JEL classification: C61, C63, D82, D91, H55
Keywords: nonfinancial defined contributions, variable retirement, adverse selection, actuarial fairness

Benefit-Retirement Age Schedules and Redistribution in Public Pension Systems*

András SIMONOVITS—Institute of Economics, CERS, Hungarian Academy of Sciences;
Mathematical Institute of Budapest University of Technology;
Department of Economics of CEU (simonovits.andras@krtk.mta.hu)

Abstract
The dependence of benefits on the retirement age (the schedule) is an important feature in any public pension system. The nonfinancial defined contribution (NDC) pension system has recently become popular mainly because of its alleged actuarial fairness. Using the framework of mechanism design with adverse selection, these systems have theoretically been criticized because they neglect the resulting regressive intracohort redistribution: longer-lived workers retire later and are rewarded as if their life expectancies were average. We document such adverse selection on Hungarian data. The resulting redistribution can be diminished but not eliminated. Giving up the framework of mechanism design, we corroborate our earlier qualitative findings in a more realistic framework.

1. Introduction

Due to aging of the population, there is growing worldwide concern with the sustainability of public (and private) pension systems. Indexing the normal retirement age (NRA) to life expectancy seems to be a powerful tool to mitigate this stress, but this should be done with the so-called actuarially fair *benefit-retirement age schedule*. Various versions of actuarially fair (or neutral) schedules, which steeply increase benefits for any additional year above the minimum retirement age, have been in use for decades. The past decade has witnessed the spread of a theoretically simple and elegant scheme called NDC. At the same time, using mechanism design, a number of theoretical papers have shown that adverse selection (workers retiring later also die later) may call for dampened schedules.

The present paper presents a novel Hungarian data set revealing strong adverse selection and discusses the pros and cons of NDC in a theoretical model. Our new set-up forsakes the mechanism design approach to allow for more realistic demographic and wage structures.

NDC is an acronym of *nonfinancial (or earlier: notional) defined contribution* and it refers to pension systems, where an unfunded system mimics a funded one without suffering from the costly transition period. The basic idea of the NDC scheme is to pay a special *life annuity*: the annual pension benefit is calculated as the ratio of the stock of accumulated nonfinancial contributions and the remaining life expectancy. This allows for a variable (flexible) retirement age: those who want to retire before/after reaching the NRA receive a properly reduced/increased annual

* I am indebted to J. Banyár for reformulating our earlier NDC benefit-retirement age schedule. I acknowledge useful comments made by K. Ágoston, B. Barabás, N. Barr, R. Holzmann, H. Fehr, B. Krémer, E. Palmer, L. Radnóti, O. Settergren, E. Sheshinski and anonymous referees on earlier versions. Any remaining error is mine.

benefit. The schedule is independent of the wage and for any given retirement age the benefit is proportional to the wage.[1] If workers differed only in their labor disutilities or in the dates of claiming their benefits (not modeled here), then the NDC scheme would provide perfect incentives, maximize social welfare and ensure the balance between revenues and expenditures (Holzmann and Palmer, 2006, and Holzmann, Palmer and Robalino, 2012).[2]

In a paradoxical way, the importance of wage heterogeneity has been stressed in the evaluation of *progressive* public pension systems. Critics of such systems (e.g. World Bank, 1994, p. 131) and their defenders (e.g. Orszag and Stiglitz, 2001) equally emphasize that "there [is] little [lifetime] redistribution from the rich to the poor, despite progressive benefit formulas, […] because [they are] earnings-related […] [and] […] upper-income people enter the labor force later in life and live longer after retirement." To see the independence of proportionality (between lifetime contributions and annual benefits) and neutrality, note that proportionality and weak actuarial adjustment coexist in a couple of countries (e.g. Germany), while in several other countries (e.g. the United States) progressivity and strong actuarial adjustment coexist.

Evidently, under the tight link between lifetime contributions and annual benefits, there is a hidden regressive lifetime redistribution from the poorer to the richer (see, for example, Breyer and Hupfeld, 2009). Krémer (2013) observed that the rather unequal distribution of the Hungarian *entry* pensions (Figure 17) becomes rather equal in the distribution of *all* pensions (Figure 16) partly because pensioners with low benefits die much earlier than the others.[3]

The designers of the NDC scheme have minimized *intercohort redistribution* of NDC by choosing an appropriate fictitious interest rate and adding a balancing mechanism to counter the impact of rising life expectancy and declining fertility (for a fresh analysis, see Knell, 2012). They claimed that NDC eliminates any intracohort redistribution.[4]

Following Diamond and Mirrlees (1978) on disability retirement, another group of analysts have criticized the excessive *intracohort redistribution* in the NDC scheme or, more generally, in actuarially fair adjustment in old-age pensions: Fabel (1994), Eső and Simonovits (2002), Diamond (2003, Chapter 7), Cremer, Lozachmeur and Pestieau (2004), Simonovits (2006), Sheshinski (2008), Bommier, Leroux and Lozachmeur (2011) and Eső, Simonovits and Tóth (2011).

