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Performance Ratios for Selecting International Portfolios: A Comparative Analysis Using Stock Market Indices in the Euro Area*

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Abstract

This paper compares the ability of alternate performance measures to support investment selection in ten-euro area stock markets. The performance ratios used in the paper are grouped in two main categories. One category comprises the performance ratios using risk measures which do not separate systematic and non-systematic risk. The performance measures of this group are Sharpe ratio, Sortino ratio, Rachev ratio and STARR ratio. The other category comprises performance ratios based exclusively on systematic risk given by asset pricing models. The performance ratios of this category are the standard Treynor ratio based on CAPM betas, and two innovations of this ratio, mentioned in the paper by $Treynor^d$ *and* $Treynor^u$*, based on the betas given by an asset pricing model, highlighted in this paper as Downside-Upside Risk Model (DURM), which estimates separate betas for downside and upside market returns. The empirical part of the paper consists of recursive portfolio selection based on each of the performance ratios mentioned above. The comparison of the ex post returns of the different portfolios shows that portfolios based on Sharpe, Sortino and STARR ratios offer better protection against losses in low return periods in the financial markets of the euro area , while Rachev ratio and Treynor,* $Treynor^d$ *and* $Treynor^u$ *ratios are more able to take advantage from high return periods.*

1. Introduction

This paper compares the ability of alternate performance measures to support investment selection in ten-euro area stock markets. A performance measure is the ratio between the expected reward of a financial asset or portfolio and a risk measure. Taking into consideration the major differences in risk measure definitions, we group the performance ratios used herein in two categories. One category comprises the performance ratios based on risk measures which do not separate systematic and non-systematic risk, while the other comprises performance ratios based on systematic risk (beta coefficients) given by asset pricing models. The performance measures of the first category are Sharpe ratio, Sortino ratio, Rachev ratio and STARR (Stable Tail Adjusted Return Ratio). The performance ratios of the second category are the standard Treynor ratio based on CAPM betas, and two innovations of this ratio, designated in the paper by $Treynor^d$ and $Treynor^u$, based on the betas given by an asset pricing model, called here the Downside-Upside Risk Model (DURM), which estimates separate betas for downside and upside market returns. The Sharpe ratio, proposed by Sharpe (1966, 1994), relates the difference between the excess of the expected (or mean) return of an

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asset (portfolio) and the risk-free interest rate, with asset's (portfolio's) return standard deviation. The Sortino ratio, proposed by Sortino and Price (1994), uses the same return reward measure as Sharpe ratio, but uses downside semi-standard deviation as risk measure. Sortino ratio considers that investors request a reward for the risk of actual return falling below the target, but they are not concerned with positive deviations from target return. The Rachev ratio, proposed by Rachev *et al.*(2008), uses as reward measure the expected excess return relative to risk-free rate on the right tail of the return distribution, and the risk measure is the expected excess return relative to risk-free rate on the left tail of the return distribution. The fourth performance measure of this group is the Stable Tail Adjusted Return Ratio, *STARR*, which combines the return reward definition used in the Sharpe ratio with the risk measure used in the Rachev ratio.

The first performance measure based on systematic risk, used in this article, is the Treynor ratio, proposed by Treynor (1965), which relates excess expected return (*i.e.* the difference between the mean return of an asset and the risk-free interest rate) to the beta coefficient given by CAPM (Capital Asset Pricing Model). The two additional Treynor ratios used in this article, are given by the downside-upside risk asset pricing model, hereinafter designated by DURM, proposed by Ang et *al.* (2006), which estimates separate betas for downside and upside market conditions. The main assumptions underlying DURM is that investors' behavior towards returns below the target is dominated by disappointment, which makes downside risk aversion more intensive than upside risk aversion. Based on the assumption of risk aversion asymmetry, DURM estimates two beta coefficients: one measuring the sensitivity of an asset or portfolio to market return above the target, and the other measuring the sensitivity to return below the target. Consequently, DURM allows the calculation of an upside and a downside Treynor ratio, each of them measuring the reward of expected return in relation to the corresponding beta coefficient.

The empirical part of the paper consists of recursive portfolio selection based on each of the performance ratios mentioned above, over moving samples of the database. The recursive portfolio selection is supported by recursive estimation of the two asset pricing models used in the paper: CAPM and DURM, required to calculate the Treynor, Treynor^d and Treynor^u ratios. The sub-samples used in asset pricing estimations are also used to calculate the reward and risk measures required for recursive estimations of Sharpe, Sortino, Rachev and STARR ratios.

The series of *ex post* returns of the portfolios based on performance ratios are subject to pairwise comparison with separation between the high return and the low return values of the euro area index. The comparison separated by high return and low returns in the euro area allows to identify which performance ratios are more able to give portfolios with good protection against losses in bearish phases of euro area financial markets, and those which help portfolios take advantage of booming periods.

