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Nonparametric Verification of GARCH-Class Models for Selected Polish Exchange Rates and Stock Indices*

Piotr FISZEDER—*corresponding author* (piotr.fiszeder@umk.pl)
Witold ORZESZKO
both authors: Faculty of Economic Sciences and Management,
Nicolaus Copernicus University, Torun, Poland

Abstract
The iid property of the model's residuals is a crucial criterion for assessing the fit of the model to the data. GARCH-class models are the most commonly used nonlinear models in financial econometrics. In this paper various uni- and multivariate GARCH-class models were applied to selected Polish financial series. In the research the iid property of the residuals and their absolute values was verified. To this end, the BDS test, the mutual information measure, and, for comparison, the Ljung-Box and Engle tests were used. To calculate p-values the bootstrap procedure was applied in each test. The results indicate that ARMA-GARCH models are generally able to capture the dependencies in the time series analyzed. However, this does not mean that every specified ARMA-GARCH model describes the existing dependencies well enough. The study shows that different parameterizations of the GARCH-class models analyzed have different abilities to describe the dynamics of financial processes. Furthermore, the research indicates that the application of higher lags in a GARCH model may have a crucial impact on the removal of the ARCH effect and, in consequence, on the nonlinearity identification.

1. Introduction

Studies of nonlinearity have become a growing area in modern econometrics. In fact, there are no theoretical or empirical reasons to state that economic systems must be linear. Consequently, nonlinearity detection methods have been developed and applied to analyze the dynamics of economic data.

Tests for nonlinearity can be divided into two classes depending on the type of alternative hypothesis. The first class consists of tests against a specific nonlinear model. In contrast, tests from the second group are carried out with an unspecified alternative. In this case, rejection of the null gives no explicit information about the dependencies identified, although pre-filtering of the data using models of specific types can provide information about their dynamics. For example, to identify nonlinear relations in stationary time series, the investigated data can be pre-filtered using an ARMA model. Next, if nonlinear dependencies are detected, a nonlinear model can be constructed and verified if it captures the dynamics found in the pre-filtered series.

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Financial time series have many distinctive features which should be included in the modeling. These important properties include volatility clustering, fat-tailed and leptokurtic distributions, the leverage effect, long memory in volatility, and a positive relation between return and risk. Due to its simple and clear construction and ease of expansion to include specific features of empirical returns, the class of univariate GARCH models is one of the most frequently applied nonlinear models of financial time series. However, empirical studies show that simple parameterizations of GARCH models are not always sufficient to describe the dynamics of financial returns (see, for instance, Andersen et al., 2006, and Bauwens et al., 2006), so there is a need to investigate methods for detecting model misspecification.

The purpose of this paper is to identify nonlinear dependencies in selected time series from the Polish financial market. Specifically, the ability of GARCH-class models to describe the dynamics of financial processes was investigated. To this end, the BDS test, the mutual information measure, and, for comparison, the Ljung-Box and Engle tests were used.

Different GARCH models differ in structure in order to capture the miscellaneous empirical features of asset returns. Consequently, the application of different kinds of GARCH models can be a key issue in detecting various types of nonlinearity. Therefore, our analysis was performed for eleven univariate specifications of GARCH models, namely, GARCH with normal, GED, Student-t, and skewed Student-t conditional innovation distributions, and GARCH-M, GJR, EGARCH, FIGARCH, IGARCH, PGARCH, and CGARCH. In financial analysis (e.g. portfolio construction) the application of univariate models rarely turns out to be sufficient, so a multivariate GARCH model—the BEKK model—was also used in the research for comparison.

The application of the BDS test and the mutual information measure to assess misspecification of GARCH models has been the subject of several studies on financial time series. The BDS test was applied, for example, to stock indices or their futures contracts by Blank (1991), Hsieh (1993), Patterson and Ashley (2000), Nieto et al. (2004), Pandey (2007), and Dong and Song (2009), to currencies or their futures contracts by Hsieh (1993), Kocenda (1995), Zivot and Wang (2006), Jirasakuldech et al. (2009), and Ullrich (2009), and to commodities or their futures contracts by Blank (1991), Chavas and Holt (1991), and Chatrath et al. (2001). The mutual information measure was applied, for instance, to stock indices by Dionisio et al. (2006), Hassani, Dionisio et al. (2010), and Hassani, Soofi et al. (2010). Studies with the BDS test or mutual information for Polish financial time series were performed, for example, by Poshokwale and Murinde (2001), Vošvrda and Žikeš (2004), and Orzeszko (2010). In most of these papers only the standard GARCH model was used, but sometimes other univariate parameterizations of the GARCH model were employed too (most often the asymmetric one). As mentioned before, different kinds of models capture different empirical features of asset returns, so unlike in other studies, several univariate GARCH models and a multivariate model were applied in our research.