For technical reasons, in modeling the socially optimal design, most of the above-mentioned critical writings assumed common earnings. Furthermore, our earlier papers used a *primitive* schedule, excluding the possibility that the shortest-lived type dies before the longest-lived one retires. Among the authors allowing for that complication, Diamond (2003) restricted his analysis to the choice between minimum and

[1] For simplicity, we neglect the age dependence of wages.

[2] The impact of *delayed claiming* of social security benefits was analyzed theoretically as well as empirically by Coile, Diamond, Gruber and Jousten (2002).

[3] The other part of this discrepancy is caused by the slow phasing-out of progressive benefits granted earlier.

[4] Note that in the model country for NDC, Sweden, the proportional NDC is modified by a pension credit system, at least after the pensioner's age overtakes the NRA.

maximum retirement ages, while Sheshinski (2008) discussed learning the survival curve. Furthermore, there were very few data to substantiate our theoretical findings. The proponents of the NDC scheme, however, have hardly noticed, let alone accepted, this type of critique on NDC's excessive intracohort redistribution. In this paper, our earlier critique of NDC is revisited and extended to other pension systems. There seems to be some hope that the present pedestrian approach may catch the proponents' attention.

The contributions of the present paper are as follows: First, nationwide Hungarian data are presented on all old-age pensioners who died in 2004 (unpublished work of Judit Marosi and Rudolf Borlói). Even though there were only very weak incentives to postpone retirement and quite strong incentives to retire early, there was very strong adverse selection in the period ending in 2004, the year when the citizens in our sample died. To give just one item of data on adverse selection: males aged 65 had a remaining life expectancy of 13.1 years, while those who retired at age 65 lived the surprisingly long period of 24.3 years on average. Second, giving up the mathematically sophisticated mechanism design approach and incorporating Banyár's (2011) critique into our earlier formulation, the original primitive NDC schedule is now replaced with a realistic one.[5] Under plausible assumptions, unless benefits are uniformly scaled down, the NDC scheme remains unbalanced, i.e. the average revenues are much lower than the average expenditures. Furthermore, the lifetime balances keep decreasing with life expectancies.

Third, to prove the inevitability of regressive intracohort redistribution, we have introduced the category of *regular* benefit—retirement age schedules. Delaying the formal definition of this category, we rest satisfied with the following outline: (i) an increase of one year in life expectancy raises the retirement age less than one year (a weakening of A1*) and (ii) the difference ratio of benefit to contribution is at least as high as the ratio of the working period to the retirement period (A5). Artificial numerical examples illustrate the analytical theorems. For example, using a 30% contribution rate and eliminating an aggregate deficit of about 2.5 years' total wages, our *balanced* NDC scheme punishes the shortest-lived (who died at 63) and lowest-paid with the denial of an equivalent 5.1-year type-specific wage. On the other hand, the longest-lived (who died at 93) and highest-paid is rewarded with nine-year type-specific wage. In the *compressed* NDC scheme, the lifetime balances are still large in absolute values and continue to decrease with life expectancies but are much more compressed, starting with a 2.6-year surplus and ending with a 3.8-year deficit.

It should be stressed that in the present approach, the mortality rates are given exogenously and are independent of the retirement age and the benefit. In reality, however, meaningful work lengthens the lifespan and increased benefits make room for better health care and longer lives.

That approach also neglects the very important gender differences, though in reality males earn much more and die much earlier than females do. Copying the practice of public pensions in our theoretical framework, we use a unisex framework, weakening the positive correlation between lifetime earnings and life expectancies prevailing for males and females separately.

[5] However, I do not follow him in his transformation of the NDC scheme into a minimum flat scheme.

Table 1 Expected Lifespan and Retirement Age, Male, Years, HU

Retirement age $L+R$	Relative frequency $100\varphi_R$	Retirement span $D_R - R$	Life expectancy $L + D_R$	Remaining life expectancy M_R	Estimation error S_R
57	7.4%	12.3	69.3	18.0	–5.7
58	6.0%	13.5	71.5	17.3	–3.8
59	4.6%	14.2	73.2	16.7	–2.5
60	60.5%	17.2	73.2	16.1	1.1
61	12.7%	18.1	79.1	15.4	1.7
62	3.9%	20.9	79.1	14.9	6.0
63	2.1%	22.4	85.4	14.3	8.1
64	1.6%	23.4	85.4	13.7	9.7
65	1.4%	24.3	89.3	13.1	11.2

Notes: $L = 20$, $S_R = D_R - R - M_R$. Number of observations: 28,500.

