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An Empirical Analysis of Relationships of Forward Exchange Rates and Present and Future Spot Exchange Rates*

Example of CZK/USD and CZK/EUR

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Abstract
The aim of this paper is to present an empirical analysis of the relationships between the forward and spot exchange rates in the Czech Republic. The forward rate unbiasedness hypothesis, the expectation hypothesis, the adaptive expectation hypothesis and the hypothesis of covered interest rate parity are formulated in this paper. To test the first two hypotheses the econometric procedure based on co-integration and weak exogeneity testing is proposed. The third and fourth hypotheses are verified by the Engle-Granger co-integration test. The estimates do not support the forward rate unbiasedness hypothesis. On the contrary, the results confirm the hypothesis of adaptive expectation and the hypothesis of covered interest rate parity.

1. Introduction

Forward exchange rates are, in accordance with an efficient market hypothesis, considered to be a suitable tool for forecasting future spot exchange rates. In comparison with a fundamental and technical analysis, the forward prediction has an indisputable comparative advantage in its low cost. The dispute is headed, in the case of the forward predictions, their level of accuracy and the causes of their failure.

The forward rate unbiasedness hypothesis claims that the forward exchange rates are the best unbiased estimators of the future spot exchange rates. Levich (1978 and 1979) tested this hypothesis on the basis of equation

$$SR_{t+n} = a_0 + a_1 \cdot FR_t^{t+n} + u_{t+n}, \quad (1)$$

where FR_t^{t+n} is the forward exchange rate quoted in time t for the term contract payable at time $t+n$, and SR_{t+n} is the spot exchange rate quoted in time $t+n$. The forward rate unbiasedness hypothesis means that the parameter α_0 is equal to 0, and that the parameter α_1 is equal to 1. According to Muth (1961), $\{u_{t+n}\}$ must be the white noise process, and cannot be correlated with forward exchange rates which represent the market expected values of future spot exchange rates.

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Levich (1978) and Frankel (1982), in the model based on the level dates, concluded (currency pairs FRF / USD, DEM / USD a GBP / USD, for the periods 1973–1978, and 1979 respectively) that forward exchange rates are unbiased estimates of future spot exchange rates. Chiang (1988) uses the same approach for the same monetary pairs (for a longer period: 1974-1983) with similar results. But he points out considerable structural instability of estimated parameters and their sensitivity to new information.

It has been argued against the above-mentioned tests that the analysed data were nonstationary and the potential problem of spurious regression was not taken into account. In this situation, Fama (1984) suggested the following model

$$(SR_{t+n} - SR_t) = \beta_0 + \beta_1 \cdot (FR_t^{t+n} - SR_t) + z_{t+n}, \qquad (2)$$

with the stationary dependent and independent variables. On the right side of this equation there is the forward premium (respectively discounts), on the left side there is the change of the spot exchange rate.[1] Under the hypothesis of the unbiased estimator, the parameter β_0 is equal to 0 and the parameter β_1 is equal to 1. Later, during the empirical verification of this relationship Cumby, Obstfeld (1984), Boothe, Longworth (1986) and Engel (1996) have concluded, surprisingly, that the estimates of parameter β_1 can be negative (close to -1). This means that in the situation where the domestic interest rate is higher than the foreign one, the forward premium leads 'irrationally' to the appreciation of the domestic currency. The results of the tests were presented as a rejection of the hypothesis about the effectiveness of foreign exchange markets (the so called the *forward premium puzzle*).

Aggarwal, Lucey and Mohanty (2006) concluded, regarding the example of currency pairs (with high market liquidity and low trading costs) GBP/USD, JPY/USD, CHF/USD and DEM/ USD (period 1973–1998, maturity 1–12 months, level data), on the basis of the co-integration analysis, that „*there is little empirical support for rational expectations in the forward rates as a forecast of the future spot rate*". Hatemi and Roca (2012) tested the *forward rate unbiasedness* hypothesis for developed country currencies (AUD, EUR, GBP, JPY and USD, during the period from 1999 to 2006), by use of the new co-integration test which allows for multiple, unknown structural breaks. Their results indicate that „*the market for these currencies is still efficient as these values may not allow the existence of arbitrage profits after taking into account the transaction costs and risk premium*".

Meredith and Ma (2002) formulated the idea that the failure of unbiased prognosis can be caused by a subjective error in expectations, as well as by the existence of the risk premium as a result of the aversity of the market subject to risk. They approximated the forward premium by the interest rate differential using the rule of the covered interest rate parity.[2] On the example of the exchange rates of the G7 countries to USD, they found that the five years rolling regression parameters estimates

[1] This forward premium on the validity of *uncovered interest parity* (without risk premiums) is equal to the interest rate differential between domestic and foreign currency. The issue will be explained further.
[2] Kočenda and Poghosyan (2009 and 2010): The analysis of macroenomic sources of foreign exchange risk premium on the base interest rate parity condition using multivariate GARCH-in-mean model.

(1984-2000) are highly unstable and move between the values -7 and 4. The average values of the estimates are in the conflict with the hypothesis close to -1.

Bansal and Dahlquist (2000), Frankel and Poonawala (2010) concluded that the forward premium puzzle is typical rather for developed economies than for developing and transition economies. But it must be stated that the results of their estimates are not very robust and suggest the existence of a time-varying risk premium.

Loring and Lucey (2013) compared the results of their own econometric analysis for the period 2004-2011 with the previous results of the analysis for the period 1996-2004 (Frankel and Poonawala, 2010). Their analysis shows, that the forward rate biasedness is less pronounced for developed country currencies than for developing country currencies. They formulated the hypothesis, that the period-specific factors were responsible for the results found in previous research.

The main objective of this paper is the analysis of the appropriateness of the forward exchange rate as a tool for the prediction of the development of the future spot exchange rate on the example of the exchange rates CZK/USD and CZK/EUR. The hypothesis of unbiased estimates as well as the other hypotheses which support, or refute, the possibility of the successful functioning of the forward exchange rate as a prediction tool (rational expectations, adaptive expectations, covered interest rate parity) will be tested.

Empirical analysis will be based on the level data and the co-integration analysis will be used. In this situation, the stationarization (suggested by Fama, 1984) is not a suitable approach because of the risk of loss of information from analyzed time series.

In the second part of paper we will make the model description of the behavior of speculators and arbitrageurs on the forward foreign exchange market. We will focus on the justification of the existence of the risk premiums and explore the possibility of their empirical verification in the case of the spot and forward exchange rates mid. We will use the example of the market makers behavior to explain why, in the case of an econometric analysis based on a spot and forward rate mid, only the presence of the risk premium for credit and liquidity risks, and not the risk premium for currency risk, can be empirically verified.

In the third part, we will formulate the economic hypothesis and solve methodological questions for the empirical econometric analysis. The nontraditional econometric approach to testing of the above mentioned economic hypothesis will be proposed. The testing procedure is based on the verification of weak exogeneity in the co-integration system. The testing of weak exogeneity in the vector error correction models has become standard procedure. But in this paper we propose a different approach; that of the method for the testing of weak exogeneity in the single-equation error correction model. The approach based on the single-equation model is given by formulations of the forward rate unbiasedness hypothesis and the expectation hypothesis (see part 3). The proposed approach is original mainly in connection with these hypotheses. Its advantage is that it can be very simply applied with the help of standard procedures which are accessible through ordinary econometric software.

