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Macroeconomic Environment and Credit Risk

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Abstract
The importance of credit-risk models has increased with the introduction of the New Basel Capital Accord (Basel II). This paper follows Merton´s approach to structural analysis, toward default-rate modeling. A latent-factor model is introduced within this framework. Estimation of this model can help further our understanding of the relationship between credit risk and macroeconomic indicators. The results have been used for stress testing the Czech banking sector.

1. Introduction

Our recent experience with the effects of economic downturn on banks´ loan portfolios in the Czech economy in the late 1990s provides an opportunity to investigate the link between macroeconomic development and credit portfolio quality.

Credit risk is one of the most important areas of risk management. It plays an important role mainly for banking institutions, which try to develop their own credit risk models in order to increase bank portfolio quality. A new wave of interest originated with the introduction of the New Basel Capital Accord known as Basel II.

Three approaches can be distinguished. The first – traditional models – are based on comparing client specific information. The objective of these models is a good prediction of future client quality. The default probability is obtained from empirical information. These models are widely used in assessment of banking clients and this approach is also very popular for transitional economies with insufficient capital markets. Models based on option pricing ("structural models") represent the second possible approach. These are based on financial pricing theory. Here the value of a firm is modeled as an option price. The firm's default is specified in relation to firm's value and leverage. The third approach is summarized in so-called reduced form models. These models use the market bond price as input, and from this information they try to derive the default probability and recovery rate. The aim of all approaches is an estimation of firm's default probability and loss given default. Together with estimation of exposure at default and effective maturity, these credit risk components can be used for determining the capital requirement – Internal Ratings-Based Approach (IRB) (Basel Committee on Banking Supervision, 2004).

One question which has become important is the relationship between credit risk models and the business cycle. Research on this relationship has found importance

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particularly during the last few years. Targets of these studies are credit risk models taking into account the macroeconomic environment. Some researches are focused on developing a macroeconomic model for credit risk estimation. In general these types of models try to estimate the default rate from macroeconomic data. These models are used for stress testing, which is emphasized by the New Basel Capital Accord. Banks with IRB models should use stress testing in the assessment of capital adequacy. Stress testing should involve identifying possible events or future changes in economic conditions that could have a negative effect on the banks' capital requirements (Basel Committee on Banking Supervision, 2004). Macroeconomic models are also a very useful tool for central banks for research and management of banking system financial stability. Through the application of these models central banks can estimate impact changes in monetary policy or expected or unexpected macroeconomic shocks.

Two basic approaches in default probability modeling can be distinguished. Banks can base assessment of borrowers on the current economic conditions. Default probability is then conditioned on the point in the cycle. When risk assessments take into account possible changes in the macroeconomic climate, then forward looking ratings can be derived. The second approach becomes important due to the possibility of implementing a different type of countercyclical policy. Macroeconomic models can help with understanding the influence of macroeconomic changes on the default events.

This paper contributes to contemporary research by applying the approach of Merton structural models to the Czech economy.[1] Our study extends empirical analyses (e.g. Virolainen (2004)) of default data by introducing latent systematic factors. In contrast to traditional empirical models, factor models can be a better way of default rate modeling, as they provide a microeconomic foundation. This research follows the study of Jakubík (2006), which develops a structural model approach for the Finnish economy.

We focus on developing macro models for default rate prediction in this paper for the financial stability purpose. The target of this study was investigation of the possible approach of default rate macroeconomic modeling in the literature and the selection of a model for the Czech economy. There are several reasons for being interested in the relationship between business cycle fluctuations and credit default. First, financial regulators need to have a good understanding of the potential credit risk in loan and corporate bond portfolios. They therefore need to be able to estimate the potential cyclical variability of default rates. Second, management and regulators want to have some idea of the likely rate of default in the near future. Macroeconomic indices are informative indicators of future default rates, requiring the direct modeling of this relationship. Third, as encouraged by the Basel committee, banks need to be able to develop stress tests of their portfolio performance in a business cycle downturn and these tests should be interpretable in terms of the magnitude of some underlying macroeconomic shock. This study can help in all these tasks. A latent factor model is a natural and popular way to estimate potential credit risks. This is why this model is the basis of Pillar 1 of the New Basel Capital Accord (Gordy, 2003). However, relatively little work has been done on estimating the cru-

[1] Merton´s models are based on the option price model, which estimates the value of a firm as the price of a put option. This idea was for the first time introduced by Merton (1974).

cial parameter, representing correlation with systematic factor. Combining this model with macroeconomic indicators provides a natural test of the specification of the macroeconomic relationship. If the macroeconomic indicators are indeed informative predictors, then the share of fluctuations explained by the latent factor will be relatively small. This unobservable factor represents the unexplained component of the macroeconomic model. We found that the latent factor remains important even with the inclusion of macroeconomic indicators. Therefore both simulation and forecasting should include allowance for this factor as well as the observed macroeconomic indicators considered.

This paper is structured as follows. Section 2 introduces related studies. Section 3 presents selected approaches to credit risk modeling. A nonlinear one-factor model, which is derived from the idea of return assets modeling by the systematic factor and idiosyncratic shocks, is described in detail. Section 4 presents the estimated one-factor macroeconomic credit risk model for the Czech economy. This model is used for the financial stability purpose in the Czech National Bank. The last section concludes and discusses possible further research topics.