To the best of our knowledge, this is the first comparison of such a wide range of univariate and multivariate GARCH models conducted using the BDS test and the mutual information measure. Because the application of higher orders in the GARCH model may have a crucial impact on the removal of the ARCH effect and, in consequence, on the nonlinearity identification, we consider models with different lags. It should be emphasized that in only two of the papers mentioned above, namely, Blank (1991) and Dong and Song (2009), were GARCH(p,q) models with lags p and q higher than one used. This could be one of the reasons why in many papers the constructed GARCH models were not able to fully capture the nonlinear dynamics of the investigated series.

There are several reasons to investigate Polish financial series. Poland is one of the largest emerging markets in Europe. It joined the European Union in 2004 and was the only member country of the EU to avoid a decline in GDP during the late-2000s recession. We should also note the relatively small number of studies on Polish financial time series in comparison with other emerging and developed markets. Furthermore, Fiszeder and Romański (2002) showed that the statistical properties of the main indices from emerging markets such as the Czech Republic, Hungary, and Poland are similar, possibly due to international portfolio flows. This implies that the results obtained for Polish financial series may to some extent be representative of the other emerging markets mentioned above.

The plan for the rest of the paper is as follows. Section 2 describes the tests applied and outlines the GARCH models and verification techniques used in the analysis. In Section 3 detailed results of the research are presented, and a summary is given in Section 4.

2. Data, Models, and Verification Techniques

The research was conducted on the exchange rates which play the key role for the Polish economy, i.e., EUR/PLN and USD/PLN, and on the stock indices quoted on the Warsaw Stock Exchange (WSE): WIG20 and SWIG80.[1] The selected indices are important indicators of behavior for, respectively, the biggest and smallest stocks quoted on the WSE. An additional factor which affected the choice of assets was the sizable differentiation of the statistical properties of the selected time series (for distinction based on the size of companies, see, for example, Lo and MacKinlay, 1988, and Fiszeder and Romański, 2002). In order to achieve stationarity, the daily log returns of the data were analyzed in the research. The period from January 2, 2001 to July 30, 2010 was considered, therefore the samples consisted of 2,403 daily log returns for the indices and 2,422 for the exchange rates.[2]

First, GARCH-class models were constructed for the series investigated. Restrictions ensuring covariance stationarity and non-negativity of the conditional variance (see Nelson and Cao, 1992) were imposed. Besides the simple GARCH model, which describes volatility clustering, the following univariate parameterizations—those most frequently used in other studies—were analyzed: GARCH-M (Engle et al., 1987), GJR (Glosten et al., 1993), EGARCH (Nelson, 1991), and FIGARCH (Baillie et al., 1996). In contrast to the simple GARCH model, these models can describe important properties of financial time series. The GARCH-M model captures the relation between the expected return and the conditional variance, the GJR

[1] WIG20 and SWIG80 are price indices which exclude dividends. The importance of dividends on the WSE is growing, but only in recent years have they begun to play a greater role in investment returns. For example, only 19% of companies paid dividends in 2001, compared to about 30% in 2011. The WSE began to calculate the WIGdiv index, which includes dividends, in 2011.

[2] The difference in the number of observations results from the fact that on certain days the exchange rates were quoted while the WSE was closed.

and EGARCH models describe the often reported negative correlation between lagged returns and the conditional variance (known as the leverage effect), and the FIGARCH model captures long-term dependencies between squared observations of a series.

Some other less frequently used parameterizations of GARCH models were also considered: IGARCH (Engle and Bollerslev, 1986), PGARCH (power GARCH; Ding et al., 1993), and CGARCH (component GARCH; Engle and Lee, 1999). These models are interesting because they describe quite different properties of the variability dynamics than do the specifications mentioned above. Current volatility has a permanent influence for forecasts of the conditional variance for the IGARCH process, but the autocorrelation function of squared observations decays exponentially to zero. The PGARCH model is able to describe autocorrelation in other functions of the observations. The volatility of the CGARCH model can be decomposed on the components, which can describe the short-term and long-term dynamics.