In the fourth part, the empirical verification of the formulated economic hypothesis on the basis of the monetary pairs CZK/EUR and CZK/USD at the maturity of the forward transactions of 3 and 6 month periods will be made (time period of the analysis: 2001-2014; daily time series frequency).

2. Factors affecting the forward exchange rate in arbitrage and speculation

Speculative thinking on the forward foreign exchange market puts the forward exchange rate and the expected spot exchange rate into some sort of relationship. In this context, Fama's paper (1984) is the most frequently cited, which from the economic and statistical point of view defined the decomposition of the forward exchange rate *FR* to the expected spot exchange rate *E*(*SR*) and the premium *P*. On the basis of the assumptions about the validity of: a) no arbitrage condition of the covered interest rate parity; b) the Fisher equation for nominal and real interest rates; c) the purchasing power parity theory; Fama derived the premium, as expected real interest rate differential between domestic and foreign economies.[3] The „perfidy" of Fama´s approach to the premium is, in our opinion, in the fact that it contains both the real gain, as well as various forms of risk premiums. Using this approach leads necessarily to a problem with model consistency, because the growth of the nominal interest rate caused by the growth of the real profit rate leads to the appreciation of the spot exchange rate, while the growth in the nominal interest rates which caused the growth of risk premium will be exchange rate neutral. Noteworthy is the fact that Fama´s premium is gradually called, and viewed as, a risk premium which can have a macroeconomic and stochastic explanation (for example, Cheung, 1993, Meredith and Ma, 2002, Loring and Lucey, 2013).

In our approach, we focus on defining the financial risks and appropriate risk premiums which affect the behavior of the market makers on the forward market. The market makers quote exchange rates and can perform arbitrage and speculative transactions. The arbitrage equilibrium is associated with the so-called condition of covered interest rate parity. We will work with domestic and foreign interest rates (IR_D and IR_F), and with the direct quotation of the forward (*FR*) and spot (*SR*) exchange rates. That is the number of units of domestic currency per unit of foreign currency (*D/F*).

The covered interest rate parity for foreign exchange rates mid and interest rates mid, abstracting transaction costs and risk premiums for the period *t* to *t*+*n* can be expressed as

$$FR_t^{t+n} = \frac{1 + IR_{D,\,t}^{t+n} \cdot n/360}{1 + IR_{F,\,t}^{t+n} \cdot n/360} \cdot SR_t\,, \tag{3}$$

where *n* in the upper index at the interest rate *IR* means the maturity. Partially contrary to Burnside, Eichenbaum and Rebelo (2007), we assume that market makers are the most knowledgeable players in the market and are averse to risk, and conduct the operations with a profit motive[4]. The market makers buy foreign currencies on the

[3] Fama (1984) defines the premium as follows „…,*the premium P_t in the forward rate expression is just the difference between the expected real returns on the nominal bonds of the two countries*. *Thus, the variables that determine the difference between the expected real returns on the nominal bonds (for example the differential purchasing power risks of their nominal payoffs) also explain the premium on the forward rates*."

[4] Each market maker in the commercial bank is supported by a strong analytical team. There is no reason to believe that there are better informed subjects (resp. teams) with better analytical skills on the market. The only exceptions are the dealers in central banks who have privileged information regarding the conduct of their own foreign exchange interventions (i.e. the strategy and tactics of exchange rate policy).

forward and spot market at the bid price, and sell them at the ask price. In arbitrage operations, they are not exposed to currency risk because they have closed a foreign exchange position. But we cannot eliminate credit risk if arbitrage is associated with a currently made deposit, loan or purchase of debt securities. The microeconomic arbitrage profit functions of the market maker in his own quotations, the bid and ask are the following

$$\pi_{A,bid} = \left[\frac{1 + IR_{D,\,d,\,t}^{t+n} \cdot n/360}{1 + IR_{F,\,l,\,t}^{t+n} \cdot n/360} \cdot \frac{1}{(1 + rp_{1,\,D})} \cdot SR_{t,\,ask} - FR_{t,\,bid}^{t+n} \right] \cdot Q_F \,, \qquad (4)$$

respectively

$$\pi_{A,\,ask} = \left[FR_{t,\,ask}^{t+n} - \frac{1 + IR_{D,\,l,\,t}^{t+n} \cdot n/360}{1 + IR_{F,\,d,\,t}^{t+n} \cdot n/360} \cdot (1 + rp_{1,\,F}) \cdot SR_{t,\,bid} \right] \cdot Q_F \,, \qquad (5)$$

where $rp_{1,D}$ and $rp_{1,F}$ are credit risk premiums when investing in domestic and foreign currencies, interest rates, *IR*, the lower indexes *d* and *l* indicate the deposit and lending interest rates, Q_F is the amount of foreign currency. The market makers maximize profits from arbitrage π_A, if

$$\frac{\partial \pi_{A,bid}}{\partial Q_F} = 0 \text{, respectively } \frac{\partial \pi_{A,ask}}{\partial Q_F} = 0 \,. \qquad (6)$$

The covered interest rate parity from the point of view of the market maker then has the form of the following equations

$$FR_{t,\,bid}^{t+n} = \frac{1 + IR_{D,\,d,\,t}^{t+n} \cdot n/360}{1 + IR_{F,\,l,\,t}^{t+n} \cdot n/360} \cdot \frac{1}{(1 + rp_{1,\,D})} \cdot SR_{t,\,ask} \,, \qquad (7)$$

$$FR_{t,\,ask}^{t+n} = \frac{1 + IR_{D,\,l,\,t}^{t+n} \cdot n/360}{1 + IR_{F,\,d,\,t}^{t+n} \cdot n/360} \cdot (1 + rp_{1,\,F}) \cdot SR_{t,\,bid} \,. \qquad (8)$$

The covered interest rate parity for exchange rates mid and interest rates mid has the form[5]

$$\ln FR_t^{t+n} = \ln SR_t + \ln ID_t^{t+n} + CRP \,, \qquad (9)$$

where $\ln ID_t^{t+n} = \ln(1 + IR_{D,t}^{t+n}) - \ln(1 + IR_{F,t}^{t+n})$ is the logarithm of the interest rates differential between the domestic a foreign currency and $CRP = [\ln(1+rp_{1,D}) - \ln(1+rp_{1,F})]/2$, where the credit risk premiums $rp_{1,D}$ and $rp_{1,F}$ will

[5] Both equations are logarithmically transformed, left and right sides of the equations are summed and divided by two and then expressed as exchange rates mid and interest rates mid.

be zero for short-term arbitrage operations between countries with a higher grades rating. From the point of view of the following empirical verification, it is an important finding that the level of credit risk premiums affects not only the forward rates bid and ask, but also forward rate mid.

The market makers can also get into the open speculative foreign exchange positions when they must compare the quoted forward exchange rates with the expected values of the future spot exchange rates $E(SR)$.

In a world without risk premiums and exchange rates mid, the speculators are in a state of equilibrium if the following condition is met $FR_t^{t+n} = E_t(SR_{t+n})$. Speculation via the forward exchange rate is associated with the currency risk and the liquidity risk at the time of the closing of the open foreign exchange speculative position. If the speculators are averse to risk, the required risk premiums will be nonzero. Microeconomic profitable functions of the market maker during a long (L) and short (S) forward foreign exchange position are as follows

$$\pi_{S,L} = \left[\frac{E_t(SR_{t+n,\,ask})}{(1+rp_{2,\,D})\cdot(1+rp_3)} - FR_{t,\,bid}^{t+n} \right] \cdot Q_F \, , \tag{10}$$

$$\pi_{S,S} = \left[FR_{t,\,ask}^{t+n} - E_t(SR_{t+n,\,bid})\cdot(1+rp_{2,\,F})\cdot(1+rp_3) \right] \cdot Q_F \, , \tag{11}$$

where $rp_{2,D}$ and $rp_{2,F}$ are liquidity risk premiums associated with buying domestic or foreign currency on the future spot market, rp_3 is the risk premium associated with the currency risk, symmetrically affecting the forward rates bid and ask.