Next the paper is organized into the following parts: Section 2 describes the asset pricing models the performance ratios used in the paper, the procedures of portfolio selection and methods of *ex post* return comparison, Section 3 presents the data and the results, and Section 4 addresses the conclusions.

2. Asset-Pricing Models and Performance Ratios

2.1 CAPM Versus Downside-Upside Risk Model

The risk assumption underlying the standard capital asset pricing model is that investors equally dislike negative and positive deviations from target return. According to this assumption, a homogeneous risk-averse utility function covers the entire return spectrum.

The standard CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966) is represented as the following equation:

$$E(R_i - r_f) = \beta_i E(R_M - r_f) \quad (1)$$

where $E(R_i - r_f)$ is the expected excess return of an individual asset or portfolio *i*, relative to the risk-free interest rate, r_f, $E(R_M - r_f)$is the expected excess return of the market portfolio, *M*, relative to the risk-free interest rate, $\beta_i$is the sensitivity of asset or portfolio *i* return to the market portfolio return. CAPM can be tested through the following OLS estimation:

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_i[R_{M,t} - r_{f,t}] + \varepsilon_{i,t} \quad (2)$$

where $R_{i,t}$ is the individual current return on asset, $R_{M,t}$ is the market portfolio current return,$r_{f,t}$is the risk-free interest rate, α_i and $\varepsilon_{i,t}$are, respectively, the abnormal return and the residual return.

Based on the disappointment utility function defined by Gul (1991), the downside-upside risk asset pricing model (DURM) assumes that risk aversion is higher in the downside of the expected return than in the upside. Hence, in DURM, the expected return of asset *i* has different sensitivities (i.e. betas) relative to upside and downside market return. Consequently, according to DURM, asset *i* pays two separate premia: the downside premium and the upside premium, and the model is represented as follows:

$$E(R_i - r_f) = \beta_i^d E(R_M - r_f)^- + \beta_i^u E(R_M - r_f)^+ \quad (3)$$

where

$$E(R_M - r_f)^- = \frac{\sum_{t=1}^{N} min(0, R_{M,t} - r_{f,t})}{N} \quad (4)$$

is the downside excess expected return of the market portfolio (N = Number of possible outcomes), and

$$E(R_M - r_f)^+ = \frac{\sum_{t=1}^{N} max(0, R_{M,t} - r_{f,t})}{N} \quad (5)$$

is the upside excess expected return of the market portfolio. The coefficient β_i^d (downside beta) is the sensitivity to downside market return and β_i^u (upside beta) is the sensitivity to upside market return.

The testable version of DURM is:

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_i^d \, min\left[0, R_{M,t} - r_{f,t}\right] + \beta_i^u \, max\left[0, R_{M,t} - r_{f,t}\right] + \varepsilon_{i,t} \tag{6}$$

According to the model's assumptions about risk aversion differences towards downside and upside risk, high value downside beta means that the corresponding asset has high downside risk sensitivity, which must be compensated by a premium in expected return that is not captured by CAPM. Conversely, high value of upside beta means that the corresponding asset has good potential in upside market conditions, hence it requires lower expected return, which is not detected by standard CAPM.

2.2 Performance Ratios Used in the Paper

As defined above, a performance ratio is the relation between the return reward of an asset or portfolio and a risk measure. The return reward most commonly used in performance ratios is the difference between the expected return of the portfolio and the risk-free interest rate, which plays the role of benchmark or target return. This definition of return reward is used in performance measures as the Sharpe ratio, the Sortino ratio, and the Treynor ratio. The Sharpe ratio, *ShR*, measures the relation between excess expected return and total risk, given by the standard deviation of return, and is represented as follows:

$$ShR = \frac{E\left(R_i - r_f\right)}{\sigma_i} \tag{7}$$

where $E\left(R_i - r_f\right)$is the excess of the expected return of asset or portfolio *i* over the risk-free interest, r_f, and $\sigma_i$is the standard deviation of asset's *i* return.

The Sortino ratio, *SoR*, measures the asset's performance relative to downside risk, i.e., downside semi-standard deviation, and is defined as follows:

$$SoR = \frac{E\left(R_i - r_f\right)}{\sigma_i^d} \tag{8}$$

where downside risk is given by:

$$\sigma_i^d = \sqrt{\frac{\sum_{t=1}^{N}\left[min\left(0, R_{i,t} - E(R_i)\right)\right]^2}{N}} \tag{9}$$

The Treynor ratio is the relation between excess return of a risky asset (or risky portfolio) and its CAPM beta, and is represented as follows:

$$TR = \frac{E\left(R_i - r_f\right)}{\beta_i} \tag{10}$$

As referred above, the downside-upside risk asset pricing model, DURM, represented in (2) to (6), permits the calculation of two Treynor ratios, one related to downside β_i^d, defined as follows:

$$TR^d = \frac{E(R_i - r_f)}{\beta_i^d}, \tag{11}$$

and the other related to upside β_i^u:

$$TR^u = \frac{E(R_i - r_f)}{\beta_i^u}. \tag{12}$$

Ang *et al.* (2006) argue that high downside risk sensitivity must be compensated by a premium in upside market circumstances that is not captured by CAPM. Conversely, high value of upside beta means that the corresponding asset has good potential in upside market conditions, hence it requires lower expected return, which is not detected by standard CAPM. Hence, an asset, or portfolio, with high β_i^d and low β_i^u has a high value of TR^u and a low value of TR^d, respectively.

