

APPENDIX

Appendix 1.

The first appendix presents in an extended fashion the methodology described in short in Section 2 of the main body of the article.

1. The inference function for margins method.

Sklar (1959) showed that multivariate distribution can be decomposed into marginal distributions and a dependence function between them. This linking function is called a copula. Formally, let H be an n -dimensional distribution function with margins F_1, \dots, F_n . Then, there exists a n -copula C such that for all x in \bar{R}^n :

$$H(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n)) \quad (1)$$

Under an additional assumption that F_1, \dots, F_n are continuous, the copula function is uniquely determined and for any $u \in [0,1]^n$ the following relation holds:

$$C(u_1, \dots, u_n) = H(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)) \quad (2)$$

where F^{-1} is the generalised inverse function $F^{-1}(z) = \inf \{x \in R | F(x) \geq z\}$ for all $z \in [0,1]$.

The assumption of continuity proves particularly convenient for the estimation of parametric distributions. Let $\mathbf{x} = \{\mathbf{x}_1, \dots, \mathbf{x}_T\}$ be a sample data matrix, where $\mathbf{x}_t = \{x_{1t}, x_{2t}, \dots, x_{nt}\}$. If the joint distribution is n times differentiable, the density $h(\mathbf{x})$ is equal to the product of marginal densities f_i characterised by parameters α_i and the copula density c with parameter θ (Patton (2006)):

$$h(\mathbf{x}_t, \alpha, \theta) = \prod_{i=1}^n f_i(x_{it}, \alpha_i) \cdot c_\theta(F_1(x_{1t}, \alpha_1), \dots, F_n(x_{nt}, \alpha_n)) \quad (3)$$

This implies that the joint log-likelihood is the sum of univariate log-likelihoods and the copula log-likelihood:

$$\begin{aligned} \log(h(\mathbf{x}, \alpha, \theta)) &= \sum_{t=1}^T \sum_{i=1}^n \log(f_i(x_{it}, \alpha_i)) \\ &+ \sum_{t=1}^T \log(c_\theta(F_1(x_{1t}, \alpha_1), \dots, F_n(x_{nt}, \alpha_n))) \end{aligned} \quad (4)$$

This form suggests an IFM estimation procedure, consisting of *separate* estimation of the parameters of marginal distributions and then copula parameter conditionally on marginal distributions' parameters fixed, rather than a computationally much more involved, though asymptotically efficient *joint* estimation of parameters for margins and copulas by maximum likelihood (ML). The IFM method was proposed by Joe and Hu (1997) and is commonly applied in similar settings (Patton,

2006, Dias and Embrechts, 2010, Christoffersen *et al.*, 2012), primarily because it is computationally much more effective than the ML method, while the IFM estimator remains asymptotically normal (see Joe, 1997). The IFM method amounts to first, maximising the likelihood for margins $\sum_{t=1}^T \sum_{i=1}^n \log(f_i(x_{it}, \alpha_i))$ over α_i -s to obtain transformed variable $\hat{u}_{it} = F_i(x_{it}, \hat{\alpha}_i)$ which is distributed uniformly on a unit interval, and second, maximising the likelihood of the copula function $\sum_{t=1}^T \log(c_{\theta}(\hat{u}_{1t}, \dots, \hat{u}_{nt}))$ over θ . In our application we use the IFM method and limit ourselves to two-dimensional distributions, that is dependence between *pairs* of variables. We use Matlab R2011b, A. Patton's Copula Toolbox and J.P. LeSage's jplv7 toolbox.

1.1. Modelling marginal distributions

Following the IFM method, in the first step we specify parametrically the marginal distributions. To this end, we need an appropriate family of models. We decide to model the data in a broad tradition of GARCH framework which captures most of stylised facts observed in financial data (volatility clustering, asymmetry of gains and losses, thick tails, etc.). In many applications, a simple GARCH(1,1) model seems to be a reasonable approximation of the underlying process' dynamics and complex specification search hardly improves forecasting abilities of the model (Hansen and Lunde, 2001). However, the IFM method requires the marginal distributions to be well-specified and may be non-robust against misspecifications (Kim *et al.*, 2007). Therefore, the right implementation of the method involves allowing for a broad family of models from which the right model will be chosen, as well as using appropriate tests to choose the best alternative from the set of competing models.

Consider the variable of interest x_t . It is a logarithmic rate of return or a difference depending on the variable under consideration (*e.g.* rate of return for currencies, but difference for interest rates). Its conditional mean is parameterised as ARMA(R,M):

$$x_t = \kappa + \sum_{i=1}^R \phi_i x_{t-i} + \varepsilon_t + \sum_{j=1}^M \theta_j \varepsilon_{t-j} \quad (5)$$

In the models we consider the maximum possible orders of autoregressive and moving average terms are low in order to favour more parsimonious representations.

We model the conditional variance of each of the variables either as a pure GARCH(P,Q) or as one of the asymmetric extensions, EGARCH(P,Q) and GJR(P,Q).