The market makers maximize profits from the speculative transactions π_S under the conditions

$$\frac{\partial \pi_{S,\,L}}{\partial Q_F} = 0 \text{, respectively } \frac{\partial \pi_{S,\,S}}{\partial Q_F} = 0 \, . \tag{12}$$

Equilibrium conditions on the forward market from the point of view of the market maker can be therefore written as

$$FR_{t,\,bid}^{t+n} = \frac{E_t(SR_{t+n,\,ask})}{(1+rp_{2,\,D})\cdot(1+rp_3)} \, , \tag{13}$$

$$FR_{t,\,ask}^{t+n} = E_t(SR_{t+n,\,bid})\cdot(1+rp_{2,\,F})\cdot(1+rp_3) \, . \tag{14}$$

For exchange rates mid[6]

$$\ln FR_t^{t+n} = \ln E_t(SR_{t+n}) + LRP \, , \tag{15}$$

[6] Both equations are logarithmically transformed, left and right sides of the equations are summed and divided by two and then expressed as the exchange rates mid.

where $LRP = [\ln(1+rp_{2,D}) - \ln(1+rp_{2,F})]/2$. The currency risk (risk premium rp_3) is a form of speculative risk and it is associated with the expected exchange rate volatility. The market maker, quoting the forward rate, responds to the change in currency risk by symmetric widening or narrowing of the spread bid - ask at the constant exchange rates mid. Therefore, the forward rates mid does not capture a currency risk change, in our opinion.[7] But at the level of exchange rate mid, the liquidity risk can be observable. The liquidity risk premium rp_2 will depend on the extent of the open forward positions in both domestic and foreign currencies. If the purchase of foreign currency (forward contracts for FR_{bid}) grows, then the liquidity risk associated with the sales of foreign currency (respectively with buying domestic currency) on the future spot market will gradually increase ($rp_{2,D}$ grows). If the sales of foreign currency (forward contracts for FR_{ask}) grow, then the liquidity risk associated with the purchase of foreign currency on the future spot market will gradually increase ($rp_{2,F}$ grows). The development of the liquidity risk will be, at the level of forward rates mid, observable, because it will affect the spread bid – ask asymmetrically.

3. Research methodology

In this part we will formulate the economic hypotheses and explain the basic ideas of the econometric analysis to be used in the next empirical part of this paper.

a) ***The forward rate unbiasedness hypothesis*** says that the time series of the future spot exchange rate SR_{t+n} depends on the time series of today´s (in time t) quoted forward exchange rate FR_t^{t+n}. This hypothesis can be fulfilled if: a) speculators are neutral to risk and simultaneously maximize their profits: b) the hypothesis of rational expectations regarding the development of future spot rates is fulfilled, and: c) speculators force the market makers (by active trading), who quote the forward rates in line with market expectations.

b) ***Expectation hypothesis*** in the forward rates[8] means that the time series of today (in time t) quoted forward exchange rate FR_t^{t+n} depends on today´s (in time t) expected future spot exchange rate $E_t(SR_{t+n})$. Because the expected spot rate is not available, it is replaced, consistently with the hypothesis of rational expectations, by the specific value of future spot exchange rate SR_{t+n}. Thus we use the substitution based on the following two relationships

$$FR_t^{t+n} = f\left[E_t(SR_{t+n})\right] + w_{t+n}\,, \tag{16}$$

$$SR_{t+n} = E_t(SR_{t+n}) + v_{t+n}\,, \tag{17}$$

c) ***Adaptive expectation hypothesis*** means that the time series of today´s quoted forward exchange rate FR_t^{t+n} depends on the time series of the known present and past values of the spot exchange rate, that is SR_t, SR_{t-1},…

[7] From our perspective, it is therefore theoretically questionable to use the historical volatility rate as a proxy variable in interpreting the risk premium on the forward market, if at testing we use the time series of forward rates mid.

[8] It is an analogy to the hypothesis of the expectations theory in the field of the yield curve (i.e. the term structure of interest rates).

d) ***Covered interest rate parity hypothesis*** claims that the forward exchange rate is based on the arbitrage equilibrium. The time series of today´s forward exchange rate FR_t^{t+n} depends on the time series of known present and past values of the spot exchange rate; that is SR_t, SR_{t-1}, … and the time series of the present or past values of interest rate differentials between the domestic interest rate $IR_{D,t}^{t+n}$ and the foreign interest rate $IR_{F,t}^{t+n}$.

The consideration was given to the possibility of the existence of risk premiums in accordance with their theoretical justification in the second part. This will also be tested and discussed, particularly the possibility of a credit risk premium in the case of the covered interest rate parity and the liquidity risk premium in the case of the unbiasedness hypothesis and the adaptive expectation hypothesis.

The first two hypotheses follow from relationship (15); the third and forth hypotheses follow from relationship (9).

The forward rate unbiasedness and the expectation hypotheses and weak exogeneity

To test these hypotheses, the following three options may arise: I) Two-sided dependence expressed by a joint model of the time series. Both the forward rate unbiasedness and the expectation hypotheses hold. II) One-sided dependence in which in the conditional model, SR_{t+n} is a dependent variable, and FR_t^{t+n} is an independent variable. This means that the direction of dependence goes from FR_t^{t+n} to SR_{t+n}, and the hypothesis of the forward rate unbiasedness holds. III) One-sided dependence in which in the conditional model, FR_t^{t+n} is a dependent variable, and SR_{t+n} is an independent variable, that is the direction of dependence goes from SR_{t+n} to FR_t^{t+n} and the hypothesis of the expectation holds.

We start from the assumption that the analyzed time series are non-stationary of I(1) types and are co-integrated. Their two-sided relationship can be expressed by a two-dimensional error correction model (for example Arlt and Arltová, 2009, Pesaran, 2015, Enders, 2010).

The forward rate unbiasedness hypothesis can be formulated as the hypothesis of weak exogeneity of the forward rate FR_t^{t+n} with respect to the parameters of the conditional error correction model for the change of the future spot rate SR_{t+n}, the expectation hypothesis can be formulated as the hypothesis of weak exogeneity of the future spot rate SR_{t+n} with respect to the parameters of the conditional error correction model for the change of the exchange rate FR_t^{t+n}. The weak exogeneity means that the relationship of the analyzed time series can be seen as one sided, where one variable has a character of condition and the second one of consequence which follows on from the condition (Engle et al., 1983). The estimates of the parameters, in which we are interested, can be then obtained from a conditional model without the loss of information. The joint model is not needed.

From a joint model of the time series y_t and x_t,it is possible to obtain the so called conditional and marginal model by a simple derivation (Arlt and Arltová, 2009, Pesaran, 2015).