We also estimate the Rachev ratio, proposed by Rachev *et al.* (2008), whose purpose is to measure the reward potential given by average (or expected) return on the right tail of the return distribution, relative to the loss risk, represented by the average (or expected) return on the left tail. The Rachev ratio is represented as follows:

$$RR = \frac{\int_u^{\infty} xf(x)dx}{-\int_{-\infty}^{l} xf(x)dx} \tag{13}$$

where the $x = R_i - r_f$ is the excess return random variable, $f(x)$ is the corresponding density function, *u* is the variable's value in the lower quantile of the top tail, and d is the variable's value in the upper quantile of the bottom tail. The numerator of Rachev ratio is the average (or expected) reward given by the top tail, and the denominator is the average (or expected) loss on bottom quantile, which is also currently called conditional value-at-risk. The denominator is multiplied by (-) to give a positive value to the ratio, since the expected loss represented in the denominator is assumed to be negative.

Following the suggestions and examples given by Rachev *et al.* (2008), in the calculations of Rachev ratio conducted in this article (Section 3), the top tail comprises values of excess return above the 95 percentile and the bottom tail comprises the values of excess return below the 5 percentile.

A mild version of the Rachev ratio is given by the Stable Tail Adjusted Return Ratio, *STARR*, which uses conditional value-at-risk as risk measure, similarly to Rachev ratio, but uses the return reward definition of the Sharpe and Treynor ratios, *i.e.*:

$$STARR = \frac{E(R_i - r_f)}{-\int_{-\infty}^{l} xf(x)dx} \tag{14}$$

The bottom tail used to calculate the conditional value-at-risk is STARR that comprises the values of excess return below the 5 percentile, just as for the Rachev ratio.

3. Portfolio Selection of Euro Area Stock Indexes Based on Alternate Performance Ratios

In this section we select portfolios composed by euro area stock indexes, each of them based on the ranking given by one of the performance ratios presented in the previous section, over successive moving samples. This procedure gives a series of out-of-sample (*ex post*) return for the portfolio based on each performance measure. The comparison of the *ex post* return of the portfolios, based on the ranking given by alternate performance ratios, indicates which performance ratios are more able to give efficient portfolios.

Before introducing the results of portfolio selection based on alternate performance ratios, we present the database used in this paper, the domestic indexes returns statistics, the asset pricing model estimations and other statistics that support performance ratio calculations.

3.1 Data Presentation

The database used in this paper is composed of daily data from 1 January 2001 to 31 December 2015, comprising 3.908 observations from MSCI index of the ten euro area countries with developed stock markets (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal and Spain), and the MSCI EMU index (the Index of the European Monetary Union). The daily series of EURIBOR 1-week maturity interest rate, covering the same period, is used as a proxy for the risk-free interest rate. The interest rate level was significantly higher in the period before 2008 than after. A significant change in the interest rate level occurred during the debt crisis which followed the 2007 sub-prime crisis. The decline of EURIBOR interest rates in 2008 was mostly a consequence of the reduction of the key ECB lending rate, the purpose of which was to counteract the negative effects of the financial crisis. Negative nominal values of the EURIBOR interest rate emerged in the second half of 2014, because of the ECB policy decision to fix a negative value for the deposit facility rate on 11 June 2014.

Table 1 Indices Return Descriptive Statistics

	Mean	Std. Deviation	Minimum	Maximum
Austria	0.000149	0.015732	-0.105637	0.136092
Belgium	0.000146	0.013739	-0.146036	0.110136
Finland	-0.000002	0.019631	-0.188857	0.115757
France	0.000073	0.014519	-0.088862	0.109184
Germany	0.000151	0.015082	-0.083026	0.117680
Ireland	-0.000004	0.017155	-0.162608	0.143202
Italy	-0.000058	0.014939	-0.082659	0.116116
Netherlands	0.000093	0.014202	-0.088346	0.102362
Portugal	-0.000162	0.012281	-0.102153	0.108907
Spain	0.000122	0.015480	-0.096077	0.156300
EMU	0.000123	0.013071	-0.069016	0.099586

Table 1 identifies descriptive statistics of the daily returns of domestic indices and EMU index during the period under study. These statistics show that the German index has the highest mean return, followed by the Austrian index, while the Portuguese index has the lowest mean return, followed by the Italian index. Halfway through this return ranking is the Spanish index, the EMU index and the Dutch index. The relative positions are different in terms of volatility (measured by standard deviation), with the Finnish index coming in first, and the Irish index in second place. The lowest standard deviation belongs to the Portuguese index, followed by the EMU index. The distance between maximum and minimum return, which is an alternate perspective about volatility, has the lowest value in the EMU index, followed by the Dutch index, while its highest value belongs to the Irish index, and the second place belongs to the Finnish index.