The GARCH(P,Q) model is given as:

$$\sigma_t^2 = K + \sum_{i=1}^P G_i \sigma_{t-i}^2 + \sum_{j=1}^Q A_j \varepsilon_{t-j}^2 \quad (6)$$

with constraints:

$$\sum_{i=1}^P G_i + \sum_{j=1}^Q A_j < 1$$

$$K > 0$$

$$G_i, A_j \geq 0$$

The GARCH(P,Q) model is symmetric in that it ignores the sign of the error term. It is now, however, a well-known phenomenon that financial variables exhibit asymmetries in response to good and bad news, which traditionally is related to the leverage effect (Black, 1976), or volatility feedback effect (Campbell and Hentschel, 1992). Thus, an appropriate model should allow for asymmetric news impact on conditional volatility, i.e. good news ($\varepsilon_{t-j} > 0$) having different effect than bad news ($\varepsilon_{t-j} < 0$). The two important parameterisations are GJR(P,Q) and EGARCH(P,Q). In the GJR(P,Q), the conditional variance is specified as:

$$\sigma_t^2 = K + \sum_{i=1}^P G_i \sigma_{t-i}^2 + \sum_{j=1}^Q A_j \varepsilon_{t-j}^2 + \sum_{j=1}^Q L_j S_{t-j} \varepsilon_{t-j}^2 \quad (7)$$

where $S_{t-j} = 1$ if $\varepsilon_{t-j} < 0$ and $S_{t-j} = 0$ otherwise, with constraints:

$$\sum_{i=1}^P G_i + \sum_{j=1}^Q A_j + \frac{1}{2} \sum_{j=1}^Q L_j < 1$$

$$K \geq 0$$

$$G_i, A_j, A_j + L_j \geq 0$$

The conditional variance in the EGARCH(P,Q) parameterisation is given by:

$$\log \sigma_t^2 = K + \sum_{i=1}^P G_i \log \sigma_{t-i}^2 + \sum_{j=1}^Q A_j \left[\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} - E \left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} \right] + \sum_{j=1}^Q L_j \left(\frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right) \quad (8)$$

We restrain the maximum orders of P and Q, analogous to the conditional mean specification. Finally, we allow the error term ε_t in each of the models to follow either normal or t -Student distribution. The models are fitted using standard quasi-maximum likelihood estimation (QMLE) method.

A significant advantage of using the IFM method is that the specifications of margins can be tested using standard diagnostics to ensure that they fit the data well. In the post-estimation analysis, we employ Ljung-Box test for autocorrelation of the standardised residuals and Engle's ARCH test for the presence of the remaining ARCH effects in the residuals ε_t . We also employ the Berkowitz (2001)

procedure to test if the hypothetical model's probability integral transform produces observations which are independently and identically distributed $U(0,1)$.

Finally, using the conditional cumulative distribution function of the selected model, we transform our variable of interest x_t into a $U(0,1)$ distributed variable which serves as an input for the second step of the IFM method. In doing this, we calculate

$$\hat{u}_t = F(x_t | I_{t-1}; \hat{\alpha}) \quad (9)$$

which we call *transformed* variable, where I_{t-1} is the information set available at time $t - 1$ comprising past realisations of the variable of interest and $\hat{\alpha}$ is the estimated vector of parameters.

1.2. Modelling the dependence structure

The second stage of the IFM method consists of exploring the sole dependence between the two random variables using copula functions.

We chose a set of standard, static, parametric functions, most popular in the literature. They allow a wide range of dependence relations, including asymmetric tail dependence particularly important for investigating contagion effect. Thus, relations ranging from complete independence to dependence of differing grade, also in stress times, can be modelled.

The definition of contagion which we employ in the present paper can be operationalised with the so-called asymptotic *tail dependence coefficients* introduced by Sibuya (1960) (hereinafter TDC), which, thus, become our measure of contagion. The coefficients describe the propensity of markets to crash or boom together, i.e. they measure the dependence between extreme outcomes of the variables. The upper (lower) TDC is a limiting probability of one variable exceeding (falling behind) a high-order (low-order) quantile, given that the other variable exceeds (falls behind) the same quantile. Formally, if (X, Y) is a vector of continuous random variables with marginal distributions F_x and F_y , respectively, then the upper and lower TDCs are defined as:

$$\lambda_U = \lim_{t \rightarrow 1^-} P(Y > F_y^{-1}(t) | X > F_x^{-1}(t)), \quad (10)$$

and:

$$\lambda_L = \lim_{u \rightarrow 0^+} P(Y \leq F_y^{-1}(t) | X \leq F_x^{-1}(t)). \quad (11)$$

If the upper or lower TDC equals zero, the respective extreme values are independent, otherwise we say that there is dependence between extreme values of the variables considered. Importantly, for the copulas considered in this paper the TDCs are simple functions of copula parameters. The choice of a particular copula may in some cases restrict admissible asymptotic dependence (e.g. Gaussian copula implies asymptotic independence). The Table below gives an overview of the copulas we employ

along with their TDCs. Recall that copula functions are defined on a unitary box, $(u, v) \in [0,1]^2$, where $u = F_x(X)$ and $v = F_y(Y)$ are distributed as $U(0,1)$.

Table A. Copula functions and their characteristics.

