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Multiscale Tail Risk Interdependence between Precious Metals

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Abstract

This paper investigates extreme risk interdependencies between four precious metal markets in different periods and in different time-horizons. Several wavelet approaches are used for this task – coherence, correlation and cross-correlation. Wavelet coherence shows strong extreme risk connection in the longer time-horizons, particularly between gold and other markets and between platinum and palladium. Wavelet correlations further strengthen wavelet coherence results, but also show that high correlation is present even in the short time-horizons for the gold-silver and platinum-palladium pairs. Wavelet cross-correlations reveal that gold and silver lead platinum and palladium in short term, whereas this situation reverses in the longer time-horizons. This indicates that investors in the bigger markets closely monitor extreme risk developments in the smaller markets in longer time-horizons and take them as a forecast what might happen in the future. On the other hand, bigger markets react faster to global shocks due to higher trading volumes, which is the reason why they lead smaller markets in short term.

1. Introduction

Precious metals are important assets for various purposes. They particularly have a role in electronic, chemical and automotive industries, while their use in the sector of jewellery is well known. Some papers, such as Pavelka and Turan (2014), Cai et al. (2020) and Shahid et al. (2020), contended that all precious metals, and especially gold, can be recognized as a safe haven and good diversification instruments in crisis periods, due to their low correlation with traditional assets, such as stocks and bonds. However, an extensive usage of precious metals worldwide for different goals instigates perpetual changes in demand and supply of these metals, making them susceptible to significant price oscillations, which is well illustrated in Figure 1. Kirkulak-Uludag and Lkhamazhapov (2016) stated that remarkable explosion in global gold demand has happened in the past decade, asserting that rise of jewellery buying in India and China and increased use of gold for hedging purposes are the two main reasons that enhanced global gold demand. Wide price

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fluctuations inevitably lead to the presence of risk in these markets, which jeopardizes running successful business with them. Therefore, accurate measurement of metal market volatility is important, as well as cross-market volatility transmission, since volatility generates uncertainty to consumers, producers and stockholders, regarding revenues, costs and margins (see Morales and Andreosso-O'Callaghan, 2011; Kirkulak-Uludag and Lkhamazhapov, 2017). Al-Yahyaee et al. (2019) added that dynamic correlations across markets and cross-market spillovers are crucial for various market participants because information spillover between markets could have huge effect on decision making in respect to hedging, asset allocation and portfolio management.

Figure 1 Empirical Price Dynamics of Four Precious Metals

Notes: Prices of all precious metals are in USD per troy ounce.

A number of recent papers documented the phenomenon of risk spillovers between precious metal markets as well as between precious metals and other markets (see Table 1). However, even more important is the question of extreme risk interdependence between precious metals, and Wang et al. (2016) explained why this issue is essential for market participants and policy-makers. For investors, traders and portfolio managers, understanding the mechanism of extreme risk interdependence between precious metals can contribute to better management of asset risk and portfolio constructions than in the case when risk is measured by a common variance. This is a vital issue because extreme risk can inflict profound loses to market participants that are difficult to recover from. As for policy makers, close monitoring of extreme risk interdependencies can improve responses of policies to market crashes and help them in formulating effective counter-crisis measures.

This paper tries to contribute to the international literature by investigating time-varying multiscale extreme risk interdependence between four precious metals –

gold, silver, platinum and palladium. Three different wavelet approaches are employed in the research – wavelet coherence, wavelet correlation and wavelet cross-correlation, which is the first time to the best of our knowledge. This is where we find a motive for this research.

According to Fernández-Avilés et al. (2020), examining extreme risk is more important than variance risk because extreme risk transmission can be catastrophic for various market participants as well as for countries that are producers and exporters of precious metals. Thus, it is of utter importance for them to know what is the true nature of extreme risk interlink between precious metals. Besides, in this way, a crucial drawback of variance can be circumvented, which is the fact that variance puts an equal weight to both gains and losses, whereas financial risk is obviously associated only with losses, not profits.

Table 1 Literature Overview in the Field of Risk Transmission Involving Precious Metals

Author(s)	*Subject of research*	*Methodology applied*
Panel A: Risk transmission between precious metal markets and other markets		
Rehman (2020)	Extreme dependence and risk spillover between Bitcoin and precious metals	ARFIMA-FIGARCH model and VaR, CoVaR and ΔCoVaR risk measures
Uddin et al. (2020)	Risk spillover under extreme market scenarios between the US stock market and precious metals	Copula approach for tail dependence and conditional value-at-risk (CoVaR) spillover measures
Iqbal et al. (2022)	Extreme risk spillover between energy, metals and agricultural commodities	Quantile-based connectedness measure
Ahmed et al. (2022)	Tail risk, systemic risk and spillover risk between oil and precious metals	Extreme value theory
Mighri et al. (2022)	Dynamic causal relationships between US stock market indices and precious metal prices	Quantile unit root test, quantile ARDL model, and Granger-causality-in-quantiles testing approach
Panel B: Risk transmission only between precious metal markets		
Hammoudeh et al. (2011)	Volatility and correlation dynamics in price returns between precious metals	GARCH-FHS model and Value-at-Risk
Wang et al. (2016)	Extreme risk spillover effects between four gold markets	Granger causality in risk and Value-at-Risk
Dutta (2018)	Implied volatility spillover between gold and silver	Bivariate VAR-GARCH approach
Wang et al. (2019)	Financial contagion between precious metals	Wavelet-based GARCH-EVT Value-at-Risk
Uddin et al. (2019)	Return and volatility spillover between precious metal markets	Generalized VAR and forecast error variance decomposition

In this respect, researchers usually use Value-at-Risk (VaR) to measure extreme (downside) risk (see e.g. Sheraz and Dedu, 2020; Tsafack and Cataldo, 2021; El Ghourabi et al., 2021). However, VaR has many drawbacks, such as the lack of subadditivity and non-convexity, which can create multiple local optima and unstable VaR rankings (Li et al., 2012). Even more serious flaw is the fact that VaR cannot measure the losses beyond the threshold amount of VaR, which could result in misleading investment decisions. This issue was addressed by Rockafellar and Uryasev (2002), who proposed parametric conditional VaR (CVaR), which controls the magnitude of losses beyond VaR. We target extreme risk of losses, so we calculate the CVaR spillover effect at the 95% probability level. This is interpreted as an average loss of the worst 5% of returns. However, calculating CVaR in empirical time-series is inadequate because they are not independently and identically distributed. In addition, since we analyse relatively long time-period, it is reasonable to assume that asymmetric effect of volatility could be present in the time-series, but also heavy tails and distribution skewness. In order to address these potential issues, we fit every time-series of precious metals in the GJR-GARCH model with the skewed Student t distribution. White noise residuals from this model are used to create dynamic VaR time-series for each precious metal.