Instead of the conditional normal innovation distribution, the Student-t, GED, and skewed Student-t distributions were also applied. These specifications are able to better describe the fat tails of the unconditional distributions of financial time series. A model with the conditional skewed Student-t distribution can additionally describe the asymmetry of the unconditional distribution.

Furthermore, the bivariate BEKK models (Baba et al., 1990) were used for all the pairs of series analyzed. Due to very similar conclusions of the research, we present the results only for the relationships between the investigated currencies and stock market indices, namely, for the following pairs: EUR/PLN-WIG20, USD//PLN-WIG20, and EUR/PLN-SWIG80.[3] The BEKK model is one of the most frequently used GARCH-class models in multivariate analysis for a small number of series and gives relatively good performance (see, for example, Osiewalski and Pipień, 2004). Additionally, it is a relatively general specification which includes many different multivariate GARCH models. Other frequently applied models, such as the Constant Conditional Correlations model of Bollerslev (1990) or the Dynamic Conditional Correlation model of Engle (2002), assume some simplifications which are not valid for many financial series.

The selection of orders in the ARMA and GARCH models was based on minimization of the Schwarz information criterion (SIC) with regard to the results of the Ljung-Box test (LB hereafter) for the presence of autocorrelation and the Engle test (LM hereafter) for the presence of the ARCH effect. The results of these tests for standardized residuals can be used as a preliminary check of the adequacy of the constructed models. Due to the relatively high complexity and parameter estimation problems of BEKK(p,q) models, only lags $p = q = 1$ and $p = 1$, $q = 2$ were considered.

Application of the adopted criterion allowed us to select the best model for each type of GARCH-class model. However, for cognitive purposes, the ARMA-GARCH(p,q) and VAR-BEKK(p,q) models with the lags most often used in practice, i.e., $p = q = 1$, were also considered in our research. Moreover, two additional models consisting of only the ARMA part and only the GARCH part of the best

[3] The exchange rates from the periods when the WSE was closed were excluded to allow multivariate analysis.

ARMA-GARCH model were analyzed. This enables us to study separately the presence of autocorrelation and the ARCH effect in the analyzed data.

The parameters of the ARMA-GARCH models were estimated using the quasi-maximum likelihood method (except for the GARCH models with the conditional Student-t, GED, and skewed Student-t distributions, for which the ML method was used). The parameters of the VAR-BEKK model with the conditional Student-t distribution were estimated using the ML method.

To assess the model fit, the BDS test, the mutual information measure (MI hereafter) and, for comparison, the Ljung-Box and Engle tests were used. The BDS test and the test based on the MI verify if the model residuals are independently and identically distributed (iid). In both tests the alternative hypothesis is unspecified, which means that rejection of the null gives no explicit information about the model which should be used to describe the dynamics of the data. The important property of the applied methods is their ability to detect dependencies of different types: linear and nonlinear ones. Therefore, to employ them as tests for nonlinearity one must remove any linear dependence from the data.

Both the BDS test and the test based on the MI were carried out for standardized residuals and for their absolute values (modules). Transformation of the investigated data into their absolute values before testing for independence can increase the power of the test against specific types of alternatives (see Stărică and Granger, 2005; Peña and Rodríguez, 2006; Diks and Panchenko, 2007).

The BDS test is a nonparametric, two-tailed test for iid-ness. For each of the residual series analyzed the test was applied with embedding dimensions $m = 2,3,4,5$ (cf. Brock et al., 1991). The value ε was set at the level of 1.5σ, where σ denotes the standard deviation of the series analyzed (see Kanzler, 1999). To verify the null hypothesis of the iid property of the data, the p-value was calculated for each series and for each value of m.[4] The p-values were evaluated through bootstrapping with 1,000 repetitions.

Next, the values of the mutual information measure were evaluated for each of the residual series.[5] The mutual information measure is one of the most important methods for detecting nonlinear dependencies in time series (cf. e.g. Granger and Lin, 1994; Maasoumi and Racine, 2002). It is defined by the following expression:

$$I(X,Y) = \iint p(x,y)\log\left(\frac{p(x,y)}{p_1(x)p_2(y)}\right)dxdy \tag{1}$$

where $p(x,y)$ is a joint probability density function and $p_1(x)$ and $p_2(y)$ are marginal densities for random variables X and Y. To measure serial dependencies in a single time series, the lagged realizations of the investigated data X should be taken as the variable Y (in our research, lag $k = 1$ was considered). To estimate the value of the MI measure, the method proposed by Fraser and Swinney (1986) was used. It can be shown that for all X and Y the measure $I(X,Y)$ takes non-negative values and $I(X,Y) = 0$ only if X and Y are independent. Thus, based on the values obtained, an independence test with the following hypothesis:

[4] To apply the BDS test the Matlab script created by Ludwig Kanzler was used.