Copula name	$\mathcal{C}(u, v)$	λ_L	λ_U
Normal	$\Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v))$, where Φ_ρ is the bivariate standardised Gaussian cdf with Pearson's correlation ρ and Φ^{-1} is the inverse of the univariate standardised Gaussian cdf	0	
Clayton	$(u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}$ $\alpha > 0$	$2^{-1/\alpha}$	0
Rotated Clayton	$u + v - 1 + C(1 - u, 1 - v)$, where C is Clayton copula	0	$2^{-1/\alpha}$
Plackett	$\frac{((1 + (\theta - 1)(u + v)) - \sqrt{(1 + (\theta - 1)(u + v))^2 - 4uv\theta(\theta - 1)}) / (2(\theta - 1))}{0 < \theta \neq 1,}$ for uv , for $\theta = 1$	0	
Frank	$\frac{1}{\alpha} \ln(1 + \frac{(e^{\alpha u} - 1)(e^{\alpha v} - 1)}{(e^\alpha - 1)})$ $\alpha \neq 0$	0	
Gumbel	$\exp(-(-\ln u)^\alpha + (-\ln v)^\alpha)^{1/\alpha}$, $\alpha > 1$	0	$2 - 2^{1/\alpha}$
Rotated Gumbel	$u + v - 1 + C(1 - u, 1 - v)$, where C is Gumbel copula	$2 - 2^{1/\alpha}$	0
t-Student	$t_{\nu, r}(t_\nu^{-1}(u), t_\nu^{-1}(v))$, where $t_{\nu, r}$ is the bivariate t -Student cdf with parameter r and degrees of freedom ν and t_ν^{-1} is the inverse of the univariate t -Student cdf with ν degrees of freedom	$2t_{\nu+1}(-\sqrt{\frac{(v+1)(1-r)}{(1+r)}})$	
Symmetrised Joe-Clayton (SJC)	$0.5(C_{\tau^U, \tau^L}(u, v) + u + v - 1 + C_{\tau^L, \tau^U}(1 - u, 1 - v))$, where $C_{\tau^U, \tau^L}(u, v) = 1 - \left\{ [(1 - (1 - u)^\kappa)^{-\gamma} + [(1 - (1 - v)^\kappa)^{-\gamma} - 1]]^{-1/\gamma} \right\}^{1/\kappa}$, for $\kappa = 1/\log_2(2 - \tau^U)$, $\gamma = -1/\log_2(\tau^L)$, and $\tau^U, \tau^L \in (0, 1)$	τ^L	τ^U
Independence copula	uv	0	

Having obtained a bi-variate pseudo-sample from any two transformed variables of interest as in eq. (9), parameters of the above copulas are obtained by maximising the respective likelihood functions.

1.3. Testing copula functions

The IFM procedure amounts to estimating θ under the assumption that the copula \mathcal{C} linking marginal distributions indeed belongs to a chosen family of copulas \mathcal{C}_0 , i.e. under $H_0: \mathcal{C} \in \mathcal{C}_0 = \{\mathcal{C}_\theta: \theta \in \Theta\}$. The goodness-of-fit tests, reviewed and compared in Monte Carlo studies by Genest *et al.* (2009) and Berg (2009), aim at the complementary issue of testing whether H_0 holds. To our knowledge, the cited papers are the latest available and most comprehensive studies of such methods in the literature. The experiments are designed to assess, in a number of different

setups, the ability of various goodness-of-fit tests to maintain the nominal levels and their power against a variety of alternatives. The only method that ranks among three best performing in both power studies is the goodness-of-fit procedure introduced in Genest *et al.* (2008), ranking first in Genest *et al.* (2009) and second in Berg (2009). It is based on the “empirical copula” (a-theoretic information on the dependence structure, to be defined below), it thus belongs to a class of “blanket tests” applicable to all copula structures and free of any strategic choices for their use or parameter fine-tuning. Its implementation involves, however, approximating *p-values* for testing H_0 with a bootstrap procedure.

The idea is to compare the distance between the “empirical copula” with the estimated parametric one. To assess whether the distance is significantly different from zero, a bootstrap procedure is implemented. As the input, the goodness-of-fit test takes the maximally invariant with respect to continuous, strictly increasing transformations of the components of bivariate distribution statistic, i.e. ranks obtained from the pseudo-sample $\{(\hat{u}_t, \hat{v}_t)\}_{t=1}^T$. The information on dependence comprised in the pseudo-sample is summarised in the “empirical copula” C_T

$$C_T(u, v) = \frac{1}{T} \sum_{t=1}^T \mathbf{1}(U_t \leq u, V_t \leq v), \quad (12)$$

for $u, v \in [0,1]$, where U_t is obtained by dividing the rank of \hat{u}_t (in a set $\{\hat{u}_t\}_{t=1}^T$) by $(T + 1)$, and V_t by dividing the respective rank of \hat{v}_t . The test statistics is based on the empirical process $\mathbb{C}_T = \sqrt{T}(C_T - C_{\hat{\theta}})$, and it is given by the Cramer-von Mises statistic

$$S_T = \int_{[0,1]^2} \mathbb{C}_T(u, v)^2 dC_T(u, v), \quad (13)$$

whose large values imply the rejection of H_0 . Asymptotic *p-values* could in theory be deduced from the limiting distribution of the above statistic. However, as the asymptotic behaviour of the empirical process depends on the family of copulas under the composite H_0 and on the unknown true parameter θ , whose estimate is used in C_T instead, the only viable way to execute statistical test is to resort to specially adapted parametric bootstrap procedure. It consists of the following steps:

- 1) Compute C_T and estimate $\hat{\theta}$
- 2) Compute $\hat{S}_T = \sum_{t=1}^T [C_T(\hat{u}_t, \hat{v}_t) - C_{\hat{\theta}}(\hat{u}_t, \hat{v}_t)]^2$
- 3) For a large B repeat the following for $b = 1, \dots, B$:
 - a. Generate a random sample from the distribution $C_{\hat{\theta}}$
 - b. Using the random sample compute $C_T^{(b)}$ and estimate $\hat{\theta}^{(b)}$
 - c. Compute $S_T^{(b)}$ analogously to 2)
- 4) Approximate *p-value* with $p = \frac{1}{B} \sum_{b=1}^B \mathbf{1}(S_T^{(b)} > \hat{S}_T)$.

The final question, if the above goodness-of-fit test admits more than one copula, concerns the choice of one particular function for further analysis. We chose the parametric copula with the lowest distance to the “empirical copula”, as measured by \hat{S}_T . Then, we compute the TDCs.

Appendix 2.

Table A. Definitions of variables and their transformations.

Variable	Definition	Transformation
EURPLN	Nominal spot exchange rate of euro expressed in Polish zloty	R
PL2Y	2-year Polish government generic bond yield (%). Currency denomination: Polish zloty.	D
PL10Y	10-year Polish government generic bond yield (%). Currency denomination: Polish zloty.	D
WIG	Warsaw Stock Exchange (WSE) WIG index, total return index which includes dividends. The index includes all companies listed on the main market and excludes foreign companies and investment funds. Currency denomination: Polish zloty.	R
PLBANKS	Sub-index of the WIG index which includes 14 banks listed on the WSE	R
VIX	CBOE volatility index which reflects a market estimate of future volatility, based on the weighted average of the implied volatilities for a wide range of option strikes for the S&P500 index. Commonly used as a market risk-aversion and uncertainty indicator	R
EURUSD	Nominal spot exchange rate of euro expressed in US dollars	R
EMCARRY	JPMorgan Income FX index tracking a strategy to generate positive returns by depositing money in a high yielding currency and borrowing money in a lower yielding currency, thereby earning the interest rate differential or “carry”. The strategy analyzes the monthly return generated by an investment in 14 currency pairs and select the 4 pairs with the highest ratio of return to risk; and then replicate an equally-weighted trading position in these 4 currency pairs. Currency of denomination: Euro.	R
EMFX	Morgan Stanley Capital International (MSCI) currency index which sets the weights of each of 25 currencies equal to the relevant country weight in the MSCI Emerging Markets equity index (see MSCI). The index measures total investment performance for included currencies stemming from appreciation/depreciation against US dollar and from return from interest earned in holding the currencies.	R
DE2Y	2-year German government generic bond (Bund) yield (%). Currency denomination: Euro.	D
DE10Y	10-year German government generic bond (Bund) yield (%). Currency denomination: Euro.	D
US2Y	2-year US government note yield (%). Currency denomination: US dollar.	D
US10Y	10-year US government note yield (%). Currency denomination: US dollar.	D
EMBI	Emerging Markets Bond Global Diversified Index measuring the total return performance of international government bonds issued by emerging market countries. In order to qualify for index membership, the debt must be more than one year to maturity and have more than USD 500 million outstanding inter alia.	D
SP500	Standard and Poor’s index of 500 stocks in US, capitalization-weighted. Currency denomination: US dollar.	R
DAX	German total return stock index of 30 stocks with largest capitalization. Currency denomination: Euro.	R
EUBANKS	Euro Stoxx Banks index, capitalization-weighted, including 32 EMU countries banking sector stocks. Currency denomination: Euro.	R
MSCI	MSCI Emerging Markets index, free float-adjusted market capitalisation index measuring equity market performance in the global emerging	R

	markets. The index covers over 800 securities across 23 markets and represents approximately 13% of world market capitalization. Currency denomination: US dollar.	
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Notes: R – rate of return between two consecutive trading days. D – difference between two consecutive trading days.

Appendix 3.

Table A. Results for the marginal distributions.