Al-Yahyaee et al. (2019) contended that correlation intensifies during turbulent periods, but also it may sustain in relatively long time, decreasing the benefits of diversification even in well-diversified portfolios. This clearly emphasizes different time-horizons. In order to address this issue, we use the wavelet technique in the form of several methodological solutions. The first wavelet-approach is wavelet coherence (WTC), which produces the colour image of coherence across time and different wavelet scales, i.e. time-horizons. Deficiency of this method is the absence of exact numerical values of coherence strength in the WTC plots. We bypass this problem by using wavelet correlation, which serves as complementary analysis. Wavelet correlation calculates exact level of correlation between two variables in different wavelet scales, but this methodology lacks time domain. However, both WTC and wavelet correlation behave as complements, correcting each other deficiencies. Combining these two approaches, we can gain an accurate insight into the strength of extreme risk interaction between the precious metal markets across the time and different wavelet scales.

In order to be thorough in the analysis, we determine pairwise lead (lag) relationship between extreme risks, which is an important segment of the interdependence. In this process, we use the wavelet cross-correlation tool, which indicates lead (lag) nexus at different frequency scales, showing the direction of extreme risk shocks (Živkov et al., 2023). More specifically, wavelet cross-correlation can tell which market is the transmitter of extreme risk and which market is the receiver of extreme risk. Wavelet cross-correlation can be very useful for international investors, since the leading asset can be used to forecast the dynamics of the lagging variable. Also, this is important for portfolio managers because it is not appropriate to combine in a single portfolio both risk transmitters and risk receivers. All these findings can efficiently serve global investors when they decide whether to enter or leave particular market and how to rebalance their international portfolio.

As for the existing studies, we present only the papers that researched risk transmission between precious metal markets (see Table 1), so we can compare different methods used and their corresponding results. Hammoudeh et al. (2011)

analysed the volatility dynamics in precious metals. They applied Value-at-Risk to study the risk of precious metals, using the calibrated RiskMetrics, different GARCH models, and the semi-parametric filtered historical simulation approach. In order to design optimal risk management strategies, they concluded that portfolio managers who want to follow a conservative strategy should calculate VaR using the GARCH-t model. Wang et al. (2016) used the ARMA-(T)GARCH-GED model and the variance–covariance method to estimate the dynamic downside and upside VaRs. In the following, they utilized VaR approach and Ganger causality in risk to investigate the extreme risk spillover effects among the four major world gold markets (London, New York, Tokyo and Shanghai). They found strong extreme risk spillover effects between London and New York, and London and Shanghai. The paper of Dutta (2018) investigated the implied volatility spillover effects between gold and silver markets. In this regard, he employed both symmetric and asymmetric bivariate VAR-GARCH models to investigate the uncertainty transmission relationship between these two markets. He reported that returns and shocks significantly run from gold market to silver market, but the opposite effect is not found to be statistically significant. Wang et al. (2019) applied a wavelet-based approach to identify multiscale financial contagion in four precious metal markets. They found that financial contagion existed in these markets at all time-scales. They reported that gold and silver had the stronger contagion impact, and they had a unidirectional contagion effect on the other three precious metal markets at all time-scales. On the other hand, platinum had a relatively weak contagion impact, and there was no contagion effect from platinum to gold and silver at some time scales. According to their results, palladium had the weakest contagion impact on other precious metals for most time scales, especially gold. The study of Uddin et al. (2019) examined the time and frequency domain spillovers among four precious metals. They implemented the spillover index of Diebold and Yilmaz (2012) and the frequency domain spillover measures of Barunik and Krehlik (2018). Their results showed that the asymmetric spillovers between the volatilities and returns of the precious metals considered are time varying, where negative and positive shocks cause the asymmetric spillovers and are more pronounced in times of financial turmoil. They claimed that the largest transmission of net spillovers is exerted by gold and silver.

Besides introduction, the paper is structured as follows. Second section explains the used methodologies – GJR-GARCH model, conditional Value-at-Risk, wavelet coherence, wavelet correlation and wavelet cross-correlation. Third section introduces data and shows how dynamic CVaR series are created. Fourth section, *via* three subsections, presents and discusses the results of wavelet coherence, wavelet correlation and wavelet cross correlation. The last section is reserved for conclusion.

2. Used Methodologies

2.1 GJR-GARCH Model

In order to appropriately model the stylized facts of precious metals, we apply the asymmetric GJR-GARCH model with the skewed Student-t distribution, capable of capturing heavy tails and distribution asymmetry. First autoregressive term AR(1) is used in the mean equation, which is enough to resolve the autocorrelation problem.

Variance equation in the GJR-GARCH model overcomes the problem of heteroscedasticity by default. The mathematical formulation of the mean and variance equations are presented in expressions (1) and (2), respectively.

$$y_t = C + \Theta y_{t-1} + \varepsilon_t; \qquad \varepsilon_t \sim z_t \sqrt{\sigma_t^2} \tag{1}$$

$$\sigma_t^2 = c + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I_{t-1}; \qquad I_{t-1} = \begin{cases} 1 \ if \ \ \varepsilon_{t-1} < 0 \\ 0 \ if \ \ \varepsilon_{t-1} > 0 \end{cases} \tag{2}$$

where *C* and *c* denote constants in the mean and variance equations, respectively. y_t stands for log-returns of the particular precious metal, while Θ is the autoregressive parameter. σ_t^2 is the conditional variance with the conditions $\alpha \geq 0$ and $\beta \geq 0$. α parameter gauges ARCH effect, β measures the persistence of volatility, whereas γ coefficient captures the asymmetric response of volatility to positive and negative shocks. Dummy variable (I_{t-1}) activates only in situation if the previous shock (ε_{t-1}) is negative. When $\gamma > 0$ then negative shocks increase the volatility more than positive shocks, while if $\gamma < 0$ then positive shocks increase the volatility more than negative shock. The error term (ε_t) describes the $i.i.d.$ process of skewed Student-t distribution: $\varepsilon \sim skSt(0, \sigma_t^2, \delta, \nu)$. Symbols δ and ν denote skew and shape parameters, respectively. All GJR-GARCH models are estimated by quasi-maximum likelihood (QML) technique.