Table 2 Expected Lifespan and Retirement Age, Female, Years, HU

Retirement age $L+R$	Relative frequency $100\varphi_R$	Retirement span $D_R - R$	Life expectancy $L + D_R$	Remaining life expectancy M_R	Estimation error S_R
52	2.3%	14.8	66.8	27.4	–12.6
53	2.0%	15.8	68.8	26.6	–10.8
54	2.3%	19.7	73.7	25.7	–6.0
55	46.2%	20.7	75.7	24.9	–4.2
56	16.8%	23.6	79.6	24.1	–0.5
57	7.9%	24.3	81.3	23.3	1.0
58	5.9%	24.7	82.7	22.5	2.2
59	4.0%	25.4	84.4	21.7	3.7
60	4.3%	26.7	86.7	20.9	5.8
61	2.6%	25.6	86.6	20.1	5.5
62	2.0%	24.5	86.5	19.3	5.2
63	1.7%	23.5	86.5	18.5	5.0
64	1.0%	23.0	87.0	17.7	5.3
65	1.0%	21.5	86.5	16.9	4.6

Notes: $L = 20$, $S_R = D_R - R - M_R$. Number of observations: 30,300.

The structure of the remainder of the paper is as follows: Section 2 displays the Hungarian data on adverse selection. Section 3 analyzes the actuarial pitfalls of the NDC system, while Section 4 proves the inevitability of some intracohort redistribution for any regular scheme. Section 5 contains numerical illustrations on the balanced and the compressed NDC systems and Section 6 concludes the paper.

2. Hungarian Data on Adverse Selection

We present *Tables 1* and *2* to contrast the remaining life expectancies with the expected pension spans as a function of the retirement age of all Hungarian male and female old-age pensioners who died in 2004. We shall see the adverse selection mentioned in the introduction. Rows with the NRA (60 for males and 55 for females, valid until 1996) are italicized.[6] Note that our statistical data are quite rudimentary: they do not distinguish between the survival profiles of subsequent cohorts; they neglect the changes in the rules of retirement occurring in various years and are silent on wage heterogeneity.

Let's look at these items one by one. Like in the usual period life expectancy (this is a specific term, AS), the entering cohorts are replaced by outgoing cohorts. In Hungary, the NRA for males increased from 60 (until 1996) to 62 (since 2001), while this parameter value for females rose from 55 (until 1996) to 62 (since 2009). The calculation of benefits (both of the full benefit and the reward for delayed retirement) has also changed substantially. The full benefit was quite progressive until 1997; since then, progressivity has been quickly phased out. There was no reward for delayed retirement until 1996 and practically no extra reward for working longer than 42 years. Since 1997, the reward has been substantial and every additional year beyond 40 years of employment (with 80% net replacement) increases the benefits by 2% of the net wage.

The weakest point of the empirical background is the lack of data on pensions (and wages). If the main driver of intracohort redistribution is the well-known strong positive correlation between incomes and life expectancies rather than that between retirement ages and life expectancies, then the foregoing criticism is somewhat misplaced.

Nevertheless, I am convinced that this unique data set still correctly reflects the adverse selection. We start the presentation with column 1, classifying the deceased according to their retirement ages between 57 (52) and 65 for males (females). Column 2 contains the relative frequencies of these groups. Column 3 is the most interesting one, depicting the expected retirement span as a function of the retirement age. Adding up columns 1 and 3 yields the life expectancy, which is displayed in column 4. In contrast to males, females retiring above age 59 appear to differ only in their labor disutilities but not in their life expectancies.[7] Column 5 contains the usual remaining (average) life expectancies, regardless of the retirement status. Column 6 displays the estimation error of government, being the signed difference of columns 3 and 5. Note that for males the estimation error drops from 5.7 (at retirement age 57) to –11.2 years (at 65), while for females the change is from 12.6 (at age 52) to –4.6 years (at age 65).

3. A Critique of the NDC Scheme

Having presented the data, we work out the model to criticize the NDC scheme, comparing symmetric information and asymmetric information. For convenience, we neglect childhood: $L = 0$, hence the type-specific adult retirement age and the length of employment are equal. We assume that the population is stationary; there is neither inflation nor personal income taxation (for an exception, see Cremer *et al.*, 2004). We also assume that the government knows the probability distribution of lifespans. Every worker chooses his own retirement age (i.e. when he stops

[6] Divényi and Kézdi (2012) provided evidence that the poor health status of the Hungarian population above the age of 50 is one important cause of low old-age employment and, by implication, of low but heterogeneous life expectancies.

[7] In this domain, NDC works perfectly.

working and claims the benefit) depending on his life expectancy, consumption utilities and labor disutility. In harmony with the practice of public pension systems, there is no sex discrimination.

3.1 Symmetric Information

Following the proponents of the NDC system, we start our analysis with the assumption of *symmetric information*: the government and the workers have the same stochastic information on lifespans (or, more accurately, life expectancies). We shall see that, in this case, the NDC is indeed actuarially fair.

Let f_D be the probability of death at age D, for $D = \alpha, \ldots, \omega$; $\sum_{D=\alpha}^{\omega} f_D = 1$, α being the earliest age that is relevant for old-age retirement, while ω is the maximum life span. We define the *remaining (average) life expectancy* of age a, where a is a positive integer:

$$M_a = \frac{\sum_{D=a+1}^{\omega} f_D \left(D-a\right)}{\sum_{D=a+1}^{\omega} f_D} \tag{1a}$$

We shall assume that M_a is decreasing but the decrease is limited: $M_a > M_{a+1} \geq M_a - 1$.