3.2 Results of Asset Pricing Model Estimations

The estimations of asset pricing models were performed on a rolling basis over moving windows of 250 observations, equivalent to approximately one year of daily observations. This procedure is applied to the entire sample, giving 3658 recursive estimations of CAPM and DURM betas for each domestic index, whose mean values are shown in Table 2. Tsai, Chen and Yang (2014) and Messis and Zapranis (2014) emphasize that rolling regressions are appropriate to identify anomalies in the behavior of betas. The other advantage of rolling regressions is to provide a permanent flow of new beta parameters and other inputs to renew portfolio composition.

In general, the mean downside betas of the domestic indices are higher than the mean upside betas. The exceptions are Finland, France and Spain, whose mean downside betas are lower than their respective upside betas.

Table 2 CAPM and DURM Mean Beta Values

	CAPM beta	DURM downside beta	DURM upside beta
Austria	0.8297	0.9052	0.7512
Belgium	0.8664	0.8784	0.8525
Finland	1.0915	1.0241	1.1629
France	1.0648	1.0636	1.0663
Germany	1.0829	1.0972	1.0702
Ireland	0.8587	0.9018	0.8181
Italy	0.9964	1.0035	0.988
Netherland	0.9761	0.981	0.9717
Portugal	0.616	0.6511	0.5774
Spain	1.0182	0.9924	1.0424

3.3 Mean Values of Performance Ratios

We calculate the performance ratios described in the previous section using the results of the rolling estimations of the asset pricing models over moving samples of 250 observations, together with other statistics computed over the moving windows: mean return, variance, downside semi-variance, and average excess return relative to the risk free rate, on the right side and left side of the return distribution. To compare the domestic indexes from the perspective of their performance ratios in the overall period under analysis, we calculate the corresponding mean values which are shown in Table 3. The comparison between mean values shows that Austrian and Belgian indices have the highest mean values of the Sharpe, Sortino, STARR and Treynor ratios, being followed by the German and the Dutch indexes. The Portuguese index is at the bottom of the ranking, with low mean values in all ratios, followed by the Italian and Irish indexes and, to a lesser extent, by the Finish and French indices, whose ratio mean values also fall predominantly on the downside part of this ranking that is occupied in the middle by the Spanish index. The most noticeably peculiarity shown by Table 3 is that the Rachev ratio often contradicts the results observed in the other ratios, both in the group of indices with high mean values and the other. The Portuguese and Italian indices are the only ones where the Rachev ratio mean value and the ranking of the other ratios converge.

Table 3 Performance Ratios Mean Values

Ratio	***Austria***	***Belgium***	***Finland***	***France***	***Germany***
Sharpe	0.029391	0.034688	0.019229	0.020968	0.028317
Sortino	0.039790	0.048624	0.027005	0.028817	0.038386
STARR	0.014263	0.017183	0.009537	0.009817	0.012842
Rachev	0.956392	0.980648	1.013361	0.977149	0.976416
Treynor	0.000347	0.000143	0.000099	0.000067	0.000140
$Treynor^+$	0.000282	0.000180	0.000120	0.000079	0.000134
$Treynor^-$	0.000408	0.000084	0.000081	0.000056	0.000146
Ratio	***Ireland***	***Italy***	***Netherlands***	***Portugal***	***Spain***
Sharpe	0.016245	0.009655	0.025967	0.003135	0.025049
Sortino	0.023154	0.012191	0.035994	0.006295	0.033913
STARR	0.008829	0.004509	0.012283	0.003570	0.011674
Rachev	0.967508	0.929608	0.983943	0.960357	1.020038
Treynor	-0.000028	-0.000072	0.000115	-0.000136	0.000124
$Treynor^+$	0.000006	-0.000046	0.000129	-0.000174	0.000125
$Treynor^-$	-0.000094	-0.000099	0.000098	-0.000042	0.000125

3.4 Portfolio Selection Based on Performance Ratios

As referred in the introduction, the performance ratios presented in section 2 are used to support the selection of portfolios composed of the domestic indices. Portfolio selection is conducted recursively daily using the performance ratios calculated over successive moving samples of 250 observations. Each of the performance ratios supports the selection of two alternate portfolios, on each date *t*: one portfolio with a 1 day holding period, *i.e.*, bought on day *t* and liquidated on day *t+1*, and one portfolio with a 10 days holding period, *i.e.*, bought on day *t* and liquidated on day *t+10*. The use of two alternate holding periods make it possible to determine whether the most favorable performance ratios maintain their advantage when the investment period is extended. For purposes of simplification the portfolios based on Rachev, Sharpe, Sortino, *STARR*, Treynor, $Treynor^d$ and $Treynor^u$ ratios, are abbreviated onwards as follows: *RR*, *ShR*, *SoR*, *STARR*, *TR*, TR^d and TR^u, respectively. Each portfolio is composed of the five highest-valued domestic indices in one of the performance ratios by applying the following formula:

$$w_i = \frac{exp(PR_i)}{\sum_{i=1}^{5} exp(PR_i)} \tag{15}$$

where w_i is the weight of country i index in the portfolio, $exp(PR_i)$ is the exponential of the corresponding value in the performance ratio, PR_i, which represents alternately