2. Related Studies

Some studies focus on business cycle effects on portfolio credit risk; others research the pro-cyclicality of credit risk measurement or relationships between financial crises and credit risk models. Four basic components are defined in the New Basel Capital Accord according to the Internal Ratings-Based Approach (Basel Committee on Banking Supervision, 2004). These are default probability, loss given default, exposure at default and effective maturity. In discussions about the relationship between the business cycle and credit risk models the most important is default probability and loss given default. Some papers solve the problem of correlation between default probability and loss given default. In general, default probability changes over time depending on the macroeconomic environment. Some models use the constant value of loss given default, but this also changes over time in practice. Many studies demonstrate this fact. The basic issue of the relationship between credit risk models and the economic cycle is the estimation of default probability as a function depending on time. Default probability is usually modeled by the default rate. This indicator is defined as a ratio between loans in default and total granted loans. This type of data on the aggregate level of the economy is sometimes very difficult to obtain. In this case some approximation must be used. These models use aggregate variables to explain the default rate. Such models are able to model the impact of a macroeconomic shock on the credit industry.

This paper is related to the literature on the influence of the macroeconomic environment on credit risk models. Few papers focus on the issue of the mutual relationship between the economic cycle and credit risk. Those studies can be divided into two groups. The first group uses company specific information and tries to research the influence of the macroeconomic environment on individual risk. Other studies use only aggregate data and investigate the default rate in relation to macroeconomic indicators. In this paper only aggregate information is used and therefore it is in the second group of papers.

In the context of the New Basel Capital Accord, there are studies investigating cyclical effects in credit risk models. These try to model the influence of cyclical policy on the bank capital requirement (e.g. (Catarineu-Rabell, Jackson, Tsomocos, 2003)). They discuss the influence of different forms of implementation of the rating system on the bank capital requirement. Lowe (2002) examined whether credit risk is low or high in economic booms. He described how macroeconomic consideration is incorporated into credit risk models and the risk measurement approach that underlies the New Basel Capital Accord. A survey of the literature on cyclical effects on default probability, loss given default and exposure at default can be found in (Allen, Saunders, 2003). They noticed that although systematic risk factors have been incorporated into both academic and proprietary models for default probability, the same is not true for loss given default and exposure at default.

There are studies that used the latent factor model for investigation business cycle effects on portfolio credit risk.[2] These models are based on the Merton model. Cipollini and Missaglia (2005) attempt to integrate market risk with credit risk using a dynamic factor model. Rösch (2003) estimated a one-factor model for the German economy. He used bankruptcy data for estimation of default probability and correlation between firms' normalized return assets. This model is estimated for the whole German economy and also for 16 industry specific sectors. The one-factor model is also employed in (Rösch, 2005). Hamerle, Liebig, Scheule (2004) also used the static factor model, but they consider the effect of different assumptions about the error distribution function. They used the logistic distribution function in contrast to (Rösch, 2003) or (Rösch, 2005), where the normal distributions function was used. They found that the inclusion of variables which are correlated with the business cycle improves the forecasts of default probabilities. Céspedes and Martín (2002) studied the two-factor model for credit risk. They compared this model with the one-factor model employed in Basel II. Tasche (2005) investigated the multi-factor extension of the asymptotic single risk factor model and derived an exact formula for the risk contributions to value-at-risk and expected shortfall. He introduced a new concept for the diversification index as an application of the risk contribution formula. A three-factor structural model is developed for example by Hui, Lo, Huang (2003).

Pesaran, Schuermann (2003) used the idea of a simple Merton-type credit model for modeling credit risk as a function of correlated equity returns of the borrower companies. These equities are linked to the correlated macroeconomic variable using an approach similar to the Arbitrage Pricing Theory. They estimated a global macroeconomic model for generating a conditional loss distribution using stochastic simulation. They analyze the impact of a shock to a set of specific macroeconomic variables on that loss distribution. Koopman and Lucas (2004) used a multivariate unobserved components framework to separate the credit and business cycles. They used this model for describing the dynamic behavior of credit risk factors in their relation to the real economy. They used data on real GDP, credit spreads and business failure for the US economy. They empirically showed a positive relationship between spreads and business failure rates and negative GDP.

[2] Latent factor model is class of the models where unobservable variables are considered as depend variable. In order to estimate such models we need to take into account some assumptions about the distribution of the unobservable factors.

Some papers try to develop a simple macroeconomic model of default rate predictions. These empirical models are mostly derived from traditional models used for prediction of individual risk. Few papers focus on the developing macroeconomic model of default rates. Virolainen (2004) estimated this kind of model for the Finnish economy based on logistic regression.[3] He used this model for stress testing and tried to investigate the influence of these shocks on the excepted and unexpected loss. Jakubík (2006) also employed Finnish economic data. Nevertheless, the latent factor model based on the Merton structural approach was used. Our study follows the results and methodology of this study.