	EUR/PLN	PL 2Y	PL 10Y	WIG	PL BANKS	VIX	EUR/USD	FXEM	FX Carry
Conditional Mean	ARMA(0,4)	ARMA(1,1)	ARMA(0,3)	ARMA(3,5)	ARMA(1,5)	ARMA(1,1)	ARMA(3,2)	ARMA(1,1)	ARMA(5,4)
K					9.91E-05 (1.13E-04)	-1.94E-03 (5.46E-04)		2.45E-04 (1.62E-04)	2.59E-04 (5.74E-05)
ϕ_1		2.30E-01 (1.56E-01)		-6.00E-01 (1.82E-01)	5.85E-01 (2.19E-01)	4.98E-01 (8.94E-02)	1.36E+00 (2.55E-02)	5.73E-01 (2.61E-01)	-1.51E-01 (1.02E-01)
ϕ_2				-7.07E-01 (1.15E-01)			-8.98E-01 (3.37E-02)		1.25E+00 (8.66E-02)
ϕ_3				-6.86E-01 (1.61E-01)			-3.21E-02 (1.83E-02)		-3.23E-01 (7.85E-02)
ϕ_4									-8.14E-01 (1.05E-01)
ϕ_5									1.01E-02 (2.42E-02)
θ_1	-3.82E-02 (2.09E-02)	-3.38E-01 (1.50E-01)	3.05E-02 (2.09E-02)	6.35E-01 (1.84E-01)	-5.62E-01 (2.19E-01)	-6.32E-01 (7.99E-02)	-1.38E+00 (1.92E-02)	-5.30E-01 (2.70E-01)	2.00E-01 (9.99E-02)
θ_2	-2.06E-02 (2.08E-02)		-2.68E-03 (1.98E-02)	7.34E-01 (1.18E-01)	-3.78E-02 (2.33E-02)		9.53E-01 (1.85E-02)		-1.24E+00 (7.58E-02)
θ_3	-1.55E-03 (1.97E-02)		-7.70E-02 (1.83E-02)	7.11E-01 (1.68E-01)	-5.27E-03 (2.52E-02)				3.00E-01 (7.57E-02)
θ_4	-3.12E-02 (2.08E-02)			5.37E-02 (2.53E-02)	2.02E-02 (2.54E-02)				8.38E-01 (9.68E-02)
θ_5				-2.52E-02 (2.57E-02)	-5.72E-02 (2.25E-02)				

(the table continues overleaf)

Conditional Variance	EGARCH(5,5)	EGARCH(4,5)	EGARCH(5,5)	EGARCH(3,5)	EGARCH(4,5)	EGARCH(5,5)	EGARCH(4,4)	EGARCH(5,3)	EGARCH(3,3)
κ	-1.54E+00 (3.56E-01)	-1.68E-01 (7.19E-02)	-1.41E-01 (5.74E-02)	-1.15E+00 (2.78E-01)	-4.85E-01 (1.63E-01)	-1.67E+00 (4.17E-01)	-1.88E-01 (8.63E-02)	-4.35E+00 (9.21E-01)	-1.53E+00 (3.39E-01)
G_1	-2.46E-01 (8.92E-02)	-4.66E-01 (1.57E-01)	-2.54E-05 (1.16E-02)	-1.02E+00 (8.40E-03)	1.63E-01 (2.46E-01)	-2.58E-01 (7.77E-02)	-2.83E-01 (1.39E-01)	-6.32E-01 (2.53E-01)	-8.27E-01 (8.86E-03)
G_2	-8.78E-01 (7.41E-02)	-7.35E-02 (4.06E-02)	7.88E-02 (7.58E-03)	9.30E-01 (1.59E-02)	-3.58E-01 (1.51E-01)	4.18E-02 (7.47E-02)	9.44E-01 (9.45E-02)	5.26E-01 (1.24E-01)	7.47E-01 (1.40E-02)
G_3	8.40E-01 (6.26E-02)	7.97E-01 (4.09E-02)	2.55E-01 (6.11E-03)	9.56E-01 (8.34E-03)	6.96E-01 (1.56E-01)	6.89E-02 (7.66E-02)	7.06E-01 (9.44E-02)	8.39E-01 (2.24E-01)	9.49E-01 (7.93E-03)
G_4	3.18E-01 (7.33E-02)	7.11E-01 (1.54E-01)	9.08E-01 (7.26E-03)		4.41E-01 (2.42E-01)	4.98E-02 (7.34E-02)	-3.85E-01 (1.36E-01)	-1.19E-01 (1.68E-01)	
G_5	8.16E-01 (8.57E-02)		-2.68E-01 (1.22E-02)			8.04E-01 (7.14E-02)		-3.77E-02 (1.74E-01)	
A_1	1.99E-01 (3.58E-02)	4.00E-01 (5.96E-02)	4.59E-01 (5.54E-02)	2.90E-02 (4.85E-02)	1.37E-01 (5.24E-02)	9.94E-02 (4.30E-02)	-1.22E-01 (5.50E-02)	3.02E-01 (5.20E-02)	2.65E-01 (2.86E-02)
A_2	2.93E-01 (4.57E-02)	4.93E-01 (6.63E-02)	3.13E-01 (2.91E-02)	2.43E-02 (7.27E-02)	1.61E-02 (7.90E-02)	1.20E-01 (4.62E-02)	-8.50E-02 (3.89E-02)	4.63E-01 (9.45E-02)	3.89E-01 (5.02E-02)
A_3	2.89E-01 (6.54E-02)	2.25E-01 (4.31E-02)	1.58E-01 (2.54E-02)	2.04E-01 (8.08E-02)	2.61E-01 (4.09E-02)	2.14E-01 (4.92E-02)	1.67E-01 (5.18E-02)	2.44E-01 (6.49E-02)	1.52E-01 (3.15E-02)
A_4	2.19E-01 (4.99E-02)	-4.96E-02 (6.01E-02)	-1.10E-02 (2.85E-02)	3.71E-01 (7.83E-02)	-9.90E-03 (7.23E-02)	2.55E-01 (5.13E-02)	2.77E-01 (5.91E-02)		
A_5	4.39E-02 (5.36E-02)	-2.64E-01 (5.74E-02)	-3.34E-01 (5.75E-02)	1.63E-01 (5.89E-02)	1.58E-01 (5.69E-02)	1.28E-01 (4.77E-02)			
L_1	4.50E-02 (2.42E-02)	3.94E-02 (4.15E-02)	3.59E-02 (3.87E-02)	-1.47E-01 (3.37E-02)	-3.64E-02 (3.43E-02)	1.13E-01 (2.71E-02)	-1.89E-03 (2.98E-02)	-1.12E-01 (2.85E-02)	-4.55E-02 (1.55E-02)