The conditional error correction model can be expressed in the form

$$\Delta y_t = c + \alpha(y_{t-1} - \beta x_{t-1}) + \rho \Delta x_t + \gamma_1(B)\Delta y_t + \gamma_2(B)\Delta x_t + u_t \, , \qquad (18)$$

where $\gamma_1(B) = \gamma_{1,1}B + \gamma_{1,2}B^2 + \ldots + \gamma_{1,p}B^p$, $\gamma_2(B) = \gamma_{2,1}B + \gamma_{2,2}B^2 + \ldots + \gamma_{2,q}B^q$ and B is the lag operator, for which $B^j z_t = z_{t-j}$. The marginal error correction model has the form

$$\Delta x_t = c_2 + \alpha_2(y_{t-1} - \beta x_{t-1}) + \gamma_{21}(B)\Delta y_t + \gamma_{22}(B)\Delta x_t + \varepsilon_{2t}\,, \tag{19}$$

where $\gamma_{21}(B) = \gamma_{21,1}B + \gamma_{21,2}B^2 + \ldots + \gamma_{21,r}B^r$, $\gamma_{22}(B) = \gamma_{22,1}B + \gamma_{22,2}B^2 + \ldots + \gamma_{22,s}B^s$, $E(u_t) = E(\varepsilon_{2t}) = 0$, $D(u_t) = \sigma_C^2$, $D(\varepsilon_{2t}) = \sigma_M^2$ and $Cov(u_t,\varepsilon_{2t}) = \sigma_{CM}$. Substituting the marginal model (19) into the conditional model (18) the second marginal model is obtained, that is

$$\Delta y_t = c_1 + \alpha_1(y_{t-1} - \beta x_{t-1}) + \gamma_{11}(B)\Delta y_t + \gamma_{12}(B)\Delta x_t + \varepsilon_{1t}\,, \tag{20}$$

where $c_1 = c + \rho c_2$, $\alpha_1 = \alpha + \rho\alpha_2$, $\gamma_{11}(B) = \gamma_1(B) + \rho\gamma_{21}(B)$, $\gamma_{12}(B) = \gamma_2(B) + \rho\gamma_{22}(B)$, $\varepsilon_{1t} = u_t + \rho\varepsilon_{2t}$, $D(\varepsilon_{1t}) = E[(u_t + \rho\varepsilon_{2t})^2] = \sigma_C^2 + 2\rho\sigma_{CM} + \rho^2\sigma_M^2$, $Cov(\varepsilon_{1t},\varepsilon_{2t}) = E[(u_t + \rho\varepsilon_{2t})\,\varepsilon_{2t}] = \sigma_{CM} + \rho\sigma_M^2$. Suppose that

$$\delta = \frac{Cov(\varepsilon_{1t},\varepsilon_{2t})}{\sigma_M^2} = \rho + \frac{\sigma_{CM}}{\sigma_M^2}\,. \tag{21}$$

Multiplying the model (19) by parameter δ and subtracting this transformed model from the model (20), the following conditional model is obtained

$$\begin{aligned}\Delta y_t &= (c + (\rho-\delta)c_2) + (\alpha + (\rho-\delta)\alpha_2)(y_{t-1} - \beta x_{t-1}) + \delta\Delta x_t + \\ &+ (\gamma_1(B) + (\rho-\delta)\gamma_{21}(B))\Delta y_t + (\gamma_2(B) + (\rho-\delta)\gamma_{22}(B))\Delta x_t + e_t\,,\end{aligned} \tag{22}$$

where $e_t = u_t + (\rho - \delta)\varepsilon_{2t}$. The conditional model (18) has parameters: $\lambda_1 = (c, \alpha, \beta, \rho, \gamma_{1,1}, \ldots, \gamma_{1,p}, \gamma_{2,1}, \ldots, \gamma_{2,p}, \sigma_C^2)$, the marginal model (19) has parameters: $\lambda_2 = (c_2, \alpha_2, \beta, \gamma_{21,1}, \ldots, \gamma_{21,r}, \gamma_{22,1}, \ldots, \gamma_{22,s}, \sigma_M^2)$.

The first condition of the weak exogeneity of Δx_t for the parameters of a conditional model λ_1, namely the condition that the parameters are functions only of the parameters of a conditional model, is fulfilled in a model (22) when a) $\sigma_{CM} = 0$, b) $\alpha_2 = 0$. Under this condition, the second condition of weak exogeneity is also met. This means that the parameters of the conditional and marginal models, i. e., λ_1 and λ_2 are variation free. (See, for example: Engle at al., 1983, Johansen, 1992, Boswijk, 1992, Urbain, 1992, Enders, 2010, Pesaran, 2015).

From the conditional model (22) it follows, that $Cov(\Delta x_t,e_t) = Cov(\varepsilon_{2t},e_t) = E(\varepsilon_{2t}(u_t + (\rho - \delta)\varepsilon_{2t})) = \sigma_{CM} + (\rho - \delta)\sigma_M^2$. If the so called „condition of simultaneity loss" is true, that is $\sigma_{CM} = 0$, then $\delta = \rho$, $Cov(\Delta x_t,e_t) = 0$ and model (22) is reduced to model (18). The parameters of interest, (apart from the long-run multiplier β) are only in the conditional model. This condition ensures that between the series Δx_t and Δy_t only the one-way relationship exists (no simultaneous relationship), namely from Δx_t

to Δy_t, and no feedback is there. To obtain the estimates, and to test the parameters of interest λ_1, the conditional model is sufficient. This corresponds generally with the predetermination condition. The strict exogeneity that leads to unbiased estimates of the parameters of the conditional model using the least squares method requires that there is no feedback in the marginal model (19); that is: $\gamma_{21,1} = \ldots = \gamma_{21,r} = 0$ (Pesaran, 2015, p. 198-9).

The condition $\alpha_2 = 0$ is refered to as the „identification condition". This means that Δx_t is not influenced by deviation from the „long-run relationship". As $\alpha = \alpha_1 - \rho\alpha_2$ in the model (18), under this condition it can be written as

$$\Delta y_t = c + \alpha_1 y_{t-1} + \beta_1 x_{t-1} + \rho\Delta x_t + \gamma_1(B)\Delta y_t + \gamma_2(B)\Delta x_t + u_t, \quad (23)$$

where $\beta_1 = -\alpha_1\beta$. Because in the marginal model the long-term multiplier β is not present, it is clear that its estimate is uniquely obtained („identified") from the estimated parameters α_1 and β_1 of conditional model (23) (Enders, 2010).

Weak exogeneity test

Assuming the non-stationarity of the analyzed time series and their co-integration, testing of the identification condition is based on model (19) where the null hypothesis is $\alpha_2 = 0$.