3. Credit Risk Models

Two basic groups of models are usually used for credit risk modeling. The first group of models tries to estimate the individual risk of debtors. These are involved in credit risk assessment of commercial banks and are called individual credit risk models. Nevertheless, banks can also incorporate some macroeconomic indicators into a model in an effort to avoid the problem of credit risk assessment pro-cyclicality.[4] The outputs of the individual credit risk models can provide inputs for capital adequacy ratio calculation as well – Internal Ratings-Based Approach (IRB) – New Basel Capital Accord (Gordy, 2003), (Finger, 2001).[5] The estimated model in this paper belongs to the group of macroeconomic credit risk models. This group of models tries to estimate aggregate credit risk, and therefore correspond to financial stability purposes. Macroeconomic credit risk models are usually related to individual risk models, which are possible to express with the following general equation:

$$p_t = f(X_t) \tag{1}$$

where p_t is individual default probability at time t and X_t are some indicators of client quality related to the financial statement in the case of the traditional model, firm's value and leverage in the case of structural models or the bond price in the case of the reduced model. Macroeconomic indicators can be part of these inputs for all types of these models. Originally macroeconomic factors were not considered, but in recent years a lot of papers research the influence of the macroeconomic environment on the credit risk model.

Some empirical macroeconomic models may be found in the literature. These models are based on the same idea as the traditional model. They try to find the empirical observed relationship between the default rate and some macroeconomic indicators. This relationship is usually modeled very simply with linear, probit

[3] The logistic regression model corresponds to linear regression applied after logit transformation of the explained variable. The logit transformation of the explained variable y is defined as $\ln \frac{y}{1-y}$. In the case of credit risk models, this expression transforms the original values from the interval [0,1] to $(-\infty; +\infty)$.

[4] That is, the problem where the credit risk of a single entity is assessed in positive terms during a period of economic growth and in negative terms during a period of economic slowdown. Credit risk models which fail to address the issue of pro-cyclicality might result in a further strengthening of the economic downturn.

[5] A one-factor model was used to calibrate risk weights for the purposes of Basel II framework (default probability, assets correlation of borrowers within risk classes).

or logit models. Static or dynamic approaches are applied for modeling. Vector autoregressive models (VAR) are often used in the case of the dynamic model. These models are able to model the mutual relationship of time series even in the case of time series nonstationarity. The vector autoregressive model can be applied for nonstationarity time series if cointegration exists. The vector error correction model (VEC) as a reformulation of the VAR model is able to distinguish long-term and short-term dependence.

The other different approach is derived from the Merton structural model, which is employed in the Basel II framework for risk weight calibration. The model is based on modeling of the assets return. The default event is defined as a fall of borrowers' return assets under some threshold. This model was originally used for estimation of individual risk, but this idea was extended to default rate estimation. The structural model was chosen for the stress test purpose of the Czech economy. The aim of the model is prediction of the possible future development of non-performing loans as a function of the negative changes in the macroeconomic environment. The selected approach follows the one-factor model which will be introduced later in this paper.

3.1 Credit Risk Models in Central Banks

Most central banks employ some form of sensitivity analysis or stress testing, but only a few of them use a macroeconomic credit risk model. Where central banks do use such macroeconomic credit models, they are mostly empirical-type models, as, for example, in the case of the United Kingdom, Germany, Belgium and Finland. The Bank of England uses an empirical model which estimates the bankruptcy rate of non-financial corporations and the default rate in mortgage and credit card portfolios (Bunn, Cunningham, Drehmann, 2004). The data collected in this manner are then entered into credit loss estimation models as explanatory variables. The default rates are estimated from real GDP, the real interest rate, unemployment, the corporate debt ratio and other aggregate indicators. Finland uses a macroeconomic model based on logistic regression which explains the default rate relationship for individual sectors of the economy using macroeconomic indicators (Virolainen, 2004). This model regards real GDP, nominal interest rates and the debt ratios of the individual sectors investigated as the explanatory variables. The default rate is modeled using the bankruptcy rate of companies in the total number of companies for the given sector of the economy. The Hungarian central bank is also preparing a credit model which uses the number of bankruptcies of companies for individual sectors of the economy, based on the approach employed by the Finnish central bank. Germany used a regression model estimated on a panel of German banks (Deutsche Bundesbank, 2005). The explanatory variable here is a logistic transformation of the proportion of provisions in the credit portfolio. This model works with the change in the risk-free interest rate, GDP growth and loan portfolio growth as the macroeconomic indicators in the role of explanatory variables. The Belgian central bank uses a model based on logistic regression estimating the aggregate default rate of the corporate sector (National Bank of Belgium, 2005). The output gap, nominal long-term interest rates and the lagged rate of aggregate corporate default are used as the explanatory variables. Generally speaking, the development of macroeconomic credit risk models has become an important area of interest of central banks as institutions pursuing financial stability. However, the topic associated with these models is undergoing very rapid development and there is no overall consensus on which model is the best.

3.2 One-Factor Model

The one-factor model is one of the variants of the latent factor model which belongs to the class of the Merton structural model. The following model appears in many papers, for example in (Rösch, 2005), (Céspedes, Martín, 2002), (Cipollini, Missaglia, 2005) and (Lucas, Klaassen, 2003). Application of the model to the German aggregate economy may be found in (Rösch, 2003) and (Hamerle, Liebig, Scheule, 2004).