2.2 Conditional Value-at-Risk

We calculate dynamic extreme risk with the parametric conditional Value-at-Risk, which is based on the Gaussian distribution. This measure indicates average amount of loss that investor might experience in a single day at certain probability. CVaR is an integral of VaR, where VaR can be expressed as $VaR_\alpha = \hat{\mu} + Z_\alpha\hat{\sigma}$. $\hat{\mu}$ and $\hat{\sigma}$ denote estimated mean and standard deviation of a particular precious metal, respectively, while Z_α indicates the left quantile of the normal standard distribution. CVaR is calculate as in equation (3):

$$CVaR_\alpha = -\frac{1}{\alpha}\int_0^\alpha VaR(x)dx \tag{3}$$

2.3 Wavelet Coherence

Both time and frequency domains are combined in wavelet coherence, indicating the strength of the connection (coherence) between two variables *via* colour surface. WTC is calculated on the continuous wavelet transformation (Torrence and Webster, 1999). Expression $W_{xy}(u,s) = W_x(u,s)W_y^*(u,s)$ explains the cross wavelet transform of the two time-series, x(t)' and y(t)', whereas W_x and W_y explains the wavelet transforms of x(t)' and y(t)', respectively. Symbols u and s are position and scale indices, while the symbol * denotes a complex conjugate. The squared wavelet coherence coefficient can be calculated as in equation (4):

$$R^2(u,s) = \frac{\left|\mathbb{S}\left(s^{-1}W_{xy}(u,s)\right)\right|^2}{\mathbb{S}(s^{-1}|W_x(u,s)|^2)\mathbb{S}\left(s^{-1}\left|W_y(u,s)\right|^2\right)} \tag{4}$$

where $\mathbb{S}(.)$ is the smoothing operator, and squared wavelet coherence coefficient ranges between $0 \leq R^2(u,s) \leq 1$. Values of $R^2(u,s)$ near zero signal weak correlation, while values near one indicate strong correlation, and this is presented *via* colour surface in the WTC plot.

2.4 Wavelet Correlation

The WTC plots cannot show exact numerical estimates, which is the drawback of this methodology. In order to overcome this deficiency, we complement wavelet coherence with wavelet correlation, which calculates the exact value of correlation across wavelet scales. Unlike wavelet coherence, wavelet correlation has no time dimension. However, when these methods are put together, both methodologies correct imperfections of the other methodology, providing the comprehensive picture of the interdependence.

In order to calculate pairwise wavelet correlations between precious metals, we use maximum overlap discrete wavelet transformation (MODWT), which computes highly redundant non-orthogonal transformation (Percival and Mofjeld, 1997). Wavelet correlation assumes the bivariate stochastic process ($\mathbb{Z}_t = (x_t, y_t)$) of two time-series, x(t)' and y(t)', where each wavelet coefficient is obtained by applying the MODWT process of $\mathbb{Z}_t$. The mathematical expression of wavelet variance for the scale j of x(t)' and y(t)' time-series is presented in the following forms: $\sigma^2_{x,j,t} = Var\left(\widehat{D}_{x,j,t}\right)$ and $\sigma^2_{y,j,t} = Var\left(\widehat{D}_{y,j,t}\right)$, whereas $\widehat{D}_{j,t} = \left(\widehat{D}_{x,j,t}, \widehat{D}_{y,j,t}\right)$ is scale j wavelet coefficient computed from $\mathbb{Z}_t$. Accordingly, time-dependent wavelet covariance for scale j is then $COV\left(\widehat{D}_{x,j,t}, \widehat{D}_{y,j,t}\right)$. When wavelet variances and wavelet covariance are combined in the same equation, then wavelet correlation coefficients ($\rho_{x,y,j,t}$) can be obtained. Expression (5) shows how wavelet correlation is computed.

$$\rho_{x,y,j,t} = \frac{COV(\widehat{D}_{x,j,t}, \widehat{D}_{y,j,t})}{\left(Var(\widehat{D}_{x,j,t})Var(\widehat{D}_{y,j,t})\right)^{1/2}} \tag{5}$$

2.5 Wavelet Cross-Correlation

Wavelet cross-correlation suggests which extreme risk leads and which one lags in different time-horizons. In this way, researchers can learn from which market extreme volatility shocks originate and which market is the recipient of these shocks.

Wavelet cross-correlation considers two time-series, as in the case of wavelet correlation, but it also calculates lagged correlation function (ρ_τ) with lag τ. In this way, wavelet cross-correlation has symmetric lagged correlation function ($\rho_\tau = \rho - \tau$). In situation when deviations between ρ_τ and $\rho - \tau$ become significant this symmetry is interrupted, which creates asymmetry in the information flow. When asymmetry happens then leading asset has predictive power over lagging asset.

According to Gencay et al. (2002), the MODWT cross-correlation equation, for scale j and lag τ can be written as follows:

$$\rho_{x,y,j,t,\tau} = \frac{COV\left(\widehat{D}_{x,j,t}, \widehat{D}_{y,j,t+\tau}\right)}{\left(Var\left(\widehat{D}_{x,j,t}\right)Var\left(\widehat{D}_{y,j,t+\tau}\right)\right)^{1/2}} \tag{6}$$

where Var and COV have the same meaning as in equation (5), and cross-correlation takes value $-1 \leq \rho_{x,y,j,t,\tau} \leq 1$.

3. Dataset and Construction of Tail Risk Time-Series

We use the daily near maturity closing futures prices of four precious metals – gold, silver, platinum and palladium. All time-series are collected from the stooq.com website, which harbours futures prices from CBOT and NYMEX markets. Data-sample ranges from January 2017 to December 2022. The sample encompasses the two tumultuous events – the corona virus pandemic and the Ukrainian war, which produced a lot of turbulence in all commodity and financial markets around the globe. Hence, it is an opportunity to inspect how extreme risk interacts between the precious metals markets in these periods. All precious metal futures prices are transformed into log-returns according to the expression: $y_{i,t} = 100 \times log(P_t/P_{t-1})$, where P is the value of a particular price. Since we research pairwise extreme risk interdependence, we synchronise each precious metal with other three, making in this way the six pairs of synchronized time-series. Synchronization is necessary because each metal is traded in different number of days in the observed period, i.e. gold (1563), silver (1554), platinum (1881), palladium (1803). Synchronization matches existing trading days, while unmatched days are discarded. After synchronization, the created pairs have following number of observations: gold-silver (1545), gold-platinum (1563), gold-palladium (1562), silver-platinum (1553), silver-palladium (1546) and platinum-palladium (1799).