The test of the simultaneity loss condition is based on the following procedure. Assume that $\omega = \sigma_{CM}/\sigma_M^2$, from relationship (21) it follows that $\rho - \delta = -\omega$. Then the model (22) under the condition $\alpha_2 = 0$ can be expressed in the form

$$\begin{aligned}\Delta y_t &= (c - \omega c_2) + \alpha_1(y_{t-1} - \beta x_{t-1}) + (\rho + \omega)\Delta x_t + \\ &\quad + (\gamma_1(B) - \omega\gamma_{21}(B))\Delta y_t + (\gamma_2(B) - \omega\gamma_{22}(B))\Delta x_t + e_t.\end{aligned} \quad (24)$$

From model (24) it follows that the tested hypothesis, which means the simultaneity loss condition is H_0: $\omega = 0$ in the model

$$\Delta y_t = c + \alpha_1(y_{t-1} - \beta x_{t-1}) + \rho\Delta x_t + \gamma_1(B)\Delta y_t + \gamma_2(B)\Delta x_t + \omega\varepsilon_{2t} + e_t. \quad (25)$$

The steps in testing the forward rate unbiasedness hypothesis and the expectation hypothesis

a) With the use of the unit root tests the stationarity of the time series FR_t^{t+n} and SR_{t+n} will be tested.
b) In the case that these time series are of I(1) type, the co-integration will be tested. If the null hypothesis that the time series are not co-integrated is rejected, the weak exogeneity, i. e., the conditions of simultaneity and identification will be tested.
c) The identification condition testing is performed in the model (19) as the testing of residuals $\hat{u}_{t-1} = y_{t-1} - \hat{\beta}x_{t-1}$ as another explanatory variable, $\hat{\beta}$ is obtained by OLS. Because all the time series used in this model, including $\hat{u}_{t-1}$ are stationary of the I(0) type (the original time series are nonstationary of the I(1) type and are

cointegrated, while in the model (19), the time series are differenced), it is possible to test the hypothesis H_0: $\alpha_2 = 0$ by the standard *t*-test.

d) The simultaneity loss condition testing is performed in the model (25) as the testing of the presence of residuals $\hat{\varepsilon}_{2t}$ from the marginal model (19). Because all of the time series in a conditional model (25) including the residuals $\hat{\varepsilon}_{2t}$ are stationary, the standard *t*-test can be used.

e) If the simultaneity loss and identification conditions are not rejected, and the result is the conditional error correction model

$$\begin{aligned}\Delta SR_{t+n} &= c + \alpha_1 (SR_{t+n-1} - \beta FR_{t-1}^{t+n}) + \rho \Delta FR_t^{t+n} \\ &+ \gamma_1(B)\Delta SR_{t+n} + \gamma_2(B)\Delta FR_t^{t+n} + u_t,\end{aligned} \tag{26}$$

then there is a co-integration relationship between the time series FR_t^{t+n} and SR_{t+n}, and the dynamics of the future spot exchange rate SR_{t+n} is conditional on the dynamics of the forward exchange rate FR_t^{t+n}, and so the hypothesis of forward rate unbiasedness is not rejected. The important role in the evaluation of the relationship between the time series has the value of parameter α_1. This is referred to as „loading". It characterizes the power that promotes the long-term relationship between the time series.

f) If testing of the simultaneity loss and identification conditions leads to the conditional error correction model

$$\begin{aligned}\Delta FR_t^{t+n} &= c + \alpha_1 (FR_{t-1}^{t+n} - \beta SR_{t+n-1}) + \rho \Delta SR_{t+n} \\ &+ \gamma_1(B)\Delta FR_t^{t+n} + \gamma_2(B)\Delta SR_{t+n} + u_t,\end{aligned} \tag{27}$$

then there is the co-integration between time series FR_t^{t+n} and SR_{t+n}, and the dynamics of the forward exchange rate FR_t^{t+n} is conditional on the dynamics of the future spot exchange rate SR_{t+n}, and the expectation hypothesis is not rejected. Here it is necessary to respect the fact that the future spot rate replaces its expected value. Again, when assessing the relationship between the time series, the value of „loading" α_1 plays an important role.

Adaptive expectation and covered interest rate parity hypotheses

The econometric situation in testing the hypotheses of adaptive expectations, and covered interest rate parity, is simpler than in the previous cases. The dependence of the time series of the forward exchange rate FR_t^{t+n} on the time series of the current spot exchange rate SR_t, or on the time series of the current spot exchange rate SR_t and interest rate differential ID_t^{t+n}, will be verified. The definition of the time series of SR_t and ID_t^{t+n} suggests that they are exogenous in the case of the validity of the analyzed hypothesis. Therefore, the exogeneity of time series need not be validated empirically, and the hypotheses of adaptive expectations and covered interest rate parity are supported by a co-integrated single-equation regression model that has the time series FR_t^{t+n} on the left side, and the time series SR_t or the couple SR_t and ID_t^{t+n}, or their lagged forms on the right side. The presence of the long-run relationships is, in this situation, tested by the Engle-Granger one-equational approach.

4. Empirical analysis - the examples of exchange rates of CZK/EUR and CZK/USD

The empirical verification of hypotheses has been made on the basis of the time series of the mid exchange rate of CZK to USD, and CZK to EUR, for the representatively long period 2001-2014 (daily frequency of data). The interest rate differencial is defined as $ID_t^{t+n} = (1 + PRIBOR_t^{t+n}) / (1 + LIBOR_t^{t+n})$. Testing was performed for the forward rates with a maturity of three months (n = 90), and six months (n = 180). Data were obtained from statistical sources of the Czech National Bank.[9] The analysis was performed for the whole period; that is from 2/5/2001 to 16/7/2014 (n = 90), or to 16/4/2014 (n = 180). Further sub-analyses were performed for the period before the financial crisis; namely from 2/5/2001 to 22/2/2007, after the financial crisis, from 26/3/2009 (n = 90), or from 27/3/2009 (n = 180) to 25/10/2012, and from 26/10/2012 till the end of time series, that is to 16/7/2014 (n = 90), or to 16/4/2014 (n = 180). In connection with the financial and economic crisis, after 2007, significant limits on trading in the interbank market with deposits for PRIBOR rates at longer maturities occurs. Vejmělek (2014) also points to the fact that after the massive foreign exchange interventions of the CNB in November 2013 to support the depreciation of CZK above the level of 27 CZK/EUR, there was a gradual loosening of the relationship within the covered interest rate parity due to excess of the CZK's liquidity. [10]

The forward rate unbiasedness hypothesis and the expectation hypothesis

When testing the forward rate unbiasedness hypothesis and the expectation hypothesis we will follow the procedure set out in the part 3. All calculations were performed in the Eviews 9 on the basis of logarithmically transformed time series. In Tab. 1a, b, there are given tests of stationarity, co-integration and weak exogeneity for the exchange rate of the CZK to EUR. In Tab. 2a, b, there are the same tests for the exchange rate of CZK to USD. For stationarity testing, the well known ADF test (Dickey and Fuller, 1979), both with constant and constant with trend, was used. The number of lags was specified in Eviews 9 automatically on the basis of the AIC criterion. It has been shown that the ADF test is a suitable test for long time series (Arltová and Fedorová, 2016), nevertheless for verification of the ADF tests results, the DFGLS; (Elliot, Rothenberg, Stock, 1996), PP (Phillips and Perron, 1988), KPSS (Kwiatowski, Phillips, Schmidt and Shin, 1992), ERS (Elliot, Rothenberg and Stock, 1996) and NP (Ng and Perron, 2001); tests were used. Generally, the results of all these tests supported the conclusions given by the ADF test. These, latter, are presented in the 2nd and 3rd columns of Tab. 1a, b and Tab. 2a, b.

In all cases, except for the forward exchange rate against the EUR with a maturity of three and six months in the second period, as well as the forward exchange rate to USD with a maturity of three and six months in the third period, the null

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[9] The official and sufficiently long time series for the forward and spot rates purchase (bid) and sell (ask) do not exist, therefore it is not possible to test the effect of risk premia on the spread rate.

[10] Some new findings on the issue covered interest rate parity, see Sushko, Borio, McCauley and McGuire (2016).

hypothesis of nonstationarity of type I(1) was not rejected by the ADF test, with a constant at the conventional 5 percent significance level. In the first two cases the null of the nonstationarity was also rejected by the PP test with a constant. However, the other unit root tests (including the ADF and PP tests with a constant and trend) did not reject the null hypothesis, and conversely, the KPSS tests did not reject the null hypothesis of stationarity. In the second two cases, the null of nonstationarity was rejected by a majority of tests at the 5 percent, the null of stationarity was not rejected by KPSS test with a constant and trend. The above-mentioned results are not surprising, the majority of the financial time series has a similar nonstationary shape.