The model assumes a homogenous portfolio of firms in the economy. A random process with a standard normal distribution is assumed for the standardized logarithmic return on assets of a firm. The discrete normal logarithmic return satisfies the following equation for each firm in the economy.

$$R_{it} = \sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} \qquad (2)$$

R denotes the normal logarithmic return on assets for each firm i at time t. F corresponds to the logarithmic return in the economy independent of firm i at time t, which is assumed to be a random variable with a standard normal distribution. This variable represents the part of the return which is not specific to the firm and can thus satisfy the general conditions for profitability of firms in the economy. U denotes the return specific to the firm i at time t, which is again assumed to be random with a standard normal distribution. The two random variables F and U are also assumed to be serially independent.

$$F_t \approx N(0,1)$$

$$U_{it} \approx N(0,1)$$

The coefficient ρ expresses the correlation between the returns on assets of any two debtors.

$$E(R_{it}) = 0 \qquad (3)$$

$$Var(R_{it}) = E(R_{it}{}^2) - E(R_{it})^2 = E\left(\rho F_t^2 + (1-\rho)U_{it}{}^2 + 2\sqrt{\rho}\sqrt{1-\rho}F_t U_{it}\right) = 1 \qquad (4)$$

Given these assumptions, the logarithmic return on assets of each firm i at time t also has a standard normal distribution – see equations (3) and (4). The model is based on the Merton approach, according to which a default event occurs if the return on a firm´s assets falls below a certain threshold. Formally,

$$P(Y_{it} = 1) \ = \ P(R_{it} \ < \ T) \qquad (5)$$

where Y denotes a random variable with the two potential state.

$$Y_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults at time } t \\ 0 & \text{else} \end{cases} \qquad (6)$$

T can be assumed as a constant or random variable depending on time. Different macroeconomic indicators can be considered if the applied variant of the model assumes that the value of this threshold changes depending on changes in the macroeconomic environment. This value can be modeled as a linear combination of macroeconomic variables. Formally,

$$T = \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} \tag{7}$$

where x_j represents j-th macroeconomic indicator and β is constant coefficients. The change of the macroeconomic environment affects the value of the default threshold in time, which is probably higher in a good time and lower in a bad time. In general, a recession decreases the value of the threshold for default events. Based on all these assumptions, the probability of default of the firm can be derived, with Ψ denoting the standard normal distribution function and x_{jt} denoting the macroeconomic indicators included in the model (e.g. gross domestic product, nominal interest rate, inflation, etc.). The default probability of firm i at time t is given by equation (8) in the case of the constant default threshold in time.

$$p_{it} = P(R_{it} < T) = P(\sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} < \beta_0) = \Psi(\beta_0) \tag{8}$$

This enables us to derive the relationship for the conditional probability of default in response to the realization of an unobservable factor (f_t denotes realization of the unobservable factor F_t).[6]

$$p_i(f_t) = P\left(U_{it} < \frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) = \Psi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) \tag{9}$$

Default probability of firm i at time t is given by equation (10) in the case when a change of the threshold in time is considered according to equation (7).

$$p_{it} = P(Y_{it} = 1) = P\left(\sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} < \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} \right) = \Psi\left(\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt}) \right) \tag{10}$$

The conditional probability of default on realization f_t of random factor at time t corresponding to the default probability (10) is given by formula (11).

$$p_i(f_t) = P\left(U_{it} < \frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) = \Psi\left(\frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) \tag{11}$$

[6] The unobservable factor, or latent factor, is a random variable representing the return on assets of firms which is common to firms in the whole economic sector studied. The realization of this random variable cannot be observed, but one can make an assumption regarding its distribution. A normal distribution of this variable is considered here, although other forms of distribution, such as a logistic distribution, could also be used.

The same result is obtained under the assumption that macroeconomic indicators are considered as a part of the factor of assets return independent of firm *i* at time *t*. This concept is used, for example, in (Hamerle, Liebig, Scheule, 2004). Formally,

$$R_{it} = \alpha F_t + \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} + \omega U_{it} \tag{12}$$

If a very high number of borrowers in the portfolio is assumed, all counterparties have the same individual probability p_i and all default events are independent, then according to the "law of large numbers" the default rate of the portfolio can be estimated as individual default probability.

$$P\big(p(f_t) = p_i(f_t) | F_t = f_t\big) = 1 \tag{13}$$

Unconditional default probability can be obtained by

$$p_t = P(Y_t = 1) = \int_{-\infty}^{+\infty} P(Y_t = 1 | F_t = f_t)\, \phi(f_t) df_t = \int_{-\infty}^{+\infty} p(\mathrm{f_t})\, \phi(f_t) df_t \tag{14}$$

where φ is the density function of the standard normal distribution.

The random factor is assumed to be independent between borrowers. The number of defaults D_t (f_t) at time *t* has binomial distribution with conditional default probability $p(f_t)$ and the given number of companies N_t.

$$D(f_t) \approx Bi\big(N_t, p(f_t)\big) \tag{15}$$

The conditional probability of having exactly d_t default at time *t* can be expressed as

$$P(D_t = d_t | F_t = f_t) = \binom{n_t}{d_t} p(f_t)^{d_t} \big(1 - p(f_t)\big)^{n_t - d_t} \tag{16}$$

Unconditional probability can be obtained as an integral over the random factor.

$$P(D_t = d_t) = \int_{-\infty}^{+\infty} \binom{n_t}{d_t} p(f_t)^{d_t} \big(1 - p(f_t)\big)^{n_t - d_t} \phi(f_t) df_t \tag{17}$$