RR, *ShR*, *SoR*, *STARR*, *TR*, TR^d and TR^u. The use of the exponential to calculate the weights precludes the disturbing effect of negative values of the performance ratio which can result from occasional negative values of the mean excess return. To achieve the purpose of selecting portfolios with good ranking, only domestic indexes with performance ratios on the upper end of the performance ranking are eligible to be included in the selected portfolios. The inclusion of five domestic indices in each portfolio instead of a lower number (e.g. only the domestic index with rank 1) prevents different performance ratios from giving portfolios with equal composition. We conducted a preliminary check on the ranking frequency, which showed that this undesirable result would occur frequently if only the top domestic index were chosen in each performance ratio, and it is significantly reduced when five domestic indices are included in a portfolio. Very often two performance ratios rank the same domestic index first, when comparing the Sharpe, Sortino and STARR ratios. The Treynor, Treynor^d and Treynor^u ratios also produce the same result, albeit to a lesser extent. However, when the top 1 to 5 indices are taken together, different performance ratios seldom produce the same top-ranking indices. For this reason, the inclusion of the five top-ranking indices proves to be the most appropriate choice in this procedure of portfolio selection.

The pairwise comparison of the *ex post* return of the *RR*, *ShR*, *SoR*, *STARR*, *TR*, TR^d and TR^u portfolios uses an approach based on the conditional Sharpe ratios of Chow and Lai (2015), whose theoretical grounds are the conditional stochastic dominance definition given by Shalit and Ytzhaki (1994) and Chow (2001). The Chow and Lai (2015) method compares two portfolios, which it calls P and B, to simplify, through the statistics of the return difference between P and B, observed below given quantiles of the return distribution of B. In the present paper we propose an alternate solution where two portfolios are compared on the basis of the statistics of their return difference observed between given quantiles of the EMU index return, which represents the market conditions in the global euro area. For this purpose, we define four states in the euro area. Downside market, called state S_1, corresponds to EMU index returns below its 25-percentile. Intermediate downside market, called state S_2, corresponds to EMU index returns between the 25-percentile and the 50-percentile values. Intermediate upside market, stated as state S_3, designated as intermediate upside market, corresponds to EMU index returns between the 50-percentile and the 75-percentile values. Finally, upside market, designated as state S_4, corresponds to EMU index returns above the 75-percentile value. Our procedure has two main purposes. The first purpose is to identify the performance ratios with the highest capacity to support the selection of portfolios offering better protection when the euro area market is down. The second purpose is to determine which performance ratios can support the selection of portfolios with a high potential of taking advantage of upside conditions in euro area financial markets. The return difference between two portfolios, P and B, observed when the EMU index is in the state S_i (i=1,…4), is represented as follows:

$$D_t^{S_i} = \left(R_{P,t} - R_{B,t}\right) \tag{16}$$

where $R_{P,t}$and $R_{B,t}$are, respectively, the return values of portfolios P and B, observed on a particular day *t*, where state S_i of EMU index occurs. The mean and standard deviation of the return differences in state S_i are, respectively:

$$\overline{D^{S_i}} = \sum_{t=1}^{T} \frac{D_t^{S_i}}{T} \tag{17}$$

and

$$\hat{\sigma}^{S_i} = \sqrt{\sum_{t=1}^{T} \frac{\left(D_t^{S_i} - \overline{D^{S_i}}\right)^2}{T-1}}. \tag{18}$$

where T is the number of observations of the return differences occurred in state S_i of the EMU index. The mean and standard deviation allow to calculate the following *information ratio*:

$$IR^{S_i} = \frac{\bar{D}^{S_i}}{\hat{\sigma}^{S_i}}. \tag{19}$$

which provides information about the comparative performance between portfolios P and B. This information ratio proposed in this paper is very similar to information ratio proposed by Israelson (2004) to compare two portfolios, which consists of a single ratio for the entire sample of portfolio return. The method we use calculates different information ratios according to the situation of financial markets. Hence the information ratios we calculate have the advantage of showing which portfolios compared perform better in different market conditions, namely, which offer better protection against loss risk in downside market conditions, and which are more able to benefit from expansion periods in financial markets. Under the assumption that the return differences $D_t^{S_i}$are identically and independently distributed, the probability distribution of the difference between the sample mean $\bar{D}^{Q_i}$ and its expected value, $E(D)$, is asymptotically normal with mean zero and standard deviation $\frac{\sigma^{Q_i}}{\sqrt{T}}$. Consequently, the information ratio being subject to the Student's t statistics test:

$$\sqrt{T} IR^{S_i} = \sqrt{T}\left(\frac{\bar{D}^{S_i}}{\hat{\sigma}^{S_i}}\right). \tag{20}$$