[5] In the calculations the Matlab script created by Alexandros Leontitsis was used.

$$\mathrm{H}_0\text{: } I(X,Y)=0 \quad \text{and} \quad \mathrm{H}_1\text{: } I(X,Y)>0 \tag{2}$$

was applied. To verify the null hypothesis the p-values were evaluated through bootstrapping with 1,000 repetitions.

Moreover, for all of the residuals analyzed the Ljung-Box test for the presence of autocorrelation (a detailed description can be found in Ljung and Box, 1978) and the Engle test for the presence of the ARCH effect (for details see Engle, 1982) were also applied. In these tests the bootstrap p-values (based on 10,000 replications) were used instead of the p-values from the asymptotic chi-squared statistic because of non-normality of the return distribution (mainly due to the fat tails of the distribution). When the ARCH effect was present in the series, instead of the Ljung-Box test, its modified version (LBM hereafter), which is adequate in the case of conditional heteroskedasticity, was applied (for details see West and Cho, 1995).

3. Results of Empirical Analysis for Selected Polish Exchange Rates and Stock Indices

The calculated bootstrap p-values are presented in *Tables A1–A8* in the *Appendix*. In each case p-values lower than 0.1 (indicating misspecification of the model) are in bold.

For EUR/PLN (as in the case of all the exchange rates and stock indices analyzed in this paper) the iid hypothesis was strongly rejected for the returns. Moreover, it can be seen that in the BDS test the p-values for the residuals are higher than those for the modules (the only exception being AR(1)-GARCH(1,2)_norm for $m = 2$ and $m = 5$; see *Table A1*). This resulted in more frequent rejection of the null hypothesis for the modules. Furthermore, it can be seen that the null was more frequently rejected in the BDS test than in the MI test.

According to both the BDS and MI tests, the strongest indication of model misspecification was obtained for AR(1) (see *Tables A1* and *A2*). Moreover, only the BDS test rejected the following models: GARCH(1,1)_Stud, VAR(3)-BEKK(1,1)_Stud, and VAR(3)-BEKK(1,2)_Stud for both the standardized residuals and their modules.[6] Furthermore, the BDS test rejected AR(1)-GARCH(1,1)_norm, AR(1)-GARCH(1,1)_Stud, and AR(1)-EGARCH(1,2)_Stud (strong rejection), and GARCH(1,2)_Stud and AR(1)-GARCH(1,2)_skStud (weak rejection) only for the modules. To explain the reasons for rejecting the iid hypothesis the LB and LM tests were applied. In the EUR/PLN returns a very weak serial dependence in the mean is present according to the LB test. It is worth noting that EUR/PLN is the only investigated series for which none of the analyzed GARCH models was able to describe fully the conditional heteroskedasticity present in the data (according to the LM test). This means that the reason for the rejection of the models by the BDS and MI tests may be that the ARCH effect remained in the residuals.

According to the SIC criterion and the LB and LM tests, the AR(1)-IGARCH(1,2)_Stud model best describes the dynamics of the EUR/PLN returns (the selection was based on the models without autocorrelation in the residuals). Neither the BDS test nor the MI test rejected this model.

[6] The abbreviations norm, Stud, GED, and skStud stand for, respectively, normal, Student-t, GED, and skewed Student-t conditional innovation distributions.

In the case of USD/PLN, the p-values for the residuals are higher than those for the modules not only in the BDS test, but also in the MI test (for most models, see *Tables A3* and *A4*). This resulted in more frequent rejection of the null hypothesis for the modules. Moreover, it can be seen that the null was more frequently rejected by the BDS test than by the MI test.

For the residuals, VAR(1)-BEKK(1,1)_Stud was the only model rejected. However, this rejection occurred only in the BDS test and was not very strong. According to the results of the LM test, this rejection could have been caused by the ARCH effect remaining in the residuals of the VAR(1)-BEKK(1,1)_Stud model.

