac{\alpha}{r^*_t + \sigma} \right)^{\frac{\alpha}{1-\alpha}}; \quad \epsilon \equiv \frac{\eta}{1-\alpha} \]

Equations (17) and (18) together with the standard transversality condition\(^14\) describe the consumption and investment behaviour in the model.

To close the model, we need to find a general equilibrium in which de-

\(^13\)Note that this is the derivative of production per employee in tradable sector, which is given by

\[ \frac{Y_T}{L_T} = B_T \left( \frac{h Z}{\gamma} \right); \quad B_T = (A_T)^{-\frac{1}{1-\alpha}} \left( \frac{\alpha}{r^*_t + \sigma} \right)^{\frac{\alpha}{1-\alpha}}; \quad \epsilon = \frac{\eta}{1-\alpha}, \text{ with respect to tradable capital, multiplied by (1-\(\alpha\)). The term } aBh^\epsilon \text{ that we effectively deduct from the reduced-form production function equals to the flow of rents from the tradable capital abroad.}
mand equals supply in both the tradable and the nontradable sector. In other words, we are looking for a situation in which

\[ \begin{align*}
    c_{T,t} + I_{T,t} &= (1-\alpha) Y_{T,t}; \\
    c_{N,t} + I_{N,t} &= Y_{N,t}
\end{align*} \]

(19)

where \( I_{T,t} \) is the amount of tradable goods used for investment into nontradable capital in time \( t \) and \( I_{N,t} \) is the volume of nontradable goods invested in nontradable capital. Note that the first equation reflects the fact that a fraction \( \alpha \) of the tradable production goes as a reward to tradable capital, which is fully foreign-owned in equilibrium.

The assumption that nontradable capital is a C-D combination of the tradable and nontradable goods helps to simplify the computations, as it means that it is optimal to use the inputs into the investment process so that:

\[ I_{T,t} = \frac{\gamma}{1-\gamma} p_{N,t} I_{N,t} \]

(20)

which is analogous to equation (15).\(^{15}\) Combining equations (15), (19), and (20) with the expressions for the tradable production and the nominal value of nontradable production, which we had to derive already when expressing the nominal GDP in equation (12) of section 3, we find that the equilibrium requires the following equality:

\[ \gamma_E = \frac{1-\alpha-\eta}{(1-\alpha-\eta) + (1-\alpha)(1-\beta-\varphi)} \]

(21)

This means that in the equilibrium of this simplified version of the model, the labour shares of the tradable and nontradable sectors are constant, determined by the technological parameters. As a result, the price level, nominal GDP, and real GDP are functions of the per-capita nontradable capital only (plus the risk premium; see section 3), which makes the analysis of the convergence process much easier. Note that with the values of technological coefficients used in Figures 1 and 2 (i.e. \( \alpha=\eta=0.4; \ \beta=\varphi=0.1 \)), the labour share of the tradable sector would be about 30 percent. This is roughly the same as the median share of agriculture and manufacturing on employment in the sample of countries in (Čihák – Holub, 2003), increasing our confidence that these calibrations are not unrealistic.\(^{16}\)

We can use our results to discuss the process of real and price convergence in the model. Convergence in the standard closed-economy (Ramsey)

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\(^{14}\) The transversality condition says that the present value of households’ assets must approach 0 as time goes to infinity. It can be written as

\[ \lim_{t \to \infty} \left[ H(t) \left( \frac{1}{1 + R(t)} \right)^t \right] = 0, \]

where \( t \) is time, and \( R(t) \) is the interest rate between now and time \( t \).

\(^{15}\) Note that in the more general case, \( \gamma \) would be replaced with \( \gamma_E \) in this equation.
growth model can be described by the equation $\log(y(t)) = e^{-Bt} \log(y(0)) + (1-e^{-Bt}) \log(y^*)$, where $B > 0$. The logarithm of output per unit of effective labour is therefore a weighted average of the initial value and the steady-state value, with the weight of the initial value declining exponentially with time. The speed of convergence, $B$, depends on parameters determining technology and preferences. For the C-D function:

$$2B = \left[ \omega^2 + 4 \left( \frac{1-\bar{\alpha}}{\theta} \right) (\rho+\delta+\theta g) \left[ \frac{\rho+\delta+\theta g}{\bar{\alpha}} - (n+g+\delta) \right] \right]^{1/2} - \omega$$

(22)

where $\omega$ is defined as $\omega = \rho - n - (1-\theta)g > 0$ and $\bar{\alpha}$ denotes the capital-intensity coefficient for a single-good closed economy.\(^{17}\) In our open economy version of the model, however, we have to replace $\bar{\alpha}$ in equation (22) with $\varepsilon = \gamma(1-\alpha)$, as defined in equation (18).