Taking into consideration the P-Value of Student's t statistics we accept that, if the information ratio$IR^{S_i}$is positive and significantly different from zero the portfolio designated by P in the return difference represented in (16) performs better than portfolio B, during state S^i of the EMU index return. On the contrary, if the information ratio $IR^{S_i}$is negative and significantly different from zero, portfolio B performs better than portfolio P under the state S^i of the EMU index return. Obviously, it is not relevant to which portfolio we give the position P or B in the pair comparison. The results are inconclusive if IR^{S_i} is not significantly different from zero. If $IR^{S_i}$is significantly different from zero on downside states of the EMU index return, we conclude that the portfolio with better performance offers better protection against downside risk in euro area markets. If $IR^{S_i}$is significantly different from zero in upside states of the EMU index return, we conclude that the portfolio with better performance has higher ability to take advantage from expansion periods of the euro area stock markets.

Table 4 displays the $\sqrt{T}IR^{S_i}$ statistics and the corresponding P-Value for all pairs of portfolios based on the alternate performance ratios, and for the four EMU index states defined above. Table 4.1 shows the results for the one-day holding period, and Table 4.2 for the ten-day holding period.

To interpret the results displayed in Table 4, we accept that the information ratio is significantly different from zero if P-Value≤5%, and not significantly different from zero otherwise. Table 4.1 shows that, for the one-day holding period, the *RR* portfolio performs worse than all the others in the downside and intermediate downside market states, respectively S_1 and S_2. The results of the comparison of the *RR* portfolio with the others in the intermediate upside market (S_3) are inconclusive. Finally, in the upside market state, S_4, the *RR* portfolio performs better that all the others. Hence, the *RR* portfolio has good potential to benefit from upside market opportunities but offers less protection than alternate portfolios in downside market conditions. Table 4.1 also shows that, in downside market state, S_1, the *ShR* portfolio performs worse than the *SoR* portfolio, better than the *TR*, TR^d and TR^u portfolios, and the result of its comparison with the *STARR* portfolio is inconclusive. In the intermediate downside and intermediate upside states, S_2 and S_3, the results of the comparison of the *ShR* portfolio with the *SoR, STARR, TR*, TR^d and TR^u portfolios are inconclusive. In the upside market, S_4, the comparison of the *ShR* portfolio with the *STARR* and the TR^d portfolios is also inconclusive, but the *ShR* portfolio performs better than the *SoR* portfolio and worse than the *TR* and the TR^u portfolios. In the downside market state, S_1, the *SoR* portfolio performs better than the *TR*, TR^d and TR^u portfolios, and the result relative to the *STARR* portfolio is inconclusive. The results of the *SoR* portfolio relative to this group of portfolios are also inconclusive in the intermediate downside and intermediate upside market states, S_2 and S_3. In the upside market state, the *SoR* portfolio performs worse than the *TR*, TR^d and TR^u portfolios, and the result relative to the *STARR* portfolio is inconclusive. The *STARR* portfolio performs better than the *TR*, TR^d and TR^u portfolios in the downside market state, S_1, worse in the upside market state, S_4, and the results are inconclusive in the intermediate states, S_2 and S_3. The *TR* portfolio performs worse than the TR^d portfolio in the downside state, S_1, better than TR^d in the upside state, and the results are inconclusive in the intermediate states. The comparison between *TR* and TR^u portfolios is inconclusive in all states. Finally, the TR^d portfolio performs better than the TR^u portfolio in the downside market state, S_1, worse than TR^u in the upside market state, S_4, and the results are inconclusive in the intermediate states.

The results of the ten-day holding period portfolios, displayed in Table 4.2, show that in downside market state, S_1, the *RR* portfolio performs worse than all the other portfolios, and are inconclusive for the intermediate downside market state, S_2. In the intermediate upside market state, S_3, the *RR* portfolio performs better than the *ShR*, *SoR* and *STARR* portfolios, and the results are inconclusive with respect to the other portfolios. In the upside market state, S_4, the *RR* portfolio performs better than all the other portfolios, except the TR^u, for which the result is inconclusive. Table 4.2 also shows that, in downside market state, S_1, the *ShR* portfolio performs better than the *STARR, TR* and TR^u portfolios, but the results are inconclusive in relation to the