The parameters of model (9) or (11) can be estimated with the log-likelihood function. The number of defaults D_t is a conditional binomial distributed random variable with the number of borrowers N_t and the conditional probability $p(f_t)$ according to equation (15). Realization d_t and n_t of random variables D_t and N_t are known. The unconditional number of defaults can be computed by an integral over the random effect (14). The log-likelihood function depends only on parameters β and ρ. Formally for model (9):

$$\mathrm{l}(\beta,\rho) = \sum_{t=1}^{T} \ln \left\{ \int_{-\infty}^{+\infty} \binom{n_t}{d_t} \Psi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)^{d_t} \left[1 - \Psi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)\right]^{n_t - d_t} \phi(f_t) df_t \right\} \tag{18}$$

The log-likelihood function for model (11) can be expressed similarly by equation (19).

$$\mathrm{l}(\beta_0,...,\beta_N,\rho) = \sum_{t=1}^{T} \ln \left\{ \int_{-\infty}^{+\infty} \binom{n_t}{d_t} \Psi \left(\frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right)^{d_t} \left[1 - \Psi \left(\frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) \right]^{n_t - d_t} \phi(f_t) df_t \right\} \tag{19}$$

The generalized version of the one-factor model is a multi-factor model which assumes M correlated factors in the economy. This framework can be interpreted as a world of the M economies or countries where factor is the common for all firms of the relevant economy or country. These M economies are related, as there is a correlation between factors. A two-factor model is discussed, for example, in (Céspedes, Martín, 2002). A continuous version of the three-factor model can be found in (Hui, Lo, Huang, 2003). However due to a lack of data, only the one-factor model was estimated in this paper for the aggregate Czech economy.

4. Macroeconomic Credit Risk Model of the Czech Economy

This paper focuses on the macroeconomic default rate model in the Czech economy (Jakubík, 2006). The aim is to produce a model allowing us to estimate the expected proportion of bad loans in the total loan portfolio of banks in response to the evolution of key macroeconomic indicators. The proportion of bad loans is one of the inputs to the stress testing model developed by the Czech National Bank (CNB).[7] It has hitherto been regarded as a constant parameter estimated from extreme historical events. The new approach enables modeling of the impacts of various macroeconomic shocks on loan portfolio quality and subsequently, in combination with the stress testing system, on the capital of the entire banking system. Such shocks may be set either expertly on the basis of historical experience or constructed in the form of alternative scenarios linked to the CNB's main macroeconomic forecasting model.

4.1 Data Used

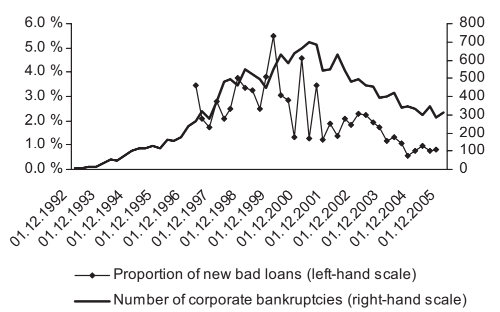
Quarterly data for the Czech economy have been used for all calculations. The model is based on time series of total aggregate bad loans in the economy and selected macroeconomic indicators.

4.2 Bad Loans

The (dependent) credit risk variable or default variable estimated in the model can be defined in several ways. A default event is commonly defined as a breach of payment discipline. A 12-month default probability is usually employed in credit risk assessments. This is defined for a given moment as the probability of a default event occurring in a 12-month period following that given moment, provided that the given person did not default in the period immediately preceding the given moment. This definition thus corresponds to new default events in the economy.

[7] The stress testing methodology is described in detail by Čihák and Heřmánek (2005).

FIGURE 1 Corporate Default Rates in the Czech Economy (quarterly data)



In our model, the default rate was modeled by the proportion of new bad loans in the total volume of loans in the economy.[8] Quarterly time series of new bad loans were available from Q1 1997 to Q3 2005. They were, however, affected by one-off measures entailing reclassification of outstanding mortgage-backed loans in 1999–2001.[9] This period saw significant deviations in the calculated proportion of newly classified loans in the banking portfolio. However, this reclassification did not in fact change the true quality of these portfolios and can be seen as a way of making the indicator of the stock of classified loans more realistic.

The special (dummy) variable used took a value of 1 for quarters when the monitored indicator saw significant deviations from the observed trend. The quarters include Q3 1999, Q4 1999, Q4 2000 and Q2 2002. In other cases, this variable takes the value of 0. The dummy variable so defined corresponds to the effect of changes in the approach to loan classification.

An alternative approach to approximating the default rate in the economy is to use time series of the number of adjudicated bankruptcies or compositions. This approach has been used, for example, to estimate the macroeconomic credit risk model of the Finnish economy.[10] For the Czech Republic, such data have been available since the start of the transformation. However, they have probably had higher information content only since the late 1990s.[11]

The quarterly development of the number of adjudicated bankruptcies in the Czech Republic is demonstrated in *Figure 1*. In practice there seems to be a lag

[8] That is, loans which became "bad" in the given quarter. The moment of default means the time when the loan was classified as substandard or worse for the first time. Shifts within the "bad" loans category (for example, a further downgrading of the loan from doubtful to loss) will not affect the default rate according to this definition. This variable does not correspond to the proportion of total non-performing loans, which are not an optimum measure of credit risk as they may include loans which were first classified a very long time ago and which remain in the loan portfolio, for example, for accounting purposes and are not related to the current economic situation.

[9] CNB Provision of 17 September 1997 stipulating the principles for classifying loan receivables and for provisioning for these receivables, as amended.