(the table continues overleaf)

L_2	7.32E-02 (2.58E-02)	-1.00E-02 (3.68E-02)	2.31E-02 (1.65E-02)	-1.80E-01 (4.68E-02)	-2.29E-02 (3.52E-02)	6.52E-02 (3.04E-02)	3.33E-03 (2.27E-02)	-1.63E-01 (4.73E-02)	-6.29E-02 (2.69E-02)
L_3	1.55E-01 (3.50E-02)	3.60E-02 (2.20E-02)	2.98E-02 (1.45E-02)	6.30E-02 (5.51E-02)	-5.90E-02 (2.68E-02)	3.96E-02 (3.22E-02)	-3.65E-02 (2.25E-02)	-9.43E-02 (2.81E-02)	-3.82E-02 (1.52E-02)
L_4	7.73E-02 (2.60E-02)	-1.97E-02 (3.75E-02)	8.69E-03 (1.62E-02)	7.51E-02 (5.13E-02)	1.28E-02 (3.47E-02)	8.71E-02 (3.05E-02)	7.08E-03 (3.03E-02)		
L_5	8.49E-02 (2.26E-02)	1.41E-02 (4.05E-02)	-1.29E-02 (3.81E-02)	-1.96E-02 (3.70E-02)	-9.49E-03 (3.76E-02)	9.85E-02 (2.66E-02)			
<i>t</i> -Student degrees of freedom	9.81E+00 (1.60E+00)	3.83E+00 (3.42E-01)	4.41E+00 (4.13E-01)	9.15E+00 (1.78E+00)	1.02E+01 (2.31E+00)	8.15E+00 (1.59E+00)	1.02E+01 (2.11E+00)	4.60E+00 (4.16E-01)	9.94E+00 (2.14E+00)
AIC	-1.67E+04	-5.37E+03	-6.15E+03	-1.34E+04	-1.25E+04	-6.41E+03	-1.66E+04	-1.67E+04	-1.98E+04

Notes: The table reports ML estimates for the univariate ARMA-GARCH models of the marginal distributions. All models have *t*-Student error terms. Standard errors are in brackets.

Table B. Results for the marginal distributions (continued).

	EMBI	DE 2Y	DE 10Y	US 2Y	US 10Y	MSCI	DAX	SP500	EU BANKS
Conditional Mean	ARMA(4,5)	ARMA(1,0)	ARMA(1,2)	ARMA(0,3)	ARMA(0,2)	ARMA(0,1)	ARMA(2,2)	ARMA(0,2)	ARMA(2,1)
K	-2.88E-02 (3.71E-01)	-6.34E-04 (7.06E-04)	-8.92E-04 (4.44E-03)		-2.47E-03 (1.11E-03)	5.11E-04 (2.29E-04)	1.58E-03 (5.62E-04)	5.87E-04 (1.61E-04)	
ϕ_1	1.34E+00 (4.25E+00)	3.09E-02 (1.98E-02)	4.95E-01 (2.50E+00)				-4.81E-01 (3.10E-03)		9.60E-01 (4.01E-02)
ϕ_2	-1.24E+00 (2.31E+00)						-9.96E-01 (3.10E-03)		4.75E-03 (2.03E-02)
ϕ_3	1.42E+00 (3.33E+00)								
ϕ_4	-6.72E-01 (3.44E+00)								
ϕ_5									
θ_1	-1.27E+00 (4.25E+00)		-4.90E-01 (2.50E+00)	-5.95E-02 (2.05E-02)	-6.00E-02 (2.02E-02)	1.58E-01 (1.93E-02)	4.85E-01 (4.37E-03)	-7.62E-02 (2.01E-02)	-9.51E-01 (3.62E-02)
θ_2	1.15E+00 (2.01E+00)		-7.59E-03 (1.97E-02)	-2.19E-02 (1.93E-02)	-2.52E-02 (2.08E-02)		9.93E-01 (4.30E-03)	-3.56E-02 (2.25E-02)	
θ_3	-1.34E+00 (3.15E+00)			4.25E-02 (2.05E-02)					
θ_4	5.71E-01 (3.22E+00)								
θ_5	4.81E-02 (2.65E-01)								