We shall need the intermediate values to be defined by linear interpolation. Let A be a positive real number and $a = [A]$ be its integer part, while $\{A\}$ is its fractional part, i.e. $A = a + \{A\}$. Then the generalized remaining life expectancy is

$$M\left(A\right) = \left(1 - \left\{A\right\}\right) M_a + \left\{A\right\} M_{a+1} \tag{1b}$$

Note that $M(A)$ is continuous and decreasing.

For simplicity, we assume that a worker earns a positive constant annual wage w during his active lifetime and the government charges a contribution rate τ, where $0 < \tau < 1$. Thus a worker retiring at age R has accumulated nonfinancial pension wealth $\tau R w$, yielding the annual NDC benefit

$$b^N\left(R\right) = \frac{\tau R w}{M\left(R\right)} \tag{2}$$

Note that delaying retirement by one year raises the numerator and decreases the denominator, thus doubly raising the annual benefit.

Then, regardless of the value of the retirement age R, his expected lifetime (pension) balance is zero:

$$z^N\left(R\right) = \tau R w - M\left(R\right) b^N\left(R\right) = 0 \tag{3}$$

We can also consider the special symmetric case, when both the government and the workers know the exact life expectancies. Then the full-information benefit rule

$$b^F\left(R\right) = \frac{\tau R w}{D - R} \tag{4}$$

also leads to a zero lifetime balance:

$$z^F(R) = \tau R w - (D - R) b^F(R) = 0 \tag{5}$$

It is remarkable that, assuming a common retirement age and using the full-information rule, Breyer and Hupfeld (2009) also totally eliminated redistribution. This solution, however, totally neglects the adverse selection; therefore, we do not follow it.

3.2 Asymmetric Information

The facts presented in *Tables 1* and *2*, namely that D_R was a steeply increasing rather than approximately constant sequence, call for replacing the assumption of symmetric information with that of *asymmetric information*. Therefore, we assume that only the workers know exactly their own lifespans (or life expectancies) but the government only knows the stochastic distribution.

We revert from D_R to R_D and add notation of annual wage w_D. Introducing the government's *estimation error of pension span* $S_D = D - R_D - M(R_D)$ the type-specific balance is

$$z_D^N = \tau R_D w_D - (D - R_D)\frac{\tau R_D w_D}{M(R_D)} = (M(R_D) - D + R_D) b^N(R_D) = -S_D b^N(R_D) \tag{6}$$

In words, the NDC lifetime balance is the product of the negative of the estimation error and the corresponding benefit.

To obtain analytical results, we must make various assumptions. They are not always elegant but are quite realistic and a little restrictive, and make the proofs easy to understand. Henceforth, we number our assumptions as A1, A2, etc.

We assume that the retirement age is a weakly increasing function of life expectancy:

$$R_D \le R_{D+1}, \qquad D = \alpha, \alpha + 1, \ldots, \omega - 1 \tag{A1}$$

Additionally, we also assume that the annual wage is a weakly increasing function of life expectancy:

$$w_D \le w_{D+1}, \qquad D = \alpha, \alpha+1, \ldots, \omega - 1 \tag{A2}$$

(A1)–(A2) are acceptable from a logical as well as an empirical point of view. A simple consequence of (A1)–(A2), $M(R_D) \ge M(R_{D+1})$ and (2) is that the NDC benefits increase weakly with life expectancy: $b_D^N \le b_{D+1}^N$.

In harmony with *Tables 1* and *2*, we also assume the existence of a positive integer $\tilde{D}$ such that for a life expectancy lower/higher than $\tilde{D}$, the estimation error is negative/non-negative:

$$S_D < 0 \text{ if } D \le \tilde{D} \text{ and } S_D \ge 0 \text{ if } D > \tilde{D} \tag{A3}$$

Due to (6), the lifetime balances of the shorter/longer-lived types are positive/ /non-positive:

$$z_D^N > 0 \text{ if } D \le \tilde{D} \text{ and } z_D^N \le 0 \text{ if } D > \tilde{D} \tag{7}$$

Strengthening (A3), we may assume that the estimation error is also an increasing fraction of the life expectancy. This assumption implies that the retirement age stays away from the life expectancy, i.e. $R_\omega << \omega$. (For example, in the US, $R_\omega = 70$ years.)

$$S_\alpha < \ldots < S_{\tilde{D}} < 0 \le S_{\tilde{D}+1} < \ldots < S_\omega \qquad (A3^*)$$

We can now formulate

Theorem 1

a) *Under (A1)–(A3*), the lifetime balance* z_D^N *of the longer-lived types* $D > \tilde{D}$ *is non-positive and decreasing with life expectancy D:*

$$0 \ge z_{\tilde{D}+1}^N > \ldots > z_{\omega-1}^N > z_\omega^N \qquad (8a)$$

b) If, in addition to a), the wages and the retirement ages are both type-invariant, then the lifetime balance z_D^N *of a shorter-lived type* ($D < \tilde{D}$) *is positive and decreasing with life expectancy D:*

$$z_\alpha^N > \ldots > z_{\tilde{D}-1}^N > z_{\tilde{D}}^N > 0 \qquad (8b)$$

Proof

a) Considering the longer-lived, by ($A3^*$), the first factor of z_D^N in (6) is negative and decreasing; by (A1)–(A2), the second factor is positive and increasing; therefore, their product is negative and decreasing.

b) If w_D and R_D are age-invariant, then the benefit b_D^N is also invariant; therefore, z_D^N is proportional to S_D, hence ($A3^*$) implies (8b).