For usual values of the other parameters,\(^{18}\) the speed of convergence is from 1.5 percent per year (for $\alpha/\bar{\eta}=0$) to about 3 percent per year (for $\alpha/\bar{\eta}=1$) and further to for instance 5 percent (for $\alpha/\bar{\eta}=3$).\(^{19}\) If we assume that less than a half of total capital in the production of tradable goods is tradable (i.e., $\alpha/\eta=1$), then the predicted speed of convergence would be in the range of 1.5–3.0 percent, which is consistent with most empirical studies (Barro–Sala-i-Martin, 1995). Note that the calibration used in Figure 1 would imply a convergence speed close to the upper end of this interval. We used such a convergence speed (in particular, 2.7 percent a year) in the convergence scenarios of section 3.

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\(^{16}\) In reality, the share of employment in tradable sector tends to be lower for richer countries. For example, in (Čihák–Holub, 2003), we found a significantly negative relationship between employment in agriculture and manufacturing and the log of GDP in PPP. The above model explains this tendency by the fact that the indebted converging countries have to allocate more labour to the tradable sector to be able to service their borrowing of tradable capital (see the term $(1-\alpha)$ in the denominator of (21)). A related – but more general – explanation is that rich countries can import more tradable goods from abroad with smaller domestic tradable sectors (we thank for this remark to an anonymous referee). This can be either due to their net creditor position (financing additional imports by interest incomes – see above) or more appreciated exchange rates, resulting for example from better terms of trade. The latter factor, however, goes beyond the scope of this section, which builds on the Blassa-Samuelson model with its simplifying assumptions. Another way to introduce the observed tendency would be to assume that $\gamma > \gamma$ in the production of nontradable capital, meaning that a converging country would have to allocate more labour to the production of tradables.

\(^{17}\) Where $\delta$ denotes capital depreciation, $n$ population growth, and $g$ exogenous technological progress. We have so far ignored these factors for simplicity, but their introduction would be straightforward.

\(^{18}\) The usual values of the parameters are $\alpha=1/3$, $n=1$ percent, $g=2$ percent, and $\delta=3$ percent. As regards the parameters of impatience, “reasonable” values include $\rho$ of about 0.02 and $\theta$ lower than 10 (Barro–Sala-i-Martin, 1995).

\(^{19}\) For $\gamma=0$, all capital in the economy would be tradable, so that the economy would effectively return to the case of an open economy with perfect capital mobility. Indeed, $\gamma=0$ means $\varepsilon=0$, and the convergence coefficient, $B$, is infinitely high. The case of $\alpha=0$, on the contrary, would correspond to a closed economy, because there is no tradable capital. In this case, $\varepsilon=\eta$, and the convergence coefficient is the same as in the case of a closed economy.
5. Balassa-Samuelson Model with More than Two Goods

One of the problems of the standard B-S approach is that the distinction between tradable and nontradable goods is largely artificial. In empirical cross-country comparisons, each commodity group generally behaves as a blend of tradable and nontradable elements rather than purely tradable or purely nontradable – see (Čihák – Holub, 2003).

To introduce such a variety of goods into the model, let us assume that there are \( x \) consumption goods, produced by combining tradable and nontradable goods – which, in turn, are subject to the production process described in the B-S model of section 3. In contrast to the standard B-S model, the tradable and nontradable goods are not consumed directly, but rather serve as intermediate inputs into the production of consumption (retail) goods. For simplicity and brevity, we limit ourselves to the case in which the shares of tradable and nontradable inputs are fixed, discussing only informally the implications of substitutability between these two inputs (more formal discussion is in (Holub – Čihák, 2003)).