(the table continues overleaf)

Conditional Variance	EGARCH(4,4)	EGARCH(4,3)	EGARCH(5,3)	EGARCH(5,5)	EGARCH(5,1)	EGARCH(5,5)	EGARCH(4,4)	EGARCH(3,5)	EGARCH(3,5)
κ	-5.72E-05 (1.25E-03)	-9.98E-02 (7.05E-02)	-5.52E-02 (2.28E-02)	-2.10E-01 (1.01E-01)	-3.78E-02 (1.78E-02)	-1.10E+00 (4.04E-01)	-7.04E-01 (2.15E-01)	-8.40E-01 (1.79E-01)	-4.32E-01 (9.87E-02)
G_1	1.18E+00 (8.57E-01)	2.50E-01 (5.64E-01)	1.47E+00 (1.81E-01)	-3.95E-01 (5.22E-03)	1.67E+00 (9.34E-02)	-8.30E-02 (4.93E-01)	2.76E-02 (2.55E-01)	-8.73E-01 (5.56E-03)	-6.25E-01 (3.81E-03)
G_2	8.17E-02 (1.59E+00)	-3.82E-01 (2.86E-01)	-3.81E-01 (3.64E-01)	-5.19E-01 (5.42E-03)	-1.20E+00 (1.06E-01)	-1.63E-02 (2.59E-01)	-9.70E-04 (1.16E-02)	8.13E-01 (9.41E-03)	5.94E-01 (5.58E-03)
G_3	7.74E-02 (9.49E-01)	8.62E-01 (2.89E-01)	-9.14E-02 (4.20E-01)	5.14E-01 (5.45E-03)	1.39E+00 (3.40E-02)	9.42E-01 (4.21E-02)	9.57E-01 (5.85E-03)	9.70E-01 (5.76E-03)	9.80E-01 (4.03E-03)
G_4	-3.42E-01 (3.54E-01)	2.56E-01 (5.61E-01)	-6.74E-01 (4.41E-01)	3.88E-01 (5.41E-03)	-1.61E+00 (1.10E-01)	-1.18E-02 (4.61E-01)	-6.32E-02 (2.45E-01)		
G_5			6.70E-01 (2.05E-01)	9.82E-01 (4.77E-03)	7.39E-01 (9.12E-02)	4.95E-02 (2.75E-01)			
A_1	2.56E-01 (5.19E-02)	1.24E-01 (5.01E-02)	7.97E-02 (3.68E-02)	2.09E-01 (2.42E-02)	7.60E-02 (1.27E-02)	1.71E-02 (5.30E-02)	1.11E-03 (4.92E-02)	-2.48E-02 (5.62E-02)	6.66E-02 (5.13E-02)
A_2	-1.19E-01 (2.40E-01)	1.00E-01 (3.45E-02)	-1.16E-01 (6.21E-02)	2.69E-01 (3.27E-02)		8.63E-02 (5.57E-02)	1.93E-01 (5.52E-02)	2.00E-01 (7.44E-02)	2.09E-01 (6.33E-02)
A_3	-5.69E-02 (2.84E-01)	1.21E-01 (7.29E-02)	1.31E-01 (3.74E-02)	2.97E-01 (4.26E-02)		1.51E-01 (8.34E-02)	1.17E-01 (5.49E-02)	3.88E-01 (7.51E-02)	1.98E-01 (5.03E-02)
A_4	-6.95E-02 (1.13E-01)			2.12E-01 (3.27E-02)		2.14E-01 (7.31E-02)	2.11E-01 (6.33E-02)	1.72E-01 (7.52E-02)	9.60E-02 (6.19E-02)
A_5				1.11E-01 (2.39E-02)		1.56E-01 (9.55E-02)		-6.43E-02 (5.79E-02)	-1.48E-02 (5.12E-02)
L_1	6.86E-02 (3.61E-02)	-9.35E-03 (1.31E-02)	-2.22E-02 (2.13E-02)	1.24E-02 (1.41E-02)	-8.34E-03 (6.00E-03)	-1.59E-01 (3.34E-02)	-1.61E-01 (3.16E-02)	-1.96E-01 (3.80E-02)	-1.15E-01 (3.33E-02)

(the table continues overleaf)