Remarks

(i) ($A3^*$) trivially holds for the primitive approach (e.g. Eső *et al.*, 2011) when $S_D = D - D^*$, where D^* is the average life expectancy.

(ii) Also, ($A3^*$) trivially holds if the retirement age is type-invariant: $R_D \equiv R^*$, when $S_D = D - R^* - M(R^*)$.

(iii) Note that without assuming constant wages and retirement ages, (8b) is not true in general (in (6), $-S_D > 0$ would decrease, while b_D^N still increases) but we will return to that in Theorem 3.

Example 1

As a simple illustration, we consider the continuous (rather than discrete) uniform lifespan distribution, where the distribution function is

$$F(x) = \frac{x-\alpha}{\omega-\alpha} \quad \text{where} \quad \alpha \le x \le \omega$$

It is also assumed that the adult retirement age is a homogeneous linear function of the adult life expectancy: $R(D) \equiv \rho D$, where $½ < \rho < 1$. Following Banyár's critique, we assume that the earliest exit precedes the latest retirement: $\alpha < \rho\omega$. Then the average life expectancy is

$$D^* = \frac{\alpha + \omega}{2}$$

while the remaining life expectancy is

$$M(A) = D^* - A \text{ if } 0 \le A < \alpha \quad \text{and} \quad M(A) = \frac{1}{2}(\omega - A) \text{ if } \alpha < A \le \omega$$

Consequently, introducing the switching point $D_S = \rho^{-1}\ \alpha$ of $M(A)$, the estimation error is

$$S(D) = D - D^* \text{ if } \alpha \le D \le D_S \quad \text{and} \quad S(D) = \frac{1}{2}\left[(2-\rho)D - \omega\right] \text{ if } D_S < D \le \omega$$

Note that $\tilde{D} = D^*$ and (A3) as well as (A3*) hold.

We can now turn to the sign of the *average* balance:

$$z^* = \sum_{D=\alpha}^{\omega} f_D z_D \tag{9}$$

To generalize our earlier result on the deficit in the NDC scheme (Theorem 3 of Simonovits, 2003), we must look for a further sufficient condition. We propose the following assumption—the earning-weighted average of the conditional estimation errors is non-negative:

$$s = \sum_{D=\alpha}^{\omega} f_D S_D w_D \ge 0 \tag{A4}$$

In the original primitive model, where $S_D = D - D^*$ and $w_D \equiv 1$ hold, $s = 0$ also holds. In the realistic model, however, if the earnings were constant (A2) and the lifespan distribution were uniform (Example 1), then (A4) would not hold: $s < 0$. But we shall see in *Table 3* that a slight rise in the wage-lifespan function would make (A4) valid. Anyway, for type-invariant retirement ages, (A4) is not only a sufficient but also a necessary condition for $z^{*N} \le 0$. In summary, (A4) seems to be satisfactory.

Theorem 2

If (A1)–(A4) hold, then the average NDC balance is negative: $z^{*N} < 0$ *except if the retirement age is type-invariant*: $R_D \equiv R^*$ *and if* $s = 0$ *holds, then* $z^{*N} = 0$.

Proof

Inserting $z_D^N s$ [(6)] into z^* [(9)] and using notation $b_D^N = b^N(R_D) = \beta_D^N w_D$, the NDC balance

$$z^{*N} = -\sum_{D=\alpha}^{\omega} f_D S_D w_D \beta_D^N \tag{10}$$

can be estimated from above. Note that the replacement rates

$$\beta_D^N = \frac{\tau R_D}{M(R_D)}$$

Table 3 Outcomes for Original and Compressed NDC Schemes

Adult life expectancy D	Balanced N-benefit $\hat{b}_D^N$	Lifetime N-balance $\hat{z}_D^N$	Compressed M-benefit b_D^M	Lifetime M-balance [year] z_D^M
42	0.212	4.579	0.371	2.371
45	0.250	4.523	0.402	2.250
48	0.295	4.307	0.436	2.046
51	0.348	3.881	0.474	1.740
54	0.412	3.175	0.515	1.307
57	0.490	2.090	0.562	0.716
60	0.588	0.480	0.616	–0.080
63	0.713	–1.878	0.679	–1.145
66	0.816	–3.964	0.726	–1.974
69	0.936	–9.964	0.777	–2.974
72	1.078	–10.032	0.834	–4.177

Notes: $\hat{b}_D^N$ from (11) and b_D^M from (13). Except for column 1, the data are given in terms of the average wage.