Table 2 Descriptive Statistics of Precious Metals Futures

	Mean	*St. dev.*	*Skew.*	*Kurt.*	*JB*	*LB(Q)*	*LB(Q^2)*	*DF-GLS*
Gold	0.014	0.394	-0.229	8.168	1752.1	0.000	0.000	-25.214
Silver	0.011	0.791	-0.555	10.053	3298.5	0.002	0.000	-6.134
Platinum	0.004	0.698	-0.371	9.887	3758.1	0.000	0.000	-4.413
Palladium	0.022	0.961	-0.653	15.737	12308.5	0.000	0.000	-4.206

Notes: JB stands for value of Jarque-Bera coefficients of normality, LB(Q) and LB(Q2) tests refer to p-values of Ljung-Box Q-statistics of level and squared log-returns of 10 lags. 1% and 5% critical values for DF-GLS test with 5 lags, assuming only constant, are -2.566 and -1.941, respectively.

Table 2 contains descriptive statistics of selected assets, including the first four moments, Jarque-Bera test, LB(Q) tests for level and squared log-returns and DF-GLS unit root test. Palladium and silver have the highest average risk whereas gold has the lowest risk, presented by standard deviation. All precious metals have negative skewness and high kurtosis, which justifies using the skewed Student t distribution in the GJR-GARCH model. The LB(Q) and LB(Q^2) tests show that all empirical time-series have problems with autocorrelation and heteroscedasticity. These findings indicate that some form of the ARMA-GARCH model might be a good solution for these issues. The DF-GLS test suggests that all assets have no problem with unit root, which is a necessary precondition for the GARCH modelling.

We use the GJR-GARCH model with the skewed Student t distribution to estimate white noise residuals, which are used subsequently for the construction of time-varying CVaR series. All α and β parameters in Table 3 are highly statistically significant, which means that the ARCH and GARCH effects are present in the time-series. Gold and silver report an asymmetric effect, where γ parameters have negative sign. Negative γ means that positive shocks increase the volatility more than negative shock. However, statistically significant γ parameters are relatively low, which indicates that positive shocks do not have notable advantage over negative shocks. All distribution parameters (δ and ν) are highly significant, suggesting that the skewed Student t distribution can recognize very well third and fourth moments of the empirical distributions. The diagnostic tests show that all created residuals do not have autocorrelation and heteroscedasticity, which is a good basis for the creation of dynamic CVaR time-series.

Table 3 Estimated GJR-GARCH Parameters

	Gold	*Silver*	*Platinum*	*Palladium*
Panel A: GARCH parameters				
α	0.047***	0.051***	0.039***	0.067***
β	0.961***	0.943***	0.955***	0.921***
γ	-0.021**	-0.025***	0.001	-0.002
Panel B: Distribution parameters				
δ	-0.71***	-0.081***	-0.094***	-0.103***
ν	4.282***	4.050***	7.478***	5.844***
Panel C: Diagnostic tests				
LB(Q)	0.666	0.199	0.683	0.418
LB(Q^2)	0.246	0.154	0.218	0.439

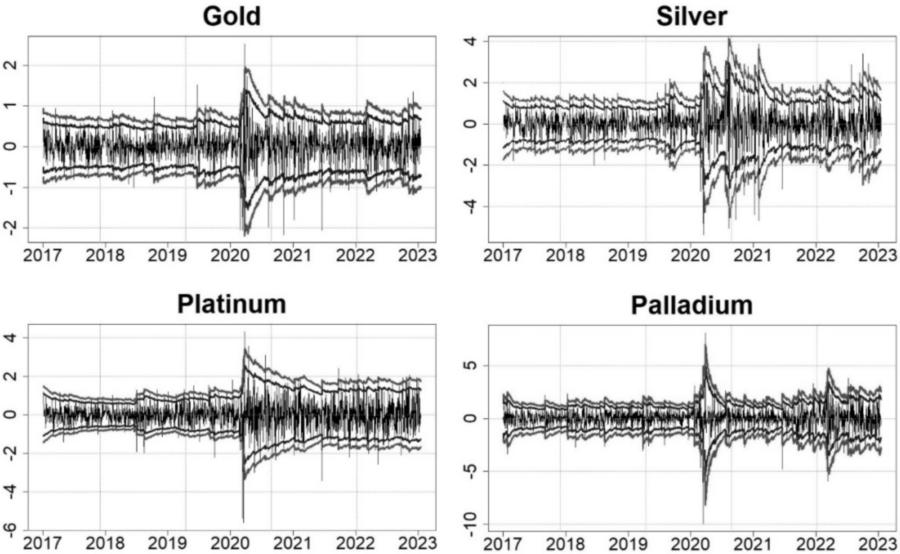
Notes: LB(Q) and LB(Q2) numbers show p-values. ***, ** indicate statistical significance at the 1% and 5% level, respectively.

Figure 2 shows log-returns of the precious metals and the two dynamic extreme downside and upside risks – VaR and CVaR, calculated at 95% probability. According to Figure 2, all precious metals have segments with extreme risk, which is particularly true since 2020, when the pandemic erupted. In the research process, we use only the downside risk of CVaR, embedding these tail risk time-series into the three different wavelet frameworks. The research addresses multiscale interdependence, so six wavelet scales are observed. The frequency scales are: scale 1 (2-4 days), scale 2 (4-8 days), scale 3 (8-16 days), scale 4 (16-32 days), scale 5 (32-64 days) and scale 6 (64-128 days)[2]. The first four scales correspond to the short-

[2] In the literature, frequency scales can also be called wavelet details, and the label of wavelet details is capital letter "D".

term horizon, whereas the fifth and sixth scales are regarded as midterm and long-term, respectively.

Figure 2 Created Extreme Downside Risk Time-Series of Precious Metals



Notes: Dark-grey (light-grey) lines denote upper and lower dynamic VaR (CVaR) time-series, calculated at 5% probability.