Table 1 a, b The forward rate unbiasedness hypothesis and the expectation hypothesis

Variables, Period			E-G test, Constant (Prob.)		Weak Exogeneity (Prob.)				Cointegration
	$FR(czk/eur)_t^{t+90}$	$SR(czk/eur)_{t+90}$	*FR*	*SR*	*Identification*		*Simultaneity Loss*		*Loading*
I. Period	2/5/2001	22/2/2007	*t* **0.0194**	*t* **0.0182**	MM(0,0) *FR*	MM(0,1) *SR*	CECM(0,0)*FR*	CECM(.,.)*SR*	CECM(0,0)*FR*
ADF_c(Prob.)	0.5632	0.5354	z **0.0121**	z **0.0113**	**0.0000**	0.9451	0.4646	-	-0.0225 (**0.0000**)
II. Period	26/3/2009	25/10/2012	*t* 0.0567	*t* 0.0631	-	-	-	-	-
ADF_c(Prob.)	**0.0376**	0.0615	z 0.0626	z 0.0349	-	-	-	-	-
III. Period	26/10/2012	16/7/2014	*t* 0.5767	*t* 0.5839	-	-	-	-	-
ADF_c(Prob.)	0.6321	0.7199	z 0.5352	z 0.5421	-	-	-	-	-
Total	2/5/2001	16/7/2014	*t* **0.0001**	*t* **0.0000**	MM(0,0) *FR*	MM(0,0) *SR*	CECM(0,0)*FR*	CECM(.,.)*SR*	CECM(0,0)*FR*
ADF_c(Prob.)	0.2446	0.2711	z **0.0001**	z **0.0000**	**0.0000**	0.8207	0.6738	-	-0.018 (**0.0000**)
	$FR(czk/eur)_t^{t+180}$	$SR(czk/eur)_{t+180}$	*FR*	*SR*	*Identification*		*Simultaneity Loss*		*Loading*
I. Period	2/5/2001	22/2/2007	*t* 0.2994	*t* 0.3184	-	-	-	-	-
ADF_c(Prob.)	0.5794	0.6356	z 0.2626	z 0.2586	-	-	-	-	-
II. Period	27/3/2009	25/10/2012	*t* 0.0479	*t* 0.1246	-	-	-	-	-
ADF_c(Prob.)	**0.0497**	0.1529	z 0.0880	z 0.0855	-	-	-	-	-
III. Period	26/10/2012	16/4/2014	*t* 0.8160	*t* 0.7795	-	-	-	-	-
ADF_c(Prob.)	0.7426	0.6434	z 0.8087	z 0.8004	-	-	-	-	-
Total	2/5/2001	16/4/2014	*t* **0.0168**	*t* **0.0172**	MM(3,3) *FR*	MM(0,1) *SR*	CECM(0,0)*FR*	CECM(.,.)*SR*	CECM(0,0)*FR*
ADF_c(Prob.)	0.2776	0.2921	z **0.0052**	z **0.0050**	**0.0000**	0.6497	0.1360	-	-0.0104 (**0.0000**)

Notes: E-G test – Engle-Granger test (Engle and Granger, 1987), MM(*k*,*l*)*x* – marginal model (19), where *k* is the number of lags of the dependent variable *x*, *l* is the number of lags of the explanatory variable, CECM(*m*,*n*)*y* – conditional error correction model (25), where *m* is the number of lags of the dependent variable *y*, *n* is the number of lags of the explanatory variable.

Source: Own calculations.

Table 2 a, b The forward rate unbiasedness hypothesis and the expectation hypothesis

Variables, Period			E-G test, Constant (Prob.)		Weak Exogeneity (Prob.)				Cointegration
	FR(czk/usd)t+90	*SR(czk/usd)t+90*	*FR*	*SR*	*Identification*		*Simultaneity Loss*		*Loading*
I. Period	2/5/2001	22/2/2007	*t* **0.0029**	*t* **0.0024**	MM(0,0) *FR*	MM(0,0) *SR*	CECM(0,0)*FR*	CECM(.,.)*SR*	CECM(0,0)*FR*
ADF_c(Prob.)	0.6194	0.4916	z **0.0009**	z **0.0008**	**0.0001**	0.1242	0.4892	-	-0.015 (**0.0000**)
II. Period	26/3/2009	25/10/2012	*t* 0.1473	*t* 0.3133	-	-	-	-	-
ADF_c(Prob.)	0.1884	0.2701	z 0.1761	z 0.2801	-	-	-	-	-
III. Period	26/10/2012	16/7/2014	*t* 0.0869	*t* 0.6038	-	-	-	-	-
ADF_c(Prob.)	**0.0313**	0.6193	z 0.0358	z 0.4808	-	-	-	-	-
Total	2/5/2001	16/7/2014	*t* **0.0000**	*t* **0.0000**	MM(0,0) *FR*	MM(0,0) *SR*	CECM(0,0)*FR*	CECM(.,.)*SR*	CECM(0,0)*FR*
ADF_c(Prob.)	0.1269	0.0782	z **0.0000**	z **0.0000**	**0.0000**	0.6996	0.8996	-	-0.0164 (**0.0000**)
	FR(czk/usd)t+180	*SR(czk/usd)t+180*	*FR*	*SR*	*Identification*		*Simultaneity Loss*		*Loading*
I. Period	2/5/2001	22/2/2007	*t* **0.0516**	*t* **0.0185**	MM(0,0) *FR*	MM(1,1) *SR*	CECM(.,.)*FR*	CECM(.,.)*SR*	CECM(0,0)*FR*
ADF_c(Prob.)	0.6153	0.4267	z **0.0387**	z **0.0133**	**0.0042**	**0.0280**	-	-	-
II. Period	27/3/2009	25/10/2012	*t* 0.3416	*t* 0.4466	-	-	-	-	-
ADF_c(Prob.)	0.1758	0.2124	z 0.3734	z 0.4438	-	-	-	-	-
III. Period	26/10/2012	16/4/2014	*t* 0.0856	*t* 0.8609	-	-	-	-	-
ADF_c(Prob.)	**0.0226**	0.6630	z 0.0417	z 0.7594	-	-	-	-	-
Total	2/5/2001	16/4/2014	*t* **0.0039**	*t* **0.0031**	MM(2,1) *FR*	MM(0,0) *SR*	CECM(0,0)*FR*	CECM(.,.)*SR*	CECM(0,0)*FR*
ADF_c(Prob.)	0.1365	0.0736	z **0.0004**	z **0.0003**	**0.0000**	0.6535	0.5719	-	-0.0091 (**0.0000**)

Notes: E-G test – Engle-Granger test (Engle and Granger, 1987), MM(*k*,*l*)*x* – marginal model (19), where *k* is the number of lags of dependent variable *x*, *l* is the number of lags of the explanatory variable, CECM(*m*,*n*)*y* – conditional error correction model (25), where *m* is the number of lags of the dependent variable *y*, *n* is the number of lags of the explanatory variable.
Source: Own calculations.