form a weakly increasing sequence. By A3, for $D \le \tilde{D}$, $S_D < 0$ and for $D > \tilde{D}$, $S_D \ge 0$. Cutting z^{*N} into two at $\tilde{D}$, the first subsum contains positive products, while the second contains negatives. Inequalities

$$\beta_D^N \le \beta_{\tilde{D}}^N \text{ for } D \le \tilde{D} \quad \text{and} \quad \beta_D^N \ge \beta_{\tilde{D}}^N \text{ for } D > \tilde{D}$$

and A4 imply

$$z^{*N} \le -\sum_{D=\alpha}^{\omega} f_D \beta_{\tilde{D}}^N S_D w_D = -\beta_{\tilde{D}}^N \sum_{D=\alpha}^{\omega} f_D S_D w_D \le 0$$

If $R_\alpha < R_\omega$ (i.e. $\beta_\alpha^N < \beta_\omega^N$) or $s > 0$, then $z^{*N} < 0$.

Of course, the resulting deficit needs to be eliminated, for example, by suitably reducing the contribution rate τ to $\hat{\tau}(<\tau)$ in the corresponding *balanced* NDC-formula:

$$\hat{b}^N(R_D) = \frac{\hat{\tau} R_D w_D}{M(R_D)} \tag{11}$$

Note that we neglected the unfavorable reaction of the adjustment on the retirement ages.

At this point we must admit that our modeling of asymmetric information is still inadequate: we assume that the government knows R_D but does not know D. Similarly, the incorporation of wage heterogeneity enables the government to infer D from w_D. The best justification for these inadequacies is as follows: in reality, there are shorter-lived types with lower labor disutility and longer-lived ones with higher labor disutility who retire at the same age. Similarly, those who die at age D may have different wages.

4. Regular Benefit-Retirement Age Schedules

We want to show that Theorem 1 can be generalized from constant wages and retirement ages and the NDC scheme to variable wages and retirement ages and some other schedules, called regular. This way we reformulate Eső *et al*. (2011, Theorem 1) from common to increasing earnings (in (A2)). Note that our critique of the NDC system is addressed not against redistribution *per se* but its degree of excess. For the time being, we leave open the matter of whether the system is balanced or not. Before entering the discussion, we present an (irregular) example without any redistribution.

Example 2

The case where there is no redistribution. Let the replacement rate be type-invariant: $b_D = \beta w_D$ ($\beta > 0$), and let the (adult) retirement age be proportional to the (adult) life expectancy: $R_D \equiv \rho D$ ($1/2 < \rho < 1$). For a balanced system (where $\tau\rho = \beta(1-\rho)$), each type's lifetime balance is zero:

$$z_D = \tau w_D \rho D - \beta w_D \left(1-\rho\right) D = 0 \quad \text{where} \quad D = \alpha,\ldots,\omega$$

This rule, however, eliminates any incentives: it does not charge any deduction for early retirement and does not pay any reward for later retirement.

We now define the concept of a *regular* schedule (and drop superscript N) for (b_D) by additional assumptions (A1*) and (A5).

The increase in the retirement age, $\Delta R_D = R_{D+1} - R_D$ is non-negative and less than or equal to the ratio of the D's benefit b_D to the sum of the contribution τw_D and the same benefit:

$$0 \le \Delta R_D \le \frac{b_D}{\tau w_D + b_D} \quad \text{where} \quad D = \alpha,\ldots,\omega-1 \qquad \text{(A1}^*\text{)}$$

We also assume that the difference quotient of the benefit to the contribution is at least as large as the ratio of the next retirement age to the next actual pension span:

$$\frac{\Delta b_D}{\tau \Delta w_D} \ge \frac{R_{D+1}}{D+1-R_{D+1}} \quad \text{where} \quad D = \alpha,\ldots,\omega-1 \qquad \text{(A5)}$$

The upper boundary on ΔR_D in (A1*) has no clear economic content except to encourage better understanding; it is worth substituting the NDC benefits (2) into A1*. Indeed, (A1*) becomes

$$\Delta R_D^N \le \frac{R_D}{M\left(R_D\right) + R_D} < 1 \qquad \text{(A1}^*\text{N)}$$

Only the relaxation of (1*) into $\Delta R_D < 1$ is a natural requirement. Indeed, why should type $D + 1$ work at least one year longer than type D just because he is going to live one year longer?

A5 on the benefit-contribution difference quotient is even more difficult to interpret. In Diamond (2003) and Eső *et al*. (2011), wages were type-independent, therefore (A5) was automatically satisfied. (A5) only means that the type-dependent wage increases much less with life expectancy than the benefit does. For example, if

$R_D \equiv \rho D$, then (A5) states that the difference quotient should be at least $\rho/(1-\rho) \geq 1$. For the NDC benefits, (A5) holds.