From the results obtained for the modules, it can be seen that both the BDS and MI tests rejected the models GARCH(1,1)_norm, GARCH(1,1)_Stud, EGARCH(2,1)_Stud, and VAR(1)-BEKK(1,2)_Stud. Moreover, only the BDS test rejected VAR(1)-BEKK(1,1)_Stud (strong rejection) and GARCH(2,1)_skStud and CGARCH(1,1)_Stud (weak rejection). According to the results of the LB test, autocorrelation was not the reason for the rejection of these models. The results of the LM test show that, except for the CGARCH(1,1)_Stud model, these rejections could have been caused by the ARCH effect remaining in the residuals.

According to the SIC criterion and the LB and LM tests, the IGARCH(2,1)_Stud model was selected as the best univariate model for USD/PLN (the selection was based on the models without autocorrelation and the ARCH effect in the residuals). Neither the BDS test nor the MI test rejected this model.

It can be seen that in the case of the WIG20 index, the p-values from the BDS test are lower for the residuals than for the modules (which is the opposite effect to the case of the exchange rates; see *Table A5*). This resulted in more frequent rejection of the null hypothesis for the residuals. Moreover, it can be seen that in the case of the modules the null was more frequently rejected in the MI test than in the BDS test (again, this result is the opposite to the case of the exchange rates).

The BDS test applied to the standardized residuals strongly rejected the following models: AR(1), GARCH(1,1)_norm, GARCH(1,1)_Stud, AR(1)-GARCH(1,1)_norm, AR(1)-GARCH(1,1)_Stud, VAR(3)-BEKK(1,1)_Stud, and VAR(3)-BEKK(1,2)_Stud. Moreover, in a weaker way it rejected AR(1)-FIGARCH(1,3)_Stud and AR(1)-CGARCH(1,1)_Stud. All these models were also rejected by the MI test (see *Table A6*). Furthermore, the MI test rejected GARCH(1,3)_Stud, AR(1)-GJR(1,3)_Stud (for both the standardized residuals and their modules), GARCH(1,3)_norm, AR(1)-GARCH(1,3)_GED, AR(1)-PGARCH(1,3)_Stud (for the standardized residuals), and AR(1)-IGARCH(1,3)_Stud (for the modules).

Only in the cases of GARCH(1,3)_norm and GARCH(1,3)_Stud can autocorrelation (detected by the LB test) be the reason for the rejection of the models. Furthermore, the rejection of the other models cannot be explained in every case by the ARCH effect either. According to the LB and LM tests, in the case of AR(1)-GARCH(1,3)_GED, AR(1)-GJR(1,3)_Stud, AR(1)-IGARCH(1,3)_Stud, AR(1)-FIGARCH(1,3)_Stud, and AR(1)-PGARCH(1,3)_Stud the low p-values (in the MI and BDS tests) are not caused by autocorrelation or by the ARCH effect.

For the WIG20 index, the AR(1)-EGARCH(1,3)_Stud model was selected as the best model according to the SIC criterion and the LB and LM tests. Neither the BDS test nor the MI test rejected this model.

As shown by the results of the LB test and the selected lags in the ARMA model, relatively high autocorrelation exists in the SWIG80 returns. This may indicate lower efficiency (in the sense of weak-form efficiency; see Fama, 1991) of the small stocks which make up the SWIG80 index, and is connected with the lower liquidity of such stocks. That is why both the BDS and MI tests univocally detected serial dependencies in the raw returns and the residuals of the model describing only the conditional variance, i.e., GARCH(1,1)_Stud (see *Tables A7* and *A8*). The *p*-values for the residuals of the linear model (i.e., ARMA(4,1)) indicate that the data are nonlinear. Most of the GARCH-class models analyzed were able to describe these nonlinear dependencies. The exceptions were ARMA(4,1)-IGARCH(1,1)_Stud, VAR(4)-BEKK(1,1)_Stud, VAR(4)-BEKK(1,2)_Stud, and ARMA(4,1)-IGARCH(1,2)_Stud (weak rejection for the last-mentioned) according to the BDS test and VAR(3)-BEKK(1,1)_Stud rejected by the MI test. Generally, the results of the LM test and the selected lags in the GARCH models indicate a weaker ARCH effect in comparison with the blue chip index.

For the SWIG80 index, the ARMA(4,1)-GARCH(1,1)_Stud model was the best univariate GARCH parameterization according to the SIC criterion and the LB and LM tests. Neither the BDS test nor the MI test rejected this model.

According to the results presented above, some general conclusions may be drawn. As expected in all cases, the iid hypothesis was rejected for the raw returns. Both models—describing only the conditional expected value for time series with autocorrelation and only the conditional variance—proved to be inadequate in describing the time series dynamics. It should be noted that the results of the BDS and MI tests showed greater similarity in this first case.