SoR and TR^d portfolios. In the intermediate downside market state, S_2, the results of performance comparison of the *ShR* portfolio with all the others are inconclusive. In the intermediate upside market state, S_3, the *ShR* portfolio performs worse than the *SoR*, *TR*, TR^u and TR^d portfolios, and the result of its comparison the *STARR* portfolio is inconclusive. In the upside market state, S_4, the *ShR* portfolio performs worse than the *TR*, TR^u and TR^d portfolios, and its results are inconclusive relative to the *SoR* and *STARR* portfolios. The results concerning the *SoR* portfolio in the downside market state, S_1, show that it performs better than the *STARR*, *TR* and TR^u portfolios, and its comparison with TR^d portfolio is inconclusive. The comparison of *SoR* portfolio with the *STARR*, *TR*, TR^d and TR^u is inconclusive in the intermediate downside market state, S_2, and also in most cases of the intermediate upside market state, S_3, where the single exception is that the *SoR* portfolio performs worse than the TR^u portfolio. In the upside market state, S_4, the *SoR* portfolio performs worse than the *TR*, TR^u and TR^d portfolios, and its comparison with the *STARR* portfolio is inconclusive. In the downside market state, S_1, the *STARR* portfolio performs better than the *TR* and TR^u portfolios, but its comparison with the TR^d is inconclusive. In the intermediate downside market state, S_2, the *STARR* portfolio has inconclusive results relative to the *TR*, TR^d and TR^u portfolios, but performs worse than them both in the intermediate upside state, S_3, and in the upside state, S_4. The *TR* portfolio performs worse than the TR^d portfolio and better than the TR^u portfolio in the downside market state, S_1, the corresponding results are inconclusive in both intermediate states, S_2 and S_3, and are reversed in the upside market state (S_4), where the *TR* portfolio performs better than the TR^d portfolio, and worse than the TR^u portfolio. Finally, the comparison between the TR^d and TR^u portfolios, in the ten-day holding period, shows that the TR^d portfolio performs better in the downside and intermediate downside market states, S_1 and S_2, the result is inconclusive in the intermediate upside market state, S_3, and the TR^u portfolio performs better in the upside market state (S_4).

The comparison between Tables 4.1 and 4.2 shows that, in general, the best-performing portfolios are the same for both the one-day and the ten-day holding periods. This means that the relative ability of performance ratios to select portfolios does not change when the holding period is extended from one to ten days. The number of inconclusive results in the intermediate upside market state, S3, in a ten-day holding period is lower than in a one-day holding period. This means that the comparative advantage that some portfolios have in this market state is clearer in the ten-day holding period than in the one-day holding period.

Table 4 Information Ratio on Pairwise Comparison of Portfolios Based on Different Performance Ratios

Table 4.1 Results for One-Day Holding Period

Euro financial markets state[(a)]	***S_1***		***S_2***		***S_3***		***S_4***	
Port. (P)-Port.(B)	$\sqrt{T}IR^{S_1}$	*P-Value*	$\sqrt{T}IR^{S_2}$	*P-Value*	$\sqrt{T}IR^{S_3}$	*P-Value*	$\sqrt{T}IR^{S_4}$	*P-Value*
Rachev-Sharpe	-6.2447	0.0000	-4.9426	0.0000	-0.9087	0.3636	6.6552	0.0000
Rachev-Sortino	-6.6424	0.0000	-5.1093	0.0000	-1.3757	0.1690	7.3101	0.0000
Rachev-STARR	-6.3656	0.0000	-4.9292	0.0000	-0.9422	0.3461	7.0861	0.0000
Rachev-Treynor	-2.7382	0.0062	-4.7097	0.0000	-1.1423	0.2534	4.6080	0.0000
Rachev-Treynor^d	-4.4403	0.0000	-4.9841	0.0000	-1.2258	0.2203	5.7201	0.0000
Rachev-Treynor^u	-2.4674	0.0137	-4.5665	0.0000	-0.9401	0.3472	4.6038	0.0000
Sharpe-Sortino	-2.0137	0.0441	-0.5529	0.5803	-1.8680	0.0618	2.9856	0.0029
Sharpe-STARR	-0.9964	0.3191	-0.4853	0.6275	-0.0818	0.9348	1.8115	0.0702
Sharpe-Treynor	5.7509	0.0000	0.5372	0.5912	-0.4795	0.6316	-3.6983	0.0002
Sharpe-Treynor^d	3.1351	0.0017	-0.2449	0.8066	-0.5653	0.5719	-1.6939	0.0904
Sharpe-Treynor^u	6.4278	0.0000	0.7238	0.4692	-0.1068	0.9150	-3.6830	0.0002
Sortino-STARR	0.4711	0.6376	-0.0885	0.9295	1.4005	0.1615	-0.2399	0.8104
Sortino-Treynor	6.2337	0.0000	0.7358	0.4619	0.3751	0.7076	-4.5385	0.0000
Sortino-Treynor^d	3.8907	0.0001	-0.0202	0.9839	0.2766	0.7821	-2.8293	0.0047
Sortino-Treynor^u	6.7383	0.0000	0.9074	0.3643	0.6829	0.4947	-4.4511	0.0000
STARR-Treynor	6.1121	0.0000	0.8581	0.3909	-0.4432	0.6576	-4.5705	0.0000
STARR-Treynor^d	3.7543	0.0002	0.0306	0.9756	-0.5860	0.5579	-2.8661	0.0042
STARR-Treynor^u	6.4918	0.0000	1.0012	0.3168	-0.0597	0.9524	-4.4767	0.0000
Treynor-Treynor^d	-4.6163	0.0000	-1.0062	0.3144	-0.1453	0.8845	3.5848	0.0003
Treynor-Treynor^u	1.0018	0.3165	0.3592	0.7195	0.6422	0.5208	-0.3832	0.7016
Treynor^d-Treynor^u	4.1033	0.0000	1.0206	0.3075	0.5410	0.5885	-2.8256	0.0047