Let us thus assume that consumption good \( i \) is produced according to the following formula:

\[
\begin{align*}
y_{i,t} &= \min \left( \frac{T_{i,t}}{w_i}, \frac{N_{i,t}}{1 - w_i} \right) ; \quad 0 < w_i < 1 \\
\end{align*}
\]  

(23)

where \( y_{i,t} \) is output of the good \( i \) \((=1, 2, \ldots, x)\) at time \( t \), \( T_{i,t} \) is the tradable input, \( N_{i,t} \) is the nontradable input, and \( w_i \) is the weight of the tradable input in the consumption good \( i \). Equation (23) is a formalization of the idea that each consumption good contains nontradable elements (such as transportation costs, wholesale margins, and retail margins) as well as tradable elements (the part of the good after the adjustment for all the nontradable elements).\(^{20}\)

Profit maximization under this production function implies that the ratio of tradable and nontradable factors in optimum has to be \( w_i/(1-w_i) \). The marginal cost is then given by:

\[
\begin{align*}
MC_i &= \frac{\partial(T_i + p_N N_i)}{\partial \frac{T_i}{w_i}} = w_i + p_N (1-w_i) \\
\end{align*}
\]  

(24)

Assuming that the consumption goods markets are perfectly competitive, the price of the consumption good \( i \), \( p_i \), would be equal to the margin-

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\(^{20}\) In (Čihák – Holub, 2003), we have attempted to proxy the "degree of nontradability" of a commodity group by the slope coefficient in a cross-country regression between the commodity group's price and GDP. We have found that in the 30 commodity groups covered under private consumption in the 1999 ICP, the empirical degrees of nontradability ranged from 10 to 85 percent. In other words, there was no purely tradable or purely nontradable commodity group.
nal costs. This means that $p_i$ is a weighted average of the price of tradable goods (normalized at 1) and the price of nontradable goods, the weights being the shares of tradable and nontradable inputs, respectively. For a given price of nontradables, the price of a consumption good would be a linear function of the weight of the tradable element, $w_i$, increasing from $p_N$ to 1 for $p_N < 1$ and decreasing from $p_N$ to 1 for $p_N > 1$ (Figure 3a).

An increase in $p_N$ would increase the price of each individual consumption good, $p_i$, thereby raising the aggregate price level. The increase will be the higher the less tradable the good, since $\frac{\partial p_i}{\partial p_N} = 1 - w_i$. For $p_N \geq 1$, an increase in $p_N$ would increase the overall dispersion of the individual prices, while for $p_N < 1$, it would decrease the price dispersion. To see this, we can consider two goods ($i$ and $j$) with different degrees of tradability. The impact of a change in $p_N$ on the difference of the prices of these goods corresponds to the difference in the weights of nontradable inputs in these two goods, namely:

$$\frac{\partial (p_i - p_j)}{\partial p_N} = \frac{\partial [w_i - w_j + p_N(w_j - w_i)]}{\partial p_N} = w_j - w_i$$

(25)

Let us take a good $i$ that is less tradable than a good $j$, that is, $w_i < w_j$. Equation (25) implies that an increase in $p_N$ always increases the price of the good $i$ more than it increases the price of the good $j$, because of its higher share of the nontradable element. For $p_N \geq 1$, it holds from (24) that $p_i \geq p_j$, so the increase in $(p_i - p_j)$ as a result of the increasing $p_N$ means an increase in the differences between the two prices. For $p_N < 1$, it holds from (24) that $p_i < p_j$, so the increase in $(p_i - p_j)$ as a result of the increasing $p_N$ means a decrease in the differences between the two prices. Figure 3b illustrates this conclusion, showing the relationship between the price level ($P$), defined as the weighted average of individual prices, and the price dispersion ($\rho$), defined as a weighted standard deviation of the individual prices, with the weights corresponding to the shares of
there is a significant negative relationship between the degree of deviations of relative prices in a given country vis-à-vis the system of relative prices in the reference country and the price level. We have also found that the relationship is still negative, but less so, if we look only at EU countries, and that the relationship is very weak when applied to a world-wide sample of countries.