L ₂	7.18E-02 (6.94E-02)	1.26E-03 (7.77E-03)	2.97E-02 (3.41E-02)	1.22E-02 (1.87E-02)		-1.31E-01 (8.89E-02)	-1.14E-01 (2.90E-02)	-3.38E-01 (4.94E-02)	-2.19E-01 (3.90E-02)
L ₃	-4.69E-02 (6.47E-02)	1.13E-04 (1.36E-02)	-1.23E-02 (2.11E-02)	-3.23E-02 (2.42E-02)		-1.15E-01 (4.29E-02)	-7.70E-02 (3.37E-02)	-9.43E-02 (4.90E-02)	-1.15E-01 (3.17E-02)
L ₄	-8.63E-02 (9.12E-02)			-6.12E-03 (1.84E-02)		7.45E-02 (5.27E-02)	6.73E-02 (4.06E-02)	1.47E-01 (5.08E-02)	2.50E-02 (4.12E-02)
L ₅				-3.27E-02 (1.37E-02)		2.36E-02 (7.88E-02)		6.74E-02 (3.84E-02)	3.72E-02 (3.36E-02)
<i>t</i> -Student's degrees of freedom	7.34E+00 (9.99E-01)	6.22E+00 (8.69E-01)	1.66E+01 (5.50E+00)	1.01E+01 (1.67E+00)	1.59E+01 (3.92E+00)	1.07E+01 (2.41E+00)	1.28E+01 (3.64E+00)	9.18E+00 (1.57E+00)	1.22E+01 (2.68E+00)
AIC	1.52E+04	-8.01E+03	-7.77E+03	-7.32E+03	-6.08E+03	-1.39E+04	-1.31E+04	-1.42E+04	-1.23E+04

Notes: The table reports ML estimates for the univariate ARMA-GARCH models of the marginal distributions. All models have *t*-Student error terms, except for PL BANKS following GJR-GARCH process with *t*-Student error term. Standard errors are in brackets.

Table C. Goodness-of-fit test results and the choice of the copula.

		Normal	Clayton	Rotated Clayton	Plackett	Frank	Gumbel	Rotated Gumbel	t-Student	SJC	Independent
EUR/PLN	VIXo								<u>0.229</u> <u>19.557</u> <u>(0.074)</u>		
EUR/PLN	EUR/USD								<u>0.003</u> <u>4.496</u> <u>(0.216)</u>		
EUR/PLN	EM CARRY				2.072 (0.098)	1.433 (0.063)	1.144 (0.054)		<u>0.233</u> <u>12.510</u> <u>(0.153)</u>		
EUR/PLN	EMFX				3.033 (0.211)	<u>2.249</u> <u>(0.083)</u>			0.354 8.474 (0.209)		
PL 2Y	VIXo			0.074 (0.256)			1.038 (0.196)			<u>0.007</u> <u>0.074</u> <u>(0.211)</u>	
PL 2Y	DE2Y	0.050 (0.289)		0.040 (0.100)	1.181 (0.452)	<u>0.336</u> <u>(0.498)</u>	1.020 (0.123)		0.050 2.7 mn (0.297)	0.000 0.000 (0.231)	
PL 2Y	US2Y	0.026 (0.762)	0.023 (0.693)	0.025 (0.852)	1.068 (0.663)	0.131 (0.663)	1.011 (0.867)	1.009 (0.714)	0.026 224.133 (0.734)	<u>0.000</u> <u>0.000</u> <u>(0.555)</u>	no param. (0.458)
PL 2Y	EMBI	0.101 (0.862)		0.099 (0.303)	<u>1.341</u> <u>(0.792)</u>	0.572 (0.783)	1.054 (0.763)	1.057 (0.097)	0.100 20.918 (0.949)	0.003 0.009 (0.97)	
PL 10Y	VIX	0.110 (0.19)		0.112 (0.185)	<u>1.400</u> <u>(0.083)</u>	0.655 (0.084)	1.071 (0.393)		0.112 17.973 (0.227)	0.006 0.010 (0.375)	
PL 10Y	DE10Y	0.124 (0.081)		0.147 (0.157)	<u>1.474</u> <u>(0.053)</u>		1.081 (0.399)		0.127 13.630 (0.101)	0.000 0.042 (0.381)	
PL 10Y	US10Y	0.066 (0.96)	0.063 (0.154)	0.065 (0.394)	<u>1.225</u> <u>(0.989)</u>	0.400 (0.983)	1.036 (0.743)	1.037 (0.417)	0.067 23.888 (0.982)	0.001 0.001 (0.982)	
PL 10Y	EMBI			0.162 (0.335)			1.083 (0.438)		0.132 16.414 (0.054)	<u>0.001</u> <u>0.049</u> <u>(0.2)</u>	
WIG	VIXo	0.405 (0.192)							<u>0.406</u> <u>34.563</u> <u>(0.188)</u>		
WIG	DAX										
WIG	SP500o		0.324 (0.055)					<u>1.187</u> <u>(0.589)</u>		0.137 0.078 (0.794)	
WIG	MSCI										
WIG BANKS	VIXo	0.337 (0.756)			2.776 (0.085)				<u>0.339</u> <u>24.156</u> <u>(0.905)</u>		
WIG BANKS	DAX										
WIG BANKS	SP500o		0.259 (0.055)					1.145 (0.589)		<u>0.099</u> <u>0.038</u> <u>(0.794)</u>	
WIG BANKS	EU BANKS										

Notes: Each cell contains parameter estimates (in case of t -Student – the first number is correlation parameter and the second is the degree of freedom, in the case of Symmetrised Joe-Clayton – τ^L and τ^U , respectively) and goodness-of-fit test's p -value (in parentheses). A cell in bold and underlined denotes the copula that additionally is \hat{S}_T -closest to the “empirical copula” for a given pair. An empty cell denotes a case of p -value lower than 0.05 and rejection of the copula class for a given pair of variables.