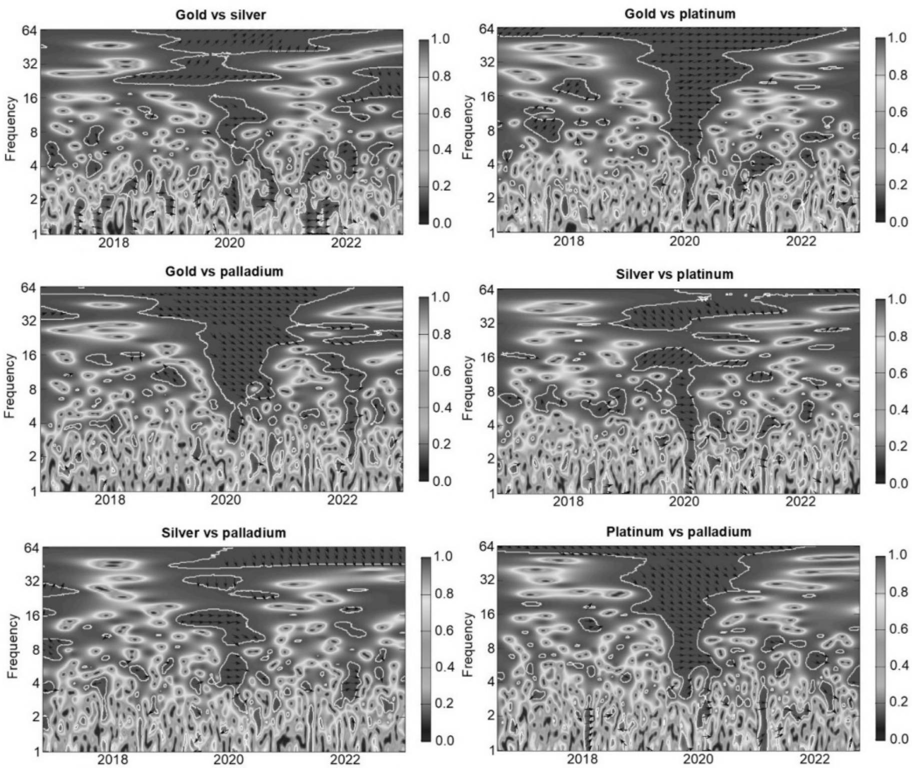
4. Empirical Results

4.1 Wavelet Coherence Results

This subsection reveals the interaction of extreme risk between precious metals, across time and wavelet scales. The research is done *via* wavelet coherence, and Figure 3 contains the six WTC plots. WTC plots can observe two dimensions simultaneously – time and frequency. Time component can be viewed on the horizontal axis, while different wavelet scales are depicted on the left vertical axis. Different time-horizons are portrayed *via* wavelet scales, which goes up to the sixth scale (64-128 days). The WTC plots show the strength of interdependence between two variables, whereas this strength is depicted in black and white surface. In particular, dark grey indicates high coherence, while light grey colours suggest low coherence. Dark grey areas with white lines suggest very strong co-movement, and these areas also contain phase arrows, indicating the direction of coherence. Right-pointing arrows suggest positive coherence, while left-pointing arrows indicate otherwise. Right vertical axis presents the colour pallet that ranges from 0 to 1, and these values correspond to the strength of coherence.

According to Figure 3, all extreme risk interdependencies are time- and scale-varying, which justifies the use of wavelet coherence methodology. In particular, all plots in Figure 3 contain both darker and lighter colours. Lighter colours are predominantly positioned at lower scales (higher frequency), i.e. up to 8 days, while darker colours dominate at higher wavelet scales (lower frequency), i.e. from 8 days onwards. This means that extreme risk interdependence is stronger at the higher wavelet scales or the longer time-horizons, while this nexus is weaker at the shorter time-horizons, which is depicted by the lower wavelet scales. The distribution of CVaR connections in Figure 3 coincide with the papers of Mensi (2019) and Bouri et al. (2020). These papers researched different topic from our own, but they also addressed VaR interdependence in the wavelet framework, reporting weaker (stronger) connection at lower (higher) wavelet scales. Besides, the paper of Wang et al. (2019) should be especially underlined because they analysed dynamic VaR interdependence using wavelet coherence. They also reported stronger coherence in higher wavelet scales, which concurs very well with our research.

Figure 3 Wavelet Coherence Plots of the Six Pairs



Notes: Left Y axis denotes frequency scales, while right Y axis indicates colour pallet.

In the shorter time-horizons, risk transmission between the markets is associated with information transfer, according to Ross (1989). In these time-horizons, a lot of activity is happening in the markets, while there is a little time for synchronization of market movements, even when it comes to the extreme price shifts. This is the reason why lighter colours prevail over darker colours in the lower frequency scales, and this is the intrinsic characteristic of all WTC plots. On the other hand, at the higher wavelet scales, darker colours take over dominance in all WTC plots. These findings can be explained by the fact that fundamental factors and global events came to the fore in the longer time-horizons, and these forces affect the commodity markets relatively equally. In other words, under the common influence of global shocks at long term, commodity markets have relatively uniform dynamics, which is recorded as high coherence areas. This is particularly true for the COVID-19 pandemic, which was an extremely bearish period on the entire planet. It is obvious that very high coherence areas are present in some plots around 2020 – gold vs platinum, gold vs palladium and platinum vs palladium, which was the year when the pandemic started. In 2020, areas of high coherence in all WTC plots descend to very low wavelet scales, which indicates that all markets behaves very synchronous, even at very short time-horizons, in the times of great distress. Also, all phase arrows in the delineated dark areas point to the right, which indicates positive coherence, and this is an argument in favour of a common global influence on commodity markets. Stronger market interconnections during global disturbances is well known phenomenon. For instance, Uddin et al. (2019) examined the spillover characteristics of returns and volatilities of four precious metals and reported that negative and positive shocks are more pronounced in times of a financial turmoil, which is in line with our findings. Besides, Umar (2021) studied the dynamic return and volatility connectedness between industrial and precious metals, and reported that net directional volatility connection increases sizably during the global COVID-19 crisis, which is in line with our results.

However, it is also evident that high coherence zones cover different areas in the WTC plots, which is a clear signal that various extreme risk interdependencies exists between different metals. It is obvious that wide areas of high coherence can be found in the gold vs platinum, gold vs palladium, platinum vs palladium and somewhat gold vs silver plots. According to the results, gold is constituent part in all WTC plots where the wide areas of high coherence is present. It is not unusual to find that gold has a high connection with other precious metals (see Hammoudeh et al., 2010), because gold market is by far the largest precious metal market in terms of daily trading volumes. This is confirmed by Table 4, which contains average daily trading volumes in the four markets in 2019[3], where can be seen that gold has significantly higher daily trading volumes compared to all other markets. In other words, the movements in smaller markets are highly connected with the happenings in the largest gold market, which is very important to know because this fact has repercussions for decision making. However, the WTC plots cannot indicate which market lead and which one lags, and this is crucial information for market participants to decide whether and how to rebalance their positions. This information is provided by wavelet cross-correlation in section 4.3.

[3] We choose 2019 for trading volumes presentation, since 2019 is the pre COVID-19 year. In this way, we can evade possible bias in empirical trading numbers due to the global pandemic.