The results of the Engle-Granger cointegration test (Engle and Granger, 1987), which is based on the residuals of a static regression with the constant (the level of the analyzed time series is very similar, but the model can be overparametrized by the constant) where the dependent time series is the forward exchange rate (*FR*), or the spot exchange rate (*SR*), are presented in the 4th and 5th columns of both tables. For residuals testing, the standard ADF testing criteria t and z, when the null hypothesis is the nonstationarity (no cointegration), are used (because the constant is included in the static regression, in the testing equation no deterministic component is used). The number of lags was specified by the minimalization of the AIC criterion. For comparison, the co-integration analysis in the VAR model with the application of the Johansen co-integration test (Johansen, 1995) was also performed. The lags of the VAR models were set on the basis of the residual diagnostic analysis (autocorrelation, heteroscedasticity, normality, AIC criterion).

In the case of the exchange rate against EUR, with a maturity of three months (Tab. 1a), and the exchange rate to USD, with a maturity of three months (Tab. 2a), the null hypothesis of no co-integration in the first period, that is from 2/5/2001 to 22/2/2007, and throughout the whole length of the time series, that is from 2/5/2001 to 16/7/2014; was rejected at 5% significance level. In other periods the null was accepted. The same results were obtained by the Johansen co-integration test (five versions of this test according to the presence of the deterministic part were applied), in the first period and throughout the whole length of the time series, the null of no co-integration was rejected at a 5 % significance level, and the alternative of one or two co-integration relationships were accepted. The situation with two relationships implies the stationarity of the analyzed time series, which is evidently not true because of the results of the unit root tests. In other periods the null was not rejected in most versions of the Johansen test. In these periods we did not continue to a further test, and both hypotheses, namely the forward rate unbiasedness and the expectation hypotheses, were considered to be not accepted. In the test of weak exogeneity we continued only in periods where the null hypothesis of no co-integration was rejected. In all such cases, the error correction part parameter estimate (loading) in the marginal model for a dependent variable of difference of the forward exchange rate (*FR*) was significantly different from (5% significance level) zero, and for the dependent variable of difference of the spot exchange rate (*SR*), it was insignificantly different from zero. The same results were obtained by testing the loadings in the VEC (Vector Error Correction) models. It is possible to conclude that the identification condition for a conditional error correction model with the dependent variable of difference of the forward exchange rate (*FR*) was not rejected. In this model we verified the condition of simultaneity loss; i.e. the presence of residuals of the marginal model as an additional explanatory variable. It was found that the parameter estimate of these residuals is not significantly different from zero (5% significance level). So, the hypothesis of weak exogeneity of the spot exchange rate with respect to the parameters of the conditional error correction model for the forward exchange rate was not rejected. We consider the expectation hypothesis in these cases as accepted. The forward rate unbiasedness hypothesis, however, was not accepted. It should be noted that in all cases the long-run relationship promotes itself very weakly, which indicates the value of estimated loading, which is listed in the last column of the tables. Very similar values were obtained from the VEC model.

In the case of the exchange rate against EUR, with a maturity of six months (Tab. 1b), and the exchange rate to USD, with the same maturity (Tab. 2b), the null hypothesis of no co-integration was rejected only throughout the whole length of the time series; that is from 2/5/2001 to 16/4/2014 for both exchange rates. In other periods the null of no co-integration was not rejected. The same results were obtained by application of the Johansen co-integration test. In both cases the parameter estimate by the error correction member in the marginal model for the dependent variable of the difference of the forward exchange rate (*FR*) was significantly different from zero (5% significance level), and for the dependent variable of the difference of the spot exchange rate (*SR*) it was statistically insignificantly different from zero. This result follows also from the VEC model. Therefore, the condition of identification for the conditional error correction model for the dependent variable of difference of the forward exchange rate (*FR*) was not rejected. In this model the condition of simultaneity loss was tested. It was found that the parameter estimate at the residuals from the marginal model is, statistically speaking, insignificantly different from zero. So, the hypothesis of weak exogeneity of the spot exchange rate for the parameters of the conditional error correction model for the forward rate was not rejected. Therefore, we consider the hypothesis of expectation in these cases as accepted, but the forward rate unbiasedness hypothesis as not accepted. But the long-run relationship promotes itself again very weakly.

The static regressions for the Engle-Granger co-integration test are characterized by statistically significant estimates of the constants. Given that these estimates are high, *t*-tests might not be distorted by a relatively big residual autocorrelation. This fact may indicate the existence of a liquidity risk premium.

Adaptive expectation hypothesis and hypothesis of covered interest rate parity

Based on an empirical analysis, the relationships of pairs of time series FR_t^{t+n} and $SR_{t\text{-}1}$ for the maturities of three and six months, and the currency pairs CZK/EUR and CZK/USD were examined first. These relationships are captured in Tab. 3. In Tab. 1a,b, and 2a,b, the ADF tests for the time series FR_t^{t+n} for the period from 2001 to 2014 are shown. In all cases, the null hypotheses that the time series are non-stationary of the type I(1) were not rejected. The third columns of Tab. 3 and Tab. 4 contain the results of the ADF test with a constant for the time series $SR_{t\text{-}1}$ for the period from 2001 to 2014. In all cases the null of nonstationarity was not rejected. These results were confirmed by other unit root tests which were mentioned above. In all four models, where FR_t^{t+n} is a dependent variable, and $SR_{t\text{-}1}$ is an independent variable, the values of the Durbin-Watson test range from 1.137404 to 1.778698, and thus exhibit a certain degree of residual autocorrelation. This does not alter the conclusion that the time series can be regarded as co-integrated, which follows from the outcome of the ADF test (without constant) of the residuals whose results (sign. level) are given in the last column of Tab. 3. These results are supported by other above-mentioned unit root tests. The presence of autocorrelation can lead to the underestimation of standard errors of parameter estimates. For their correction, the HAC parameter covariance estimator (Newey and West, 1987) was used. The estimates of the regression parameters are close to one and significantly different from zero (5% significance level), similar to the estimates of the constants (except for the first model), which are distinctly lower.

The presence of the constant would indicate the existence of a risk premium. The reason for the residual autocorrelation can be a time-varying risk premium or missing explanatory variable, which motivates us to estimate another model, the so called covered interest rate parity model.

In Tab. 4 there are the models from Tab. 3 extended by the time series of the interest rate differentials, in accordance with the hypothesis of the covered interest rate parity. The third column of this table contains the results of the ADF test with the constant for these time series. In all cases, the hypotheses of nonstationarity of type I(1) were not rejected (the same results were obtained by other unit root tests). The quality of the models was improved from the point of view of the residual autocorrelation. The values of the Durbin-Watson test range from 1.616246 to 1.997813. The inclusion of the interest rate differential has noticeably increased these values. Also, in these models, the HAC parameter covariance estimator was used, but the correction of the standard errors was very small. Based on the ADF test (and supported by the above-mentioned unit root tests), the null hypothesis of the residuals nonstationarity was rejected, so it can be stated that the analyzed time series are co-integrated. The estimates of the parameters of the interest rate differentials are relatively high. Their inclusion in the model did not significantly change the estimates of the parameters of the variables $SR_{t-1},$ but the estimates of the constants were lowered and are closer to zero. In three cases, they are statistically insignificant. In the case of CZK/EUR (6 months), the estimate of the constant is significantly different from zero (5 percent), but its value is also very low. Therefore, the existence of a higher risk premium (together with credit risk) in the model of covered interest parity was not confirmed.[11] The reason is probably the fact that the rating of the Czech Republic is in the long term not significantly different from the rating of the USA, and that the average rating of the Eurozone countries (probability of default within one year is practically zero).