Table 4.2 Results for Ten-Day Holding Period

Euro financial market state [(a)]	S_1		S_2		S_3		S_4	
Port. (P)-Port.(B)	$\sqrt{T}IR^{S_1}$	*P-Value*	$\sqrt{T}IR^{S_2}$	*P-Value*	$\sqrt{T}IR^{S_1}$	*P-Value*	$\sqrt{T}IR^{S_2}$	*P-Value*
Rachev-Sharpe	-5.9967	0.0000	-0.5887	0.5561	3.1305	0.0018	4.9865	0.0000
Rachev-Sortino	-6.0911	0.0000	-0.8497	0.3956	2.5101	0.0121	5.0535	0.0000
Rachev-STARR	-5.1597	0.0000	-0.6119	0.5407	2.9307	0.0034	5.2846	0.0000
Rachev-Treynor	-3.5147	0.0004	-0.6204	0.5351	1.8166	0.0694	2.0275	0.0427
Rachev-Treynor^d	-4.7487	0.0000	-1.2576	0.2086	1.7040	0.0885	3.7008	0.0002
Rachev-Treynor^u	-2.6468	0.0082	-0.1884	0.8506	1.4832	0.1381	1.1509	0.2499
Sharpe-Sortino	-0.1728	0.8628	-1.6020	0.1092	-2.7272	0.0064	0.6837	0.4942
Sharpe-STARR	2.3414	0.0193	-0.1318	0.8951	-0.8053	0.4207	1.1867	0.2354
Sharpe-Treynor	4.0791	0.0000	-0.0689	0.9451	-3.2441	0.0012	-5.7585	0.0000
Sharpe-Treynor^d	1.8961	0.0580	-1.3586	0.1744	-3.1635	0.0016	-2.9519	0.0032
Sharpe-Treynor^u	5.5608	0.0000	0.8560	0.3921	-3.5249	0.0004	-6.6449	0.0000
Sortino-STARR	2.6052	0.0092	1.1901	0.2341	1.8007	0.0718	0.7968	0.4256
Sortino-Treynor	3.9070	0.0001	0.5528	0.5805	-1.7422	0.0816	-5.8194	0.0000
Sortino-Treynor^d	1.8774	0.0606	-0.7551	0.4503	-1.8608	0.0629	-3.1571	0.0016
Sortino-Treynor^u	5.3340	0.0000	1.3830	0.1668	-2.1006	0.0358	-6.5572	0.0000
STARR-Treynor	2.7908	0.0053	-0.0085	0.9933	-2.6818	0.0074	-6.2768	0.0000
STARR-Treynor^d	0.5895	0.5556	-1.2835	0.1994	-2.8085	0.0050	-3.7199	0.0002
STARR-Treynor^u	4.1116	0.0000	0.8636	0.3879	-2.9410	0.0033	-6.8353	0.0000
Treynor-Treynor^d	-3.5452	0.0004	-1.9152	0.0556	-0.3134	0.7540	4.2590	0.0000
Treynor-Treynor^u	2.3092	0.0210	1.4857	0.1375	-0.8472	0.3970	-3.0526	0.0023
Treynor^d-Treynor^u	4.3206	0.0000	2.4255	0.0153	-0.3740	0.7085	-5.1237	0.0000

4. Conclusions

This article compares the ability of alternate performance measures to support investment choices in ten-euro area stock markets. There are two types of performance measures. One comprises the performance ratios using risk measures, which do not distinguish between systematic and non-systematic risk: Rachev ratio, Sharpe ratio, Sortino ratio and STARR. The other type comprises performance ratios whose risk measures are based on systematic risk only. The second type is composed of three definitions of Treynor ratios: the standard Treynor ratio given by CAPM, and two Treynor ratios, $Treynor^u$ and $Treynor^d$, given by DURM (Downside-Upside Risk Model). The results of portfolios based on alternate performance ratios are compared separately for four alternate market conditions in the euro area, described according to the level of the EMU index return: downside market, intermediate downside market, intermediate upside market and upside market. In the two intermediate market conditions the results are mostly inconclusive. On the contrary, in two extreme conditions, downside and upside market, there is clear evidence of performance differences between alternate portfolios. The results show that portfolios based on the Rachev, Sharpe, Sortino and STARR ratios, in general, perform better than the others in downside-market conditions, while portfolios based on the three specifications of the Treynor ratios perform better in the upside market.

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