[10] Macroeconomic models of the credit risk of the Finnish economy using the number of corporate bankruptcies can be found in (Virolainen, 2004), (Jakubík, 2006).

[11] The time series of bankruptcies shows that the number of bankruptcies at the start of the 1990s was very low, probably as a result of inadequate legislation.

between the filing of a petition for bankruptcy and the actual adjudication, and the default event in the loan portfolio usually precedes the adjudication of bankruptcy. The application of such time series for the Czech economy may also be limited by the frequent amendments made to the relevant legislation. Given these facts, the time series of bankruptcies in the end was not used to estimate the macroeconomic credit model for the Czech economy. Nevertheless, Figure 1 confirms the similar development of this time series and the share of growth in classified loans in the loan portfolio.

4.3 Considered Macroeconomic Indicators

Various macroeconomic indicators are used as explanatory variables relating to the indicator of the default rate in the economy. Interest rates and gross domestic product are most commonly considered in this context in the literature.[12] Gross domestic product (GDP) is a basic indicator of the cyclical position of the economy. A decline or low growth in GDP affects credit risk, for example via a negative effect on corporate earnings, wage growth, unemployment or prices of assets (such as real estate), which, in turn, leads to a deterioration in loan portfolio quality. A rise in interest rates affects the loan portfolio in a similar way, increasing the costs of corporate and household financing, decreasing the market value of assets, etc.

In the case of GDP, annual real GDP growth was applied. One-month and one-year PRIBOR[13] interbank rates were considered as nominal interest rates. Real interest rates were deflated ex post by the consumer price index.

The real effective exchange rate and the nominal koruna-euro and koruna--dollar rates[14] were also considered among the explanatory variables. They are important for credit risk given the nature of the Czech economy as a small open economy where the financial condition of the corporate sector in particular strongly depends on the exchange rate. The last indicator used was the level of indebtedness of the economy, measured by the ratio of client loans to GDP, which approximates the exposure of the financial sector to the rest of the private sector.

In selecting the set of macroeconomic indicators, the issue of the interpretability of the results obtained was also taken into account. Emphasis was put on determining the relationship between credit risk, represented by growth in bad loans in the banking portfolio, and the macroeconomic indicators which are already entered in the stress testing scenarios.[15] Another partial limitation on the selection of the variables was the effort to link this credit risk model to the results of the CNB´s macroeconomic forecast. Although many macroeconomic indicators were considered, finally only GDP, the interest rate and inflation were included in the model.[16]

[12] For a discussion of the issue of explanatory macroeconomic indicators, see, for example, (Virolainen, 2004), (Deutsche Bundesbank, 2005), (Rösch, 2003) and (Jakubík, 2006).

[13] Prague Interbank Offered Interest Rate

[14] An internal CNB calculation based on CPIs and continuous weights corresponding to the average previous annual trade turnover was used to calculate the real exchange rate.

[15] These indicators thus affect the resulting capital adequacy in the stress testing through two channels. The first acts directly via their effect on banks´ balance sheets, while the other operates indirectly via the estimate of credit risk.

[16] Other macroeconomic indicators such as the indebtedness of the non-financial sector (corporate and households), the unemployment rate, nominal and real exchange rate, money aggregate, etc. were considered. However none of them increased performance of the model and the relevant estimated coefficients were not significant.

TABLE 1 Macroeconomic Credit Risk Model (10) of the Czech Economy

Description of variable corresponding to estimated coefficient	Denoted by	Estimate	Standard error	*Pr*>|*t*|
Constant	*c*	-2.0731	0.1019	<0.0001
Gross domestic product	*hdp*	-4.9947	1.9613	0.0162
Nominal interest rate	R_{t-4}	2.7839	0.9076	0.0045
Inflation	π_{t-2}	-2.4364	1.0994	0.0344
Dummy	*dum*	0.3296	0.06629	<0.0001
Effect of latent factor	ρ	0.01211	0.003243	0.0008

Source: CNB

4.4 Model Estimation

We employed the concept of the one-factor model. However, the total number of firms and the number of firms in default in the economy were not available for individual periods for the model estimation (formula 15). Aggregate data on growth in banks´ bad loans were employed in the estimation of the model for individual quarters in place of bankruptcy data. To this end, we made the following additional assumptions: Each koruna of a loan was considered an individual loan of a single client. In such case, therefore, the random variable D corresponds to the number of new bad koruna loans, or the growth in the volume of bad loans, while N stands for the total volume of loans granted. A default event is represented here by non-repayment of a loan of CZK 1. Under these assumptions, the volume of bad loans can be modeled by means of the relation (15).[17] The model was estimated by maximization a likelihood function containing a random latent factor, which was assumed to have a standard normal distribution.

Taking into account the criteria for the selection of variables relating to the stress testing scenarios and the outputs of the CNB´s macroeconomic forecast, we selected the statistically best model containing GDP, the nominal interest rate, inflation and the dummy variable for the purposes of a change in methodology with a subsequent one-off impact on reclassification of the loan portfolio.[18] The selected model is in line with macroeconomic stress test scenarios and outputs of the macroeconomic prediction model of the Czech National Bank.