There are significant differences in the dynamic properties of the returns between the currencies and the stock indices. Much stronger autocorrelation and a weaker ARCH effect exist in the returns of the stock indices especially of small stocks (considerably higher values of the LB statistic and higher lags in the ARMA model indicate stronger serial correlation of the stock indices returns; similarly, higher values of the LM statistic and higher lags in the GARCH model indicate a stronger ARCH effect for the exchange rate returns). These features cause differences in the results. In the case of the currencies, the *p*-values for the BDS test are generally higher for the standardized residuals than for their modules. This implies that the iid hypothesis was more frequently rejected for the modules. The opposite situation occurs for the stock indices.

There are also big differences in the statistical properties of the returns between the large and small stock indices. Compared to the WIG20 returns, the SWIG80 returns are characterized by stronger autocorrelation, a weaker ARCH effect, and fatter tails of distribution.[7] This implies that it is much easier to describe the nonlinearity in the returns of the small stock index using the GARCH model with a more parsimonious parameterization.

Incomplete explanation of autocorrelation or the ARCH effect (detected by, respectively, the LB and LM tests) by the models considered was often the reason for rejection of the iid hypothesis by the BDS or MI tests. On the other hand, in some cases the BDS or MI tests detected nonlinear dependencies which are not captured by

[7] The detailed parameters estimation results are available from the authors upon request.

the applied ARMA-GARCH model (when the LB and LM tests do not reject the null hypothesis for residuals). This means that these models are not sufficient to describe the structure in the data. This was the case with CGARCH(1,1)_Stud for USD/PLN, AR(1)-GARCH(1,3)_GED, AR(1)-GJR(1,3)_Stud, AR(1)-IGARCH(1,3)_Stud, AR(1)-FIGARCH(1,3)_Stud, and AR(1)-PGARCH(1,3)_Stud for the WIG20 index (however, it should be noted that in the case of only one model, namely AR(1)-PGARCH(1,3)_Stud, was the *p*-value below 0.05).

When the ARCH effect was detected in the residuals by the Engle test, the BDS and MI tests applied to their modules rejected the iid hypothesis more often than those applied to the raw residuals (the BDS results for WIG20 being an exception). On the other hand, when a very strong ARCH effect was detected (a *p*-value below 0.01) the iid hypothesis was always rejected by the BDS or MI tests for the modules of the residuals (however, the application of only one test did not guarantee proper detection of the existing ARCH effect in all cases).

The analysis showed that the GARCH model with lags equal to one—the GARCH model most often considered in empirical studies—was frequently not able to explain the nonlinearity observed in the returns. Moreover, the results imply that different parameterizations of GARCH models have different abilities to describe the nonlinearity of financial time series. The IGARCH and EGARCH models often did not fully explain the nonlinearity in the returns. The same goes for the multivariate BEKK model. It is well known that the VAR-BEKK model allows for the description of time-varying conditional correlations between time series, which is not possible for univariate GARCH models. On the other hand, our research showed that the VAR-BEKK model was not always able to describe the conditional variances as well as some univariate GARCH specifications, especially when a more expanded parameterization with higher lags is necessary. However, this result could be due to the fact that the lags applied in the BEKK models were not sufficiently large. Problems with parameters estimation in the case of GARCH models such as IGARCH, EGARCH, and BEKK might be another reason for this misspecification. It is worth noting that these models rank among those for which ML estimation often causes many problems, especially for specifications with higher lags (in our research, the likelihood functions were flat and the number of parameters was large, so it was a problem to find the global maximum).

The parameters responsible for the existence of the leverage effect (in the GJR and EGARCH models) were significant for all of the series analyzed.[8] Moreover, the parameter that indicates a long memory in volatility (the FIGARCH model) was significant for EUR/PLN and the WIG20 index. Furthermore, the parameter responsible for a positive relation between the return and the conditional variance (the GARCH-M model) was significant only for USD/PLN and the WIG20 index. The results of the analysis indicate that the omission of these three features in time series modeling (i.e., the application of the standard GARCH model instead of its extensions) had no relevant influence on the results of the BDS and MI tests.