As a further generalisation, we could assume that producers in some (or all) markets face a downward sloping demand function and are able to set their price above marginal costs, depending on the price elasticity of demand. Again, we report the results in detail in (Holub – Čihák, 2003). The implications of this case depend on whether the differences in demand elasticities have any systematic pattern or not. As regards the relationship between the price level \( P \) and the price dispersion \( \rho \), though, the introduction of imperfect competition would not change the basic conclusion that for lower price levels, the relationship tends to be negative, while for higher price levels, it tends to be positive, even though both the price level and the price dispersion would tend to be higher under imperfect competition than under perfect competition.

The introduction of imperfect competition brings additional insights by allowing us to study, for example, the impact of a systematic relationship between the price of nontradables in a country, and the price elasticities, which is indeed consistent with empirical evidence. In particular, we could assume that the elasticity of demand for the same

\[ \rho = \frac{1}{\mu} \sqrt{\sum_i w_i (P_i - \mu)^2}, \]

where \( w_i \) are weights of the individual goods in the consumption basket and \( \mu \) is the average price level of consumption. This is consistent with the definition of the price dispersion in the theoretical calculations in Figure 3b. The only difference is that the empirical comparable price levels are measured as ratios to the same prices in a reference country, whilst here we measure all prices in terms of a single theoretical numeraire, i.e. the price of tradable input.

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\(^{21}\) To plot this curve, we have generated 10,000 consumption goods with the \( w_i \)'s drawn from a uniform distribution over the interval (0,1).

\(^{22}\) The degree of deviations of relative prices was measured as a weighted standard deviation of comparable price levels of individual goods in the given country relative to the average comparable price level,
good differs in individual countries and that companies are – due to their monopolistic power – able to set different prices in different countries. As a result, the price of the same good would be higher in the countries with a less elastic demand. Demand functions are likely to be less elastic in more advanced countries (i.e., countries with higher prices of nontradables), as higher income levels enable consumers to put more emphasis on non-price factors such as the brand or the perceived quality of the good. This effect would tend to decrease the measured price dispersion for countries with low price levels (and high price elasticity of demand), while increasing it for countries with high price levels (and low elasticity of demand). However, there are also factors attenuating this effect, in particular the fact that goods exported from less developed countries to developed countries tend to face a more elastic demand function, because of their perception as being “inferior”. In (Čihák – Holub, 2001a,b; 2003) we have indeed found empirical evidence supporting the existence of such effects for the case of EU and CEE countries.

Finally, cross-country price differences may be also influenced by government policies such as the competition policy and taxation, which may influence the mark-ups over marginal costs. As a result, a part of the price convergence process may be related to all these additional effects described in this section, rather than to the B-S effect associated with the changing relative price of the tradable and nontradable inputs.

6. Conclusion

In this article, we provided a theoretical reference point for the discussions on adjustments in the price level and relative prices. We presented a model integrating the Balassa-Samuelson model of real equilibrium exchange rate with a model of accumulation of capital consistent with the new growth literature, and with the demand side of the economy. We have also shown how the model can be generalized to a case of more than two goods.

The presented extensions to the B-S model provide several useful insights into the likely price adjustment process in the Central European economies. For instance, the simulations based on the model with nontradable capital, presented in this article, allow to assess in a consistent framework the links between the risk premium, GDP growth, and real exchange appreciation, which are typically the key equilibrium variables in the macroeconomic forecasting process.

The extension of the model to the case of more than two goods also provided intriguing insights into the nature of the relative price adjustment process. The calculations suggest that for countries with relatively low price

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23 In a review of the literature, it appears that the car industry is particularly affected by price discrimination. For example, Verboven (1996) finds that European car markets are segmented, with producers gaining higher markups in their home country than in export destination countries.
levels, there should be a negative relationship between the price dispersion and price levels; the relationship should become less negative for countries with higher price level, and eventually turn positive with increasing price levels. This prediction is consistent with empirical findings based on data for CEE and EU countries.

REFERENCES


The article contributes to the theory of convergence in the price level and relative prices. The authors present a model integrating the Balassa-Samuelson model of real equilibrium exchange rate with a model of capital accumulation and with the demand side of the economy. They also show how the Balassa-Samuelson model can be extended to the case of more than two goods. The predictions of the Balassa-Samuelson model are generally consistent with empirical findings in Central and Eastern European countries. The authors show how the model can be used toward projecting price convergence in a transition economy.