Theorem 3

a) *Under (A1)*, (A2) and (A5), for any regular schedule, the lifetime balance is a weakly decreasing sequence of life expectancy:*

$$z_\alpha \geq \ldots \geq z_D \geq z_{D+1} \geq \ldots \geq z_\omega \tag{12}$$

b) If the system is balanced, then there exists an age $\hat{D}$ such that

$z_{\hat{D}} \geq 0 \geq z_{\hat{D}+1}$

Proof

a) Starting from $z_{d+1} = (\tau w_{D+1} + b_{D+1}) R_{D+1} - b_{D+1}(D+1)$ we shall arrive at z_D by increases. Introducing $b_{D+1} = b_D + \Delta b_D$ and $w_{D+1} = w_D + \Delta w_D$,

$$z_{D+1} = \left[\tau(w_D + \Delta w_D) + b_D + \Delta b_D\right] R_{D+1} - (b_D + \Delta b_D)(D+1) =$$
$$= (\tau w_D + b_D) R_{D+1} - b_D(D+1) + \left[\tau \Delta w_D R_{D+1} - \Delta b_D (D+1-R_{D+1})\right]$$

By (A5), the third term in [ ] is negative or zero; therefore, we can drop it. Introducing also $R_{D+1} = R_D + \Delta R_D$, the difference between the first and the second terms can be estimated by (A1*) as

$$z_{D+1} \leq (\tau w_D + b_D) R_{D+1} - b_D(D+1) = z_D + (\tau w_D + b_D)\Delta R_D - b_D \leq z_D$$

b) Trivial.

At the end of this section we discuss an alternative solution underlying the previous approaches that neglected wage heterogeneity. This approach has the advantage over the present one that it avoids the artificial restriction that wages are determined by life expectancy. Let us assume that we have a discrete two-dimensional life-expectancy-wage distribution $g_{D,w}$ and form the conditional life expectancy probability distribution for a given w, whose probability is denoted by $h_w > 0$:

$$\hat{f}_{D|w} = \frac{g_{D,w}}{h_w}$$

In theory, we could separate the population into separate w-classes and solve the schedule problem separately.

But no government has the necessary information on the two-dimensional $\hat{f}_{D|w}$ for a fine enough resolution. Moreover, our technical assumption that wages do not change during the working stage of the types would prevent any practical application.

5. Numerical Illustrations

Different pension rules imply different retirement ages, and this should be taken into account in the comparisons (cf. Eső *et al.*, 2011). Nevertheless, here we only display two simple numerical examples, illustrating the balanced and the compressed NDC systems, respectively, without taking into account their impact on the choice of retirement age.

We use the simplest analytical lifespan distribution function, namely the uniform one in Example 1. We shall use $\alpha = 42$ and $\omega = 72$ years, omitting the 21 years of childhood.[8] (In other words, people die between ages 63 and 93.) Furthermore, we assume a linear retirement age-adult lifespan function $R \equiv 2D/3$ and a linear wage-adult lifespan function $w_D = w_\alpha + \Delta(D - \alpha)$ so that the expected wage is equal to 1, for $w_\alpha = 0.9$, implying $\Delta = 0.0066....$ We apply $\tau = 0.3$.

We provide useful aggregate statistics on the balanced NDC scheme, denoting by **E** the expectations: $\mathbf{E}D = 57$ years, $\mathbf{E}w = 1.00$, $\mathbf{E}R = 38$ years, $\mathbf{E}b = 0.558$, $\mathbf{E}z = 0$, $\mathbf{D}z = 4.8$ and $s = -0.007$. (We have chosen to make the minimum wage w_α so high that $s \geq 0$ A4 just holds.) To eliminate the deficit, the benefits were proportionally reduced by 18.7% by choosing $\hat{\tau} = 0.244$ in (11). The size of the corresponding standard deviation can only be judged by means of comparison to the second simulation.

The first column of *Table 3* displays the life expectancies. The next two columns provide the type-specific benefits and balances for the balanced NDC scheme, respectively. Note that the shortest-lived type's annual benefit is quite low: about 34 percent of the lowest net wage, while the longest-lived type's annual benefit is quite high: about 140 percent of the highest net wage. The lifetime balances decrease, starting from higher than 5.1 years' type-specific wage and ending with lower than –9.1 years' type-specific wage.

Turning to the compressed NDC scheme, we *dampen* the incentives of the original NDC benefit function by taking only its power θ, where $0 < \theta < 1$ and multiply it by an appropriate constant $(b^*)^{1-\theta}$ (Simonovits, 2003):

$$b_D = \left(b_D^N\right)^\theta \left(b^*\right)^{1-\theta} \tag{13}$$

where $\theta = 0.5$ and $b^* = 0.527$, making the expected lifetime balance zero. We renounce the discussion of the incentive effect on retirement ages.

The aggregate statistics of the compressed NDC scheme are as follows: $\mathbf{E}R = 38$ years, $\mathbf{E}b = 0.501$, $\mathbf{E}z = 0.05$ and $\mathbf{D}z = 2.166$. The size of the corresponding standard deviation is only 45% of the balanced one, which is a great improvement.

The last two columns of *Table 3* display the type-specific benefits and lifetime balances of the compressed run. Note that the shortest-lived type's annual benefit is quite low but higher than it was originally: about 59% of the lowest net wage, while the longest-lived type's annual benefits are quite high but much less than before: about 108% of the highest net wage. The lifetime balances still decrease but they are much more compressed, starting with a 2.6-year surplus and ending with a 3.8-year deficit, all expressed in the type-specific wages.