The estimated values of the degrees of freedom and the shape parameter for, respectively, the Student-t and GED distributions indicate fat tails of the conditional innovation distributions for all the series analyzed. Furthermore, the parameter that

[8] The detailed parameters estimation results are available from the authors upon request.

indicates asymmetry of the conditional innovation distribution (in the models with a skewed Student-t distribution) was not significant for the WIG20 index only. Despite these facts, for all the time series there were no essential differences in the results of the BDS and MI tests between the GARCH models with the Student-t and normal conditional innovation distributions. On the other hand, in the case of the GARCH models with the skewed Student-t and GED conditional distributions, small differences in the results of the BDS and MI tests were revealed for the currency returns and the WIG20 index.

Summing up the results of this research, in the time series analyzed, dependencies were detected which were well described (according to the tests applied) by the selected ARMA-GARCH models. The exception was the EUR/PLN exchange rate, for which the models failed to remove completely the ARCH effect present in the data (according to the LM test). However, it should be emphasized that for the other time series analyzed, not every specified ARMA-GARCH model was able to describe the existing dependencies well enough. In many cases this fact was indicated by the results of the BDS and MI tests and confirmed by the results of the LB and LM tests. This may mean that in such cases, autocorrelation or the ARCH effect remaining in the residuals was responsible for the rejection of the iid hypothesis by the BDS or MI tests. However, there were cases where the model was rejected by the BDS or MI tests despite the fact that, according to the LM and LB tests, there was no reason to reject it. It should be noted that such situations were quite rare (two for the 0.05 significance level and an additional seven for 0.1).

4. Conclusion

ARCH-class models are commonly used to describe the nonlinear dynamics of financial time series. In the literature one can find that pre-filtering financial data using ARMA-GARCH models is often applied since it can provide information about the dynamics of the data. In this paper we assessed the fit of GARCH-class models to selected Polish exchange rates and stock indices. To this end we applied the BDS test, the mutual information measure, and the Ljung-Box and Engle tests. We detected a strong evidence of nonlinear dynamics in all the series analyzed. We found that different parameterizations of the GARCH-class models analyzed have different abilities to describe these nonlinearities. The fact that not every specified ARMA-GARCH model was able to describe the existing dependencies well enough implies that in a nonlinearity identification process, one should attach great importance to filtering with ARMA-GARCH models. In particular, the residuals should be tested for the presence of autocorrelation and the ARCH effect (e.g. by applying the LB and LM tests) before the BDS or MI tests are applied.

The results of many financial applications indicate that some extensions of the GARCH model have an advantage over the standard GARCH model. Surprisingly, our research did not show any advantages of other GARCH parameterizations in the nonlinear modeling of time series. In fact, the results indicate that the standard GARCH model may be sufficient to describe the nonlinearity in the series investigated (only in the case of the EUR/PLN exchange rate was this type of model not able to remove the existing ARCH effect, although none of the remaining models was able to perform this). However, this conclusion is valid only on the condition that an appropriate lag structure is considered. Our research has shown that the application of higher lags in the GARCH model may have a crucial impact on the removal of the ARCH effect and, in consequence, on the nonlinearity identification. This could be one of the reasons why the iid property of residuals from the GARCH model has been rejected in many other studies described in the literature. Nevertheless, the problem of proper selection of the GARCH specification needs additional analyses of more series and also simulation studies.

According to the results of our analysis, we recommend investigation of not only the standardized residuals, but also their absolute values. Such a procedure may increase the power of the tests for nonlinearity, especially when the ARCH effect is present in the data. Similarly, the combined application of the BDS and MI tests (instead of using only one of these tests) increases the chances of proper identification of nonlinear dependencies.

Appendix

Results of Verification of the iid Property for the Residuals from the Fitted GARCH-Class Models