In the case of GDP, non-lagged annual real GDP growth was used. The statistically most significant interest rate was the nominal 1Y PRIBOR lagged by four quarters. In the case of inflation, the annual rate of growth of the average quarterly CPI lagged by two quarters was the most significant. The model was also tested without the dummy variable. This gave very similar results, although it slightly over--estimated the default rate at the end of the period under review, showing that the chosen model has some degree of robustness. *Table 1* demonstrates the results of the estimated

[17] The assumption regarding koruna loans is somewhat simplified, as koruna loans are not in fact independent. The ideal solution would be to use the real default rates of clients in default divided by the total number of clients. However these data are not available. Nevertheless all tests of the model show that the model is robust enough and the utilized assumption mentioned above does not destroy the result of the regression.

[18] We tried to use the real interest rate instead of lagged nominal interest rate and inflation. However the performance of such estimated model decreased.

model of the aggregate default rate in the Czech economy. All the coefficients were significant at the 5% confidence level. The default rate in the economy is negatively related to gross domestic product, hence higher GDP growth leads to lower credit risk. By contrast, the level of credit risk is positively related to interest rates, which is also consistent with economic intuition. Including inflation in the model reduces the effect of nominal interest rates lagged by four quarters by real inflation lagged by two quarters. For this reason, the estimate of the coefficient representing inflation in the model is negative. The combination of nominal interest rates and inflation demonstrates that the credit default rate in the Czech economy depends on real interest rates rather than nominal rates, although the estimated coefficients are not exactly the same and have different lags. The statistical significance of the effect of the unobservable component shows that this factor is still necessary for explaining the dependent variable, despite the inclusion of macroeconomic indicators.[19] This result implies that the default rate in the economy is also affected by factors other than the macroeconomic indicators considered.

The following equation (20) of the one-factor model (10) expresses the estimated relation for the aggregate default rate in the Czech economy.[20]

$$df_t = \Psi(-2.0731 - 4.9947 gdp_t + 2.7839 R_{t-4} - 2.4364 \pi_{t-2} + 0.3296 dum_t) \tag{20}$$

The dummy variable will continue to take the value of zero for the credit risk prediction. This implies that relationship (20) can be simply rewritten in the form of (21) for the purposes of the quarterly default rate prediction.

$$df_t = \Psi(-2.0731 - 4.9947 gdp_t + 2.7839 R_{t-4} - 2.4364 \pi_{t-2}) \tag{21}$$

The coefficients from equations (20) and (21) cannot be simply interpreted as the commonly used elasticities of impacts of the relevant macroeconomic factors on credit risk, as they are further recalculated using the cumulative distribution function of a normal distribution; hence their impact is not linear. A simple sensitivity analysis of the impacts of changes in macroeconomic variables is given in subsection 4.3.

The ability to explain the quarterly default rate by means of the estimated model (20) is shown in *Figure 2*. The estimated model is a version of the binary choice model,[21] to which the standard approaches to measuring the statistical significance of an estimate cannot be applied. However, there are numerous less common indicators which can be applied and which suggest that the model has good performance.

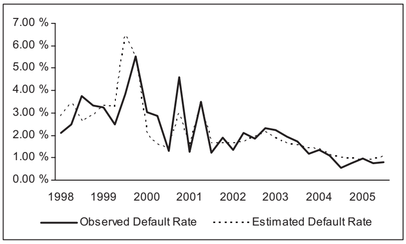
One of the tests of model quality is a test of the hypothesis that all the coefficients β_j except the constant member are zero $(\mathrm{H}_0\colon \beta_1 = \beta_2 = \ldots = \beta_N = 0)$. This hypothesis can by tested by likelihood ratio $\lambda = L_C / L_U$, where L_C denotes the likelihood function of the constrained model and L_U the likelihood function of the unconstrained model. The known result says that $-2\ln\lambda$ is an asymptotic chi-squared distributed

[19] The latent factor expressed the unobservable part of the macroeconomic risk in the model, which cannot be explained by macroeconomic indicators.

[20] Ψ denotes the standard normal distribution function, *df* denotes the quarterly default rate, index *t* denotes the relevant time.

[21] Binary models generally consider situations with two possible realizations of a dependent variable (0,1).

FIGURE 2 Performance of the One-Factor Model for the Czech Economy



variable with N degrees of freedom.[22] The result of the test rejected the hypothesis at a significance level of less than 1 %.

The observed criteria of pseudo-coefficients of determination based on the likelihood function also bear out the high quality of the model. These coefficients should be in the interval [0,1], with results close to 1 attesting to the very high quality of the model.

$$R_E^{\;2} = 1 - \left(\frac{\ln L_U}{\ln L_C)} \right)^{-\frac{2}{n}\ln L_C} = 0.97 \qquad \text{(Estrella, 1998)} \qquad (22)$$

$$R_{CU1}^{\;\;2} = 1 - \left(\frac{L_C}{L_U} \right)^{\frac{2}{n}} = 0.95 \qquad \text{(Cragg, Uhler, 1970)} \qquad (23)$$

$$R_{CU2}^{\;\;2} = \frac{1 - \left(\frac{L_C}{L_U} \right)^{\frac{2}{n}}}{1 - L_C^{\;\frac{2}{n}}} = 0.95 \qquad \text{(Cragg, Uhler, 1970)} \qquad (24)$$

$$R_{VZ} = \frac{2(\ln L_U - \ln L_C)}{2(\ln L_U - \ln L_C) + n} \frac{2\ln L_C - n}{2\ln L_C} = 0.80 \qquad \text{(Veall, Zimmermann, 1992)} \qquad (25)$$

4.5 Use of the Model in Stress Testing

Using the estimated model, the impacts of macroeconomic shocks on the default rate of the banking portfolio can be tested at the level of the aggregate economy. The estimated model is based on quarterly time series, so the estimated default rate is also a quarterly figure, which needs to be annualized for the purposes of stress testing. Two approaches are possible for solving this problem. First, the quarterly default rate is multiplied by four, which is the upper estimate of the annual default rate. Second, the four quarterly default rates and their sum under the assumption

[22] The known result of the distribution is mentioned, for example, by Rao (1973).