The same analysis was conducted also for individual periods considered in the previous study. It was found that, with the exception of the last period, i.e. from 26/10/2012 to 16/07/2014, or from 26/10/2012 to 16/04/2014 in the case of CZK/EUR (3 months), CZK/USD (3 months) and CZK/EUR (6 months), and with the exception of the third and fourth period, i. e.: from 27/03/2009 to 25/10/2012, and from 26/10/2012 to 16/04/2014 in the case of CZK/USD (6 months); the results are similar to the results for the whole time series period; i.e. from 2001 to 2014.

In the last periods: i. e. from 2012 to 2014, the estimates of the parameters of interest rate differentials are not statistically significant. These results are consistent with the fact that the interest rates CZK, EUR and USD on the interbank market fell close to zero after 2011. The interest rate differentials have therefore very low volatility and approach zero.

[11] Another analysis procedure was chosen in the same currency pairs by Kukal and Van Quang (2014). Risk premium was calculated as the residuals in the uncovered interest parity, and then it was attempted to explain its development using the non-parametric and parametric model GARCH-M.

Table 3 Adaptive expectation hypothesis

Dep. Variable	Variable	ADF$_c$(Prob.)	Coefficient	Std. Error	*t*-statistic	Prob.	Stat.
$FR(czk/eur)_t^{t+90}$ *Observations:* 3070	c		-0.005437	0.00341	-1.59243	0.1114	R^2 = 0.998107 *F*(Prob.) **0.0000** *DW* = 1.778698 *ADF*(Prob) **0.0000**
	$SR(czk/eur)_{t-1}$	0.2435	1.001573	0.00102	979.603	**0.0000**	
$FR(czk/usd)_t^{t+90}$ *Observations:* 3063	c		-0.010306	0.00293	-3.52406	**0.0004**	R^2 = 0.998563 *F*(Prob.) **0.0000** *DW* = 1.654372 *ADF*(Prob) **0.0000**
	$SR(czk/usd)_{t-1}$	0.1274	1.003286	0.00094	1072.85	**0.0000**	
$FR(czk/eur)_t^{t+180}$ *Observations:* 2774	c		-0.021062	0.00598	-3.52208	**0.0004**	R^2 = 0.997502 *F*(Prob.) **0.0000** *DW* = 1.287084 *ADF*(Prob) **0.0002**
	$SR(czk/eur)_{t-1}$	0.2916	1.006222	0.00180	559.874	**0.0000**	
$FR(czk/usd)_t^{t+180}$ *Observations:* 2774	c		-0.024320	0.00493	-4.93771	**0.0000**	R^2 = 0.997952 *F*(Prob.) 0.0000 *DW* = 1.137404 *ADF*(Prob) **0.0001**
	$SR(czk/usd)_{t-1}$	0.1341	1.007778	0.00158	636.354	**0.0000**	

Source: Own calculations.

Table 4 Hypothesis of covered interest rate parity

Dep. Variable	Variable	ADF$_c$(Prob.)	Coefficient	Std. Error	*t*-statistic	Prob.	Stat.
$FR(czk/eur)_t^{t+90}$ *Observations:* 3070	c		-0.00102	0.00244	-0.41868	0.6755	R^2 = 0.998317 *F*(Prob.) **0.0000** *DW* = 1.997813 *ADF*(Prob) **0.0001**
	$SR(czk/eur)_{t-1}$	0.2435	1.00021	0.00073	1377.06	**0.0000**	
	$ID(czk/eur)_{t-1}{}^{t+90}$	0.2632	0.88735	0.04404	20.1492	**0.0000**	
$FR(czk/usd)_t^{t+90}$ *Observations:* 3063	c		-0.00246	0.00206	-1.19335	0.2328	R^2 = 0.998797 *F*(Prob.) **0.0000** *DW* = 1.969341 *ADF*(Prob) **0.0001**
	$SR(czk/usd)_{t-1}$	0.1274	1.00053	0.00066	1527.69	**0.0000**	
	$ID(czk/usd)_{t-1}{}^{90}$	0.6583	0.96199	0.03668	26.2245	**0.0000**	
$FR(czk/eur)_t^{t+180}$*Observations:* 2774	c		-0.01095	0.00277	-3.94808	**0.0001**	R^2 = 0.998331 *F*(Prob.) **0.0000** *DW* = 1.918478 *ADF*(Prob) **0.0000**
	$SR(czk/eur)_{t-1}$	0.2916	1.00309	0.00083	1216.25	**0.0000**	
	$ID(czk/eur)_{t-1}{}^{t+180}$	0.1532	0.88658	0.02419	36.6479	**0.0000**	
$FR(czk/usd)_t^{t+180}$*Observations:* 2774	c		-0.00273	0.00380	-0.71915	0.4721	R^2 = 0.998570 *F*(Prob.) **0.0000** *DW* = 1.616246 *ADF*(Prob) **0.0000**
	$SR(czk/usd)_{t-1}$	0.1341	1.00044	0.00119	840.525	**0.0000**	
	$ID(czk/eur)_{t-1}{}^{t+180}$	0.6251	0.78514	0.03314	23.6899	**0.0000**	

Source: Own calculations.

4. Conclusion

The main aim of the paper was the empirical analysis of the relationships between the forward and spot exchange rates in the Czech Republic in the period from 2001 to 2014. In this context four hypotheses were formulated: the forward rate unbiasedness hypothesis, the expectation hypothesis, the adaptive expectation hypothesis and the hypothesis of covered interest rate parity. To test the first two hypotheses, the innovative econometric procedure based on testing co-integration and weak exogeneity was proposed. In stationarity testing, the results obtained by the ADF test were presented in tables and were confirmed by other unit root tests. The results of the Engle-Granger co-integration test were presented, for confirmation also the Johansen co-integration testing approach was applied. The weak exogeneity in the co-integrated system was verified by the testing of the simultaneity loss and identification conditions. The third and fourth hypotheses were verified by the standard Engle-Granger co-integration test based on the single-equation model. Its application was supported by the factual justification of the exogeneity of the explanatory variables.

The estimates made for the forward exchange rates CZK/EUR and CZK/USD mid with maturities of three and six months for the period 2001-2014 (daily frequency of data), in most cases do not support the forward rate unbiasedness hypothesis. The expectation hypothesis should be the preferred one. But the long-run relationship promotes itself very weakly.

The estimates, on the contrary, confirmed the competing hypotheses of adaptive expectation and especially the hypothesis of covered interest rate parity. In practice, this means that in the case of CZK, the market makers quoted the forward exchange rates according to the known „past" spot exchange rates taking the interest rate differential into account. Our empirical analysis based on the hypothesis of covered interest rate parity has identified a „structural break" in the period since September 2012. This result is consistent with the fact that the interest rates CZK, EUR and USD on the interbank market fell close to zero after 2011. Therefore, the interest rate differentials have very low volatility, and approach zero.

The estimates of the constants are in the most cases statistically insignificant. Only in the case of the forward exchange rate CZK/EUR (6 months), the constant estimate is significantly different from zero, but it has a very low value. Therefore, we conclude that our analysis has not demonstrated the existence of „higher values" of credit risk premiums on the covered interest rate parity. The reason is probably the fact that these are the short maturity contracts for currencies whose countries have a high rating. The unavailability of the necessary data on the forward rates bid and ask does not allow us to test the hypothesis that the change of currency risk is reflected in the width of the spread bid – ask at the forward exchange rate.

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