Table 4 Average Daily Trading Volumes of the Precious Metal Futures Markets in 2019

Gold	*Silver*	*Platinum*	*Palladium*
343,688	95,941	23,282	5,045

Source: Authors' calculation based on data from stooq.com website.

It is interesting to note that the platinum-palladium plot also reveals the wide area of high coherence, which indicates the existence of close correlation between the extreme risks of these two markets. However, the reason for this finding is not the same as in the cases of gold and other precious metals, because these two markets are the smallest. Explanation probably lies in the fact that these two metals are used for similar industrial purposes. Namely, in recent years, they are used predominantly for the production of catalytic converters, which reduce carbon emissions. It is also known that platinum and palladium are used in the manufacturing of dental applications, electronic components industry and jewellery sector. Therefore, due to the fact that these two metals can be regarded as substitutes in many ways, it is possible that extreme price changes in one market could cause large price movements in another market. We detect the extreme risk synchronization of these two metals particularly during the COVID-19 crisis, when extreme price changes are apparent (see Figure 2).

4.2 Wavelet Correlation Results

In order to complement the WTC plots, we also calculate wavelet correlations, which show exact estimates in different wavelet scales, but they have no time component. When WTC and wavelet correlations are combined together, we can obtain an accurate picture about the strength of multiscale interdependence between two variables. Table 5 contains wavelet correlations, while Figure 4 illustrates these findings.

According to the results, all wavelet correlations gradually increase with the rise of wavelet scales up to 32 days (D5), whereas, in four out of six cases, they are a little bit lower in the D6 scale, but also very high. This confirms previous findings that extreme risk interlink increases in the longer time-horizons, which can be attributed to the influence of global events and fundamentals. Since wavelet correlations offer exact values, it can be seen that these links are very strong in the long-term horizons, with levels even above 80% in the case of gold-platinum. High wavelet correlations are also detected in midterm, with values beyond 70% in all the cases. These findings fit very well with the WTC results, which adds to the robustness of the overall results. It can be noticed that the wavelet correlations between gold and silver are very high even in very short time-horizons, which is not so evident in the WTC plot. This confirms the previous assertion that smaller markets are well connected with the biggest one.

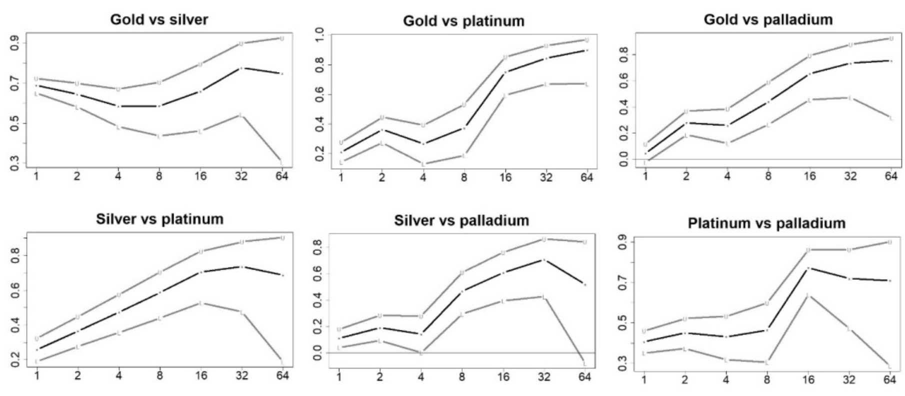
Table 5 Estimated Wavelet Correlations of the Six Pairs

	Gold vs silver	*Gold vs platinum*	*Gold vs palladium*	*Silver vs platinum*	*Silver vs palladium*	*Platinum vs palladium*
1 day – (raw)	0.688	0.212	0.045	0.257	0.111	0.408
2 days – (D1)	0.644	0.364	0.279	0.366	0.191	0.451
4 days – (D2)	0.584	0.267	0.257	0.473	0.143	0.432
8 days – (D3)	0.586	0.374	0.441	0.588	0.468	0.465
16 days – (D4)	0.659	0.752	0.654	0.706	0.610	0.773
32 days – (D5)	0.777	0.845	0.735	0.737	0.709	0.719
64 days – (D6)	0.748	0.900	0.754	0.687	0.518	0.710

Notes: Symbols D1-D6 refer to wavelet details or six wavelet scales.

As for the lower wavelet scales, we find that four out of six examined pairs have relatively low extreme risk interdependence in the very short-term horizon, i.e. up to 2 days, which corresponds to the lighter colours in WTC plots. On the other hand, the gold-silver and platinum-palladium pairs report relatively high average correlations at very high frequency scales, which is not easily visible in the WTC plots. The WTC plots provide a good indication that strong extreme risk interdependence exists between these markets, but it is not apparent that strong links are present even at very short time-horizons. This is a clear indication that wavelet correlations can improve the WTC results and correct some conclusions that might be drawn solely on the WTC findings. These nuances in the results speak in favour of a multi-methodological approach.

Figure 4 Wavelet Correlation Plots of the Six Pairs



Notes: X axis presents different wavelet scales, while Y axis denotes levels of wavelet correlations.

4.3 Wavelet Cross-Correlation Results

This section presents findings regarding the lead (lag) connection between the markets. In other words, the results can tell from which market extreme volatility shocks originate, and on which market these shocks transfer. The wavelet cross-correlation methodology is designed for this purpose. This type of knowledge can be very useful for various market participants because it can indicate how traders, investors, portfolio managers should behave in situations when extreme risk spills over between the markets. Table 6 contains results of 20 daily lags in the wavelet cross-correlation procedure, while Figure 5 gives graphical presentation. In commenting the cross-correlation findings, we only observe the values at fifth lags (bolded values in Table 6).