TABLE 2 Sensitivity Analysis of the Model
(quarterly change in bad loans in response to the value of exogenous variables) [a]

CPI (in %)	R (in %)	GDP Growth Rate (in %)							
		-1,0	**0,0**	**1,0**	**2,0**	**3,0**	**4,0**	**5,0**	**6,0**
1	2	2,3	2,1	1,8	1,6	1,4	1,2	1,1	1,0
	3	2,5	2,2	2,0	1,7	1,5	1,3	1,2	1,0
	4	2,6	2,4	2,1	1,8	1,6	1,4	1,3	1,1
	5	2,8	2,5	2,2	2,0	1,8	1,5	1,4	1,2
	8	3,4	3,0	2,7	2,4	2,1	1,9	1,7	1,5
2	3	2,3	2,1	1,8	1,6	1,4	1,3	1,1	1,0
	4	2,5	2,2	2,0	1,7	1,5	1,4	1,2	1,0
	5	2,7	2,4	2,1	1,9	1,6	1,5	1,3	1,1
	8	3,2	2,9	2,6	2,3	2,0	1,8	1,6	1,4
3	4	2,4	2,1	1,9	1,6	1,4	1,3	1,1	1,0
	5	2,5	2,2	2,0	1,8	1,6	1,4	1,2	1,1
	8	3,0	2,7	2,4	2,2	1,9	1,7	1,5	1,3
4	5	2,4	2,1	1,9	1,7	1,5	1,3	1,1	1,0
	6	2,5	2,3	2,0	1,8	1,6	1,4	1,2	1,1
	8	2,9	2,6	2,3	2,0	1,8	1,6	1,4	1,2

Note: [a] The sensitivity analysis uses non-lagged GDP growth, CPI inflation lagged by two quarters and nominal interest rates lagged by four quarters.

that the observed portfolio does not change are calculated. In order to forecast the default rate, we have to set the inputs to the macroeconomic credit risk model, which will also be used as the stress testing parameters. These include the non--lagged annual real GDP growth rate, nominal annual interest rates lagged by four quarters and annual inflation lagged by two quarters relative to the forecast horizon. These values can be set either expertly or as a percentage deviation from the macroeconomic forecasts drawn up by the CNB or as outputs from the CNB´s macroeconomic model under an assumption of significant, improbable, but not entirely impossible, negative macroeconomic shocks. *Table 2* gives the results of the macroeconomic credit risk model for different combinations of values for GDP growth rate, the nominal interest rate and inflation rate. These are merely illustrative examples of the sensitivity of the credit risk indicator for different combinations of the explanatory variables, and are not the actual values entering the stress testing.

Table 2 shows that the sensitivity of credit risk to, for example, a change in GDP growth of 1 percentage point differs *ceteris paribus* depending on the rate of such growth. For higher GDP growth rates, the impacts of a decline in growth of 1 percentage point are lower than for lower growth rates. The underlying reason for this is that the chosen variant of the model or estimation of the model (21) uses a calculation based on the standard normal distribution function. A similar conclusion applies to the other variables in the model.

The results of the macroeconomic credit model are used in the current version of stress testing for estimating the proportion of bad loans in the portfolio, which is then entered in the stress testing as an input parameter. The credit risk model allows us to generate bad loans in the banking portfolio as a result of a shock in the form of a change in real GDP growth, nominal interest rates or inflation.

5. Conclusion

We have investigated macroeconomic models of default rate estimation. The concept of a latent factor model which is based on the Merton idea was followed. These models were originally employed in individual credit risk modeling. Unobservable factors are an integral part of the models. The standard normal distribution of the unobservable factor is usually assumed. A static version of this model was considered for all of the estimations in this paper. Coefficients can be estimated using the likelihood function. The solution of a maximization problem leads to the integral over the random effects.

In order to develop a macroeconomic credit model for the Czech economy, we used a one-factor Merton-type model estimated for the aggregate economy. The model confirmed a very strong link between bank portfolio quality and the macroeconomic environment. The estimated macroeconomic credit risk model was incorporated into the existing version of stress testing.

Despite the good performance of the aggregate model, sectoral models would be desirable. The impact of the macroeconomic indicators on household and corporate sector credit risk should be different. Sectoral analysis could help to distinguish these effects. The others difficulties of the model are related to the incorporation into stress testing. The current framework assumes the worst possible scenario for a variable referred to as "loss given default", i.e. a 100% loss. The modeling of the impact of macroeconomic shocks on the volume of bad loans in the portfolio could be made more precise by estimating a model of loss given default as a function of the probability of default based on aggregate data. A further possible improvement to the default rate modeling of the Czech economy would be to make the model dynamic. This approach is able to capture non-constant assets volatility. Nevertheless, numerical solution of such models is fairly complicated.

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