In the end we note that with a more realistic life expectancy distribution function (with a hump-shaped rather than a constant density function), the distortion would probably be less but it would not be negligible. The replacement of proportional retirement ages with a more concentrated schedule would also reduce the distortion.

[8] We changed the usual $L = 20$ to 21 to have all numbers divisible by 3.

6. Conclusions

The NDC scheme is basically a reasonable pension system, achieving automatic rewarding/punishment of late/early retirement. Nevertheless, it neglects the impact of life expectancy on the choice of retirement age and the strong positive correlation between life expectancy and (lifetime average) wages. Therefore, this system achieves an excessively strong regressive redistribution from the expectedly shorter-lived and worse-paid workers to the expected longer-lived and better-paid ones. We showed that qualitatively similar redistribution occurs for all regular (and other) schemes but its size can be made much smaller.

In the international arena, there were or there are various types of interactions among progressivity and rewards/punishments. On the one hand, there were pension systems (like in Germany until 1992), where the benefit was proportional to individual lifetime contributions but there was hardly any reward/punishment. On the other hand, there are pension systems (like in the US), where the benefit is a strongly concave function of the lifetime contribution, but there is a strong reward/ /punishment for delaying/bringing forward retirement. The problem is made even more difficult by taking into account gender differences. As an anonymous referee remarked on the NDC, while there is a redistribution from poor males to rich males through the reward, there is another redistribution from rich males to poor females through progressivity.

Further research with calibrated data is needed to determine the socially optimal modification of the NDC system via mechanism design. At any rate, use of the NDC scheme requires much caution, even if means-testing or pension credit softens its impact.

REFERENCES

Banyár J (2012): Proposal for an Optimal Benefit Formula (in Hungarian). *Szigma*, 42:105–124.

Bommier A, Leroux M-L, Lozachmeur J-M (2011): Differential Mortality and Social Security. *Canadian Journal of Economics*, 44:273–289.

Breyer F, Hupfeld S (2009): Fairness of Public Pensions and Old-Age Poverty. *Finanz Archiv/ /Public Finance Analysis*, 65:358–380.

Coile C, Diamond P, Gruber J, Jousten A (2002): Delays in Claiming Social Security Benefits. *Journal of Public Economics*, 84:357–385.

Cremer H, Lozachmeur J-M, Pestieau P (2004): Social Security, Retirement Age and Optimal Income Taxation. *Journal of Public Economics*, 88:2259–2281.

Diamond P (2003): Taxation, Incomplete Markets and Social Security. *Munich Lectures*, Cambridge (MA), MIT Press.

Diamond P, Mirrlees J (1978): A Model of Social Insurance with Variable Retirement. *Journal of Public Economics*, 10:295–336.

Divényi J, Kézdi G (2012): On the Causes of Low Employment of Population above 50 in Hungary. The Role of Incentives, Cognitive Abilities and Health State (in Hungarian). In: (Eds.: Kolosi T, Tóth I Gy): *Social Report*. Budapest, Tárki (Social Research Institute), pp. 190–208.

Eső P, Simonovits A (2002): Designing Optimal Benefit Rules for Flexible Retirement. *Northwestern University*, Evanston (IL), *Discussion Paper CMS-EMS*, no. 1353.

Eső P, Simonovits A, Tóth J (2011): Designing Benefit Rules for Flexible Retirement: Welfare and Redistribution. *Acta Oeconomica (Hungary)*, 61:3–32.

Fabel O (1994): *The Economics of Pensions and Variable Retirement Schemes*. New York, Wiley.

Holzmann R, Palmer E (Eds.) (2006): *Pension Reform through NDCs: Issues and Prospects for Non-Financial Defined Contribution Schemes*. Washington D.C., World Bank.

Holzmann R, Palmer E, Robalino D (Eds.) (2012): *NonFinancial Defined Contribution Pension Schemes in a Changing Pension World.* Washington D.C., World Bank.

Knell M (2012): Increasing Life Expectancy and Pay-As-You-Go Pension Systems. *Österreichische Nationalbank, Working Paper*, no. 179.

Krémer B (2013): Why do Societies are Afraid of Longevity? (in Hungarian). *Sociological Review*, 23(3):51–83.

Orszag P, Stiglitz JE (2001): Rethinking Pension Reform: Ten Myths about Social Security Systems. In: Holtzmann R, Stiglitz JE (Eds.): *New Ideas about Old-Age Security: Toward Sustainable Pension Systems in the 21st Century.* Washington, D.C., World Bank.

Sheshinski E (2008): *The Economic Theory of Life Annuities*. Princeton, Princeton University Press.

Simonovits A (2003): Designing Optimal Linear Rules for Flexible Retirement. *Journal of Pension Economics and Finance*, 2:273–293.

Simonovits A (2006): Optimal Design of Pension Rule with Flexible Retirement: The Two-Type Case. *Journal of Economics*, 89:198–222.

World Bank (1994): *Averting the Old-Age Crisis*. New York (N.Y.), Oxford University Press.