Table 6 Wavelet Cross-Correlation Results of the Six Pairs

Pairs of metals	*Wavelet scales*	*Negative lagged correlations*				*Positive lagged correlations*			
		-20	*-15*	*-10*	*-5*	*5*	*10*	*15*	*20*
Gold vs silver	D1	-0.019	0.056	0.040	**0.041**	**0.041**	-0.024	0.020	-0.068
	D2	-0.011	0.093	0.016	**-0.046**	**0.082**	-0.048	0.071	-0.076
	D3	-0.049	0.181	-0.071	**-0.385**	**-0.039**	-0.223	0.162	-0.089
	D4	-0.072	-0.010	-0.241	**0.100**	**0.444**	-0.048	-0.079	-0.053
	D5	-0.269	-0.079	0.204	**0.496**	**0.695**	0.520	0.246	-0.020
	D6	0.091	0.307	0.504	**0.658**	**0.772**	0.719	0.602	0.439
Gold vs platinum	D1	-0.007	0.042	0.013	**0.049**	**0.054**	0.062	0.114	0.062
	D2	0.026	0.172	-0.028	**-0.081**	**-0.038**	-0.176	0.041	0.040
	D3	-0.021	0.138	-0.051	**-0.254**	**0.201**	-0.194	0.017	0.015
	D4	-0.070	-0.010	-0.123	**0.012**	**0.531**	0.097	-0.147	-0.184
	D5	-0.229	-0.005	0.292	**0.582**	**0.726**	0.494	0.154	-0.159
	D6	0.310	0.499	0.662	**0.777**	**0.796**	0.691	0.528	0.327
Gold vs palladium	D1	0.025	-0.057	-0.057	**0.013**	**-0.035**	-0.009	0.053	0.038
	D2	-0.026	0.045	-0.133	**-0.085**	**-0.146**	-0.110	0.032	0.055
	D3	-0.003	0.087	-0.201	**-0.034**	**-0.054**	-0.037	-0.008	0.033
	D4	-0.097	-0.193	-0.232	**0.145**	**0.338**	0.039	-0.059	-0.076
	D5	-0.367	-0.194	0.097	**0.418**	**0.672**	0.511	0.243	-0.017
	D6	-0.055	0.165	0.382	**0.571**	**0.780**	0.780	0.714	0.598
Silver vs Platinum	D1	0.011	-0.007	0.020	**0.071**	**-0.006**	0.020	0.081	0.049
	D2	0.051	0.062	-0.018	**0.007**	**-0.122**	-0.112	0.019	-0.036
	D3	0.028	0.044	-0.140	**-0.144**	**-0.054**	-0.190	0.102	-0.020
	D4	-0.098	-0.167	-0.185	**0.211**	**0.346**	-0.085	-0.147	-0.180
	D5	-0.187	0.004	0.292	**0.567**	**0.592**	0.297	-0.061	-0.337
	D6	0.404	0.547	0.654	**0.706**	**0.603**	0.455	0.264	0.054
Silver vs palladium	D1	0.022	-0.033	-0.037	**0.013**	**-0.023**	-0.052	0.015	0.034
	D2	-0.010	-0.012	-0.109	**0.030**	**-0.074**	-0.070	0.001	0.044
	D3	-0.001	-0.021	-0.155	**0.074**	**-0.026**	-0.101	0.042	0.038
	D4	-0.203	-0.310	-0.170	**0.275**	**0.228**	-0.049	-0.047	-0.062
	D5	-0.338	-0.200	0.085	**0.400**	**0.584**	0.384	0.102	-0.142
	D6	0.117	0.291	0.452	**0.579**	**0.663**	0.610	0.501	0.351
Platinum vs palladium	D1	0.018	-0.056	-0.048	**-0.007**	**-0.047**	0.021	-0.042	0.008
	D2	0.013	-0.053	0.117	**-0.121**	**-0.073**	0.120	-0.021	0.006
	D3	-0.035	-0.181	0.168	**-0.179**	**-0.126**	-0.039	-0.073	0.038
	D4	-0.303	-0.261	0.034	**0.391**	**0.223**	-0.157	-0.194	-0.064
	D5	-0.410	-0.112	0.279	**0.603**	**0.607**	0.333	0.034	-0.186
	D6	-0.005	0.201	0.398	**0.559**	**0.687**	0.642	0.539	0.400

Notes: Symbols D1-D6 refer to wavelet details or six wavelet scales.

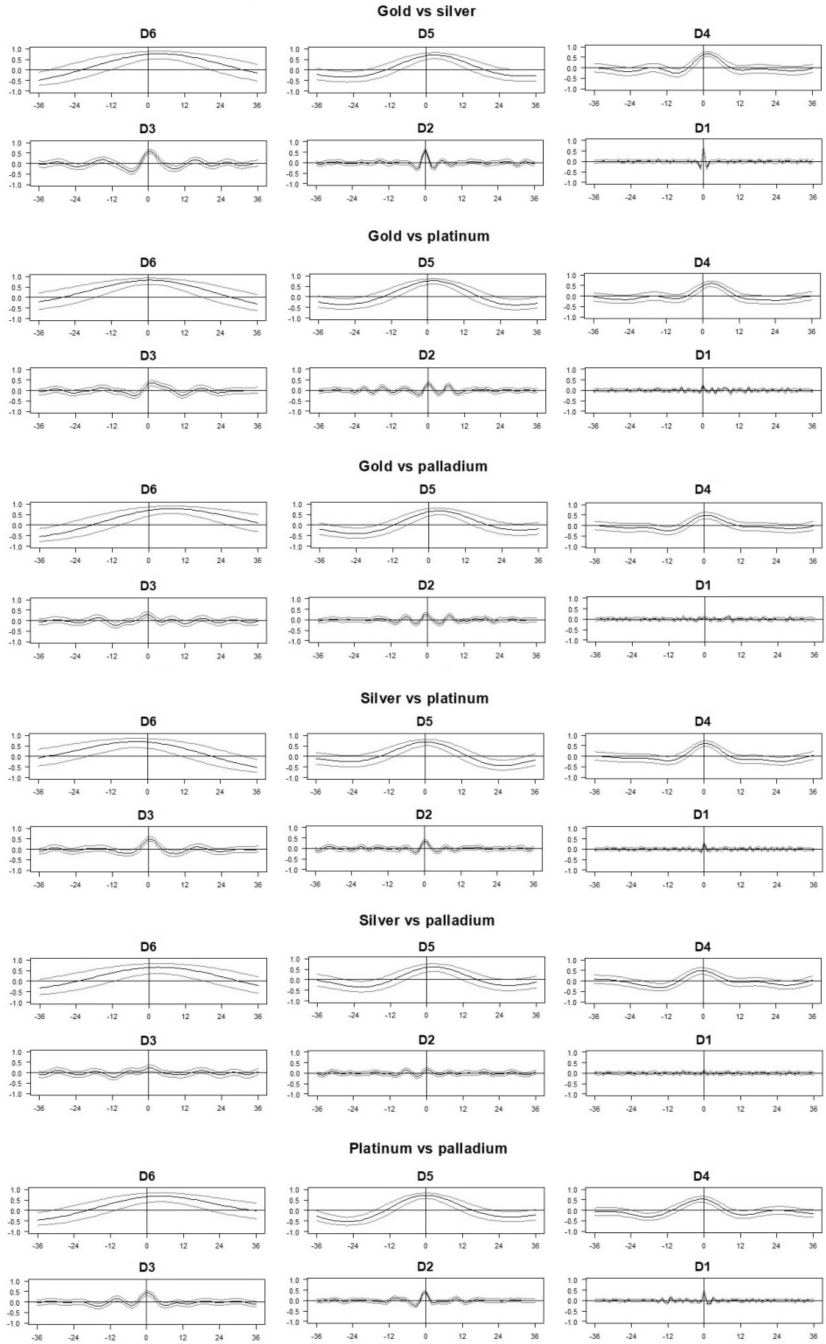
Based on Table 6 and Figure 5, we can inspect whether the extreme risk pulling effect exists between the selected precious metals at contrasting time lags. First name of some metal in Table 6 or in the plots in Figure 5 is the first variable that enters this computational process. Accordingly, the left side of the wavelet cross-correlation plots depicts the lagged correlation of the first metal, while the right part of the plots portrays the lagged correlation of the second metal. The cross-correlation curve in Figure 5 plots determines extreme risk lead-lag nexus between precious metals. More specifically, if cross-correlation curve is tilted in the left side of the graph, then it means that the first time-series leads the second, and *vice-versa* (see Bhandari, 2017). At the lower wavelet scales, tilt of the cross-correlation curve is not clearly visible, so we also present the cross-correlation values in Table 6, which can give a better indication which metal leads and which one lags in the six wavelet scales.

Our results indicate that gold, in the short run (up to D3 scale), mostly leads other metals. The same applies for silver, i.e. silver leads platinum and palladium up to D3 scale. Explanation for this finding probably lies in the fact that gold and silver are significantly bigger markets than platinum and palladium. This suggests that investors in smaller markets follow up extreme price changes in bigger markets in the short run and use them for their future actions.

This is expected and in line with the previous findings. For instance, Uddin et al. (2019) asserted that the largest transmission of net return and volatility spillovers is exerted by gold and silver, while palladium and platinum are mainly spillover receivers. Sensoy (2013) reported results that concur with the previous one. He claimed that gold has unidirectional volatility shift contagion effect on all other precious metals, while silver has similar effect on platinum and palladium. However, the latter two do not have volatility spillover effect on the former two. Researching only the gold and silver markets, Dutta (2018) concluded similarly as the previous two papers, asserting that return and shocks significantly run from gold VIX to silver VIX, but not the other way around.

However, our investigation is richer in findings because we examine risk transmission in different time horizons, where we find an evidence that this effect alters completely in the longer time-horizons. In other words, in all the cases, situation changes diametrically, meaning that smaller market takes over leading role in the longer time-horizons (from D4 scale onwards). This suggests that investors in the bigger markets, with long-term positions, keep tracking the extreme risk developments in the smaller markets and react subsequently to these changes. Smaller markets lag in short term because bigger markets react faster to global shock due to higher trading volumes (see Table 4). On the other hand, it seems that investors in the bigger markets closely monitor extreme risk developments in the smaller markets in the longer time-horizons and take them as an omen what might happen in the future. This type of findings is consistent across the markets, which adds to the credibility of the results.

Figure 5 Wavelet Cross-Correlation Plots of Six Pairs



Notes: X axis denotes lags expressed in days, while Y axis stands for wavelet cross-correlation.

5. Summary and Conclusion

This paper researches the extreme risk interdependence between four futures precious metal markets. Contribution of this study reflects in the fact that we investigated the nexus *via* both time and frequency, using different elaborate methodological approaches in this process. In particular, biasfree time-varying CVaR series are created by the GJR-GARCH model with the skewed Student t distribution, while multiscale interdependence is examined with different wavelet solutions – coherence, correlation and cross-correlation.

Several noteworthy findings can be reported. First, wavelet coherence indicates that the extreme risk interdependence is time- and scale-varying, which supports our approach to use wavelet methodologies. Strong wavelet coherence is mostly distributed in the longer time-horizons in all WTC plots, which suggests that extreme risk interlinks are highly correlated in the longer time-spans. The reason for such findings probably lies in the fact that common global events and fundamentals that came to the fore in longer terms, have strong and homogenous influence on commodity markets worldwide, making them moving in the same direction. The WTC results reveal that high coherence exist between gold and other metals, because gold is the biggest and most influential precious metal market. Besides, high coherence is also found between platinum and palladium, probably because these metals can be regarded as substitutes for various purposes.

Second, wavelet correlations further strengthen the WTC results, but also show that high correlation is present even at the short time-horizons in the gold-silver and platinum-palladium pairs, which is not so obvious to notice in the WTC plots.

Third, wavelet cross-correlations reveal that bigger markets (with higher daily trading volumes) have upper hand in short term over smaller markets (with lower daily trading volumes), when it comes to the extreme risk shock transmission. On the other hand, situation changes in the longer time-horizons, when smaller markets take over leading role over bigger markets.

The paper is well in line with the existing studies, such as Hammoudeh et al. (2010), Uddin et al. (2019), Sensoy (2013), Dutta (2018). However, the study of Wang et al. (2019) should be especially emphasized because they combined dynamic VaR series of precious metals in the wavelet coherence framework, similar as we do. However, our paper is different in terms of using CVaR instead of VaR, employing other wavelet techniques (wavelet correlation and cross-correlation), and also, comprising the pandemic and the war in Ukraine, which is novelty compared to the research of Wang et al. (2019), who observed the period up to 2019.

Results from this paper can be useful for investors, portfolio managers, but also for policymakers, who work with precious metals in different time-horizons. Based on the results, investors can learn how to make proper investment decisions, i.e. to choose appropriate time to enter or leave particular market. More specifically, short-term investors in the platinum and palladium markets should have a close eye on the extreme risk levels in the gold and silver markets because these markets lead the other two markets. In other words, when extreme price swings occur in the gold and silver markets, this should be a signal for market participants in the platinum and palladium markets to allocate their investments or to hedge their positions. The opposite happens in long-term, when the smaller markets have a leading role over bigger markets.

Also, portfolio managers can use the results for optimal portfolios construction in various time-horizons, combining the metals that have the weakest links between extreme risks. This means that the silver-platinum and silver-palladium pairs are appropriate candidates to be found in the same portfolio because they have relatively low extreme risk connections across different time-horizons. Policymakers in countries with significant revenues from precious metal exports can better understand how extreme risk shocks transmit between the markets, which leaves them more room to devise better strategies that will diminish risk contagion in the periods of economic downturns.

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