Table 1 Bootstrap *p*-Values for the BDS Test for EUR/PLN

Model	BDS							
	Standardized residuals				Modules of standardized residuals			
	***m* = 2**	***m* = 3**	***m* = 4**	***m* = 5**	***m* = 2**	***m* = 3**	***m* = 4**	***m* = 5**
Returns	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
AR(1)	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
GARCH(1,1)_Stud	**0.034**	**0.074**	0.190	0.404	**0.008**	**0.020**	**0.034**	**0.082**
GARCH(1,2)_Stud	0.830	0.998	0.956	0.914	0.176	**0.098**	0.128	0.226
AR(1)-GARCH(1,1)_norm	0.264	0.392	0.688	0.868	**0.030**	**0.046**	**0.098**	0.308
AR(1)-GARCH(1,1)_Stud	0.106	0.120	0.258	0.604	**0.012**	**0.012**	**0.032**	0.136
AR(1)-GARCH(1,2)_norm	0.162	0.336	0.430	0.390	0.522	0.272	0.302	0.444
AR(1)-GARCH(1,2)_Stud	0.788	0.950	0.980	0.818	0.180	0.132	0.178	0.354
AR(1)-GARCH(1,2)_GED	0.380	0.668	0.692	0.512	0.298	0.210	0.248	0.446
AR(1)-GARCH(1,2)_skStud	0.776	0.892	0.954	0.866	0.126	**0.068**	0.122	0.248
AR(1)-GARCH-M(1,2)_Stud	0.814	0.846	0.884	0.874	0.164	0.120	0.132	0.292
AR(1)-GJR(1,2)_Stud	0.968	0.868	0.928	0.804	0.174	0.116	0.156	0.314
AR(1)-EGARCH(1,2)_Stud	0.794	0.624	0.716	0.966	**0.024**	**0.020**	**0.024**	**0.058**
AR(1)-IGARCH(1,2)_Stud	0.674	0.924	0.886	0.696	0.184	0.136	0.198	0.336
AR(1)-FIGARCH(1,2)_Stud	0.682	0.702	0.540	0.332	0.246	0.242	0.386	0.596
AR(1)-PGARCH-(1,2)_Stud	0.408	0.714	0.772	0.636	0.504	0.320	0.386	0.608
AR(1)-CGARCH(1,1)_Stud	0.886	0.848	0.676	0.450	0.230	0.278	0.384	0.642
VAR(3)-BEKK(1,1)_Stud	**0.008**	**0.032**	0.102	0.160	**0.000**	**0.000**	**0.004**	**0.024**
VAR(3)-BEKK(1,2)_Stud	**0.022**	**0.042**	0.118	0.182	**0.000**	**0.000**	**0.006**	**0.016**

Notes: The symbol *m* stands for the embedding dimension in the BDS test. The VAR-BEKK model was constructed for the EUR/PLN exchange rate and the WIG20 index.

Table 3 Bootstrap *p*-Values for the BDS Test for USD/PLN

Model	BDS							
	Standardized residuals				Modules of standardized residuals			
	***m* = 2**	***m* = 3**	***m* = 4**	***m* = 5**	***m* = 2**	***m* = 3**	***m* = 4**	***m* = 5**
Returns	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
GARCH(1,1)_norm	0.578	0.850	0.630	0.564	**0.030**	0.154	0.178	0.304
GARCH(1,1)_Stud	0.452	0.652	0.440	0.408	**0.018**	0.112	**0.098**	0.144
GARCH(2,1)_norm	0.634	0.740	0.836	0.672	0.186	0.308	0.234	0.302
GARCH(2,1)_Stud	0.816	0.960	0.638	0.504	0.128	0.188	0.138	0.220
GARCH(2,1)_GED	0.638	0.814	0.730	0.574	0.164	0.272	0.218	0.280
GARCH(2,1)_skStud	0.938	0.736	0.396	0.296	**0.085**	0.116	**0.090**	0.136
GARCH-M(2,1)_Stud	0.804	0.956	0.598	0.462	0.198	0.252	0.210	0.292
GJR(2,1)_Stud	0.718	0.884	0.702	0.542	0.112	0.212	0.158	0.222
EGARCH(2,1)_Stud	0.936	0.716	0.374	0.276	**0.020**	**0.026**	**0.018**	**0.018**
IGARCH(2,1)_Stud	0.694	0.872	0.792	0.704	0.186	0.260	0.232	0.310
FIGARCH(1,1)_Stud	0.532	0.456	0.994	0.724	0.218	0.408	0.278	0.328
PGARCH(2,1)_Stud	0.832	0.966	0.654	0.518	0.146	0.248	0.182	0.226
CGARCH(1,1)_Stud	0.920	0.742	0.928	0.812	**0.076**	0.296	0.246	0.334
VAR(1)-BEKK(1,1)_Stud	0.166	0.138	**0.058**	**0.068**	**0.002**	**0.002**	**0.010**	**0.026**
VAR(1)-BEKK(1,2)_Stud	0.230	0.246	0.176	0.236	**0.012**	**0.036**	**0.044**	**0.074**

Notes: The symbol *m* stands for the embedding dimension in the BDS test. The VAR-BEKK model was constructed for the USD/PLN exchange rate and the WIG20 index.

Table 5 Bootstrap *p*-Values for the BDS Test for the WIG20 Index

Model	BDS							
	Standardized residuals				Modules of standardized residuals			
	***m* = 2**	***m* = 3**	***m* = 4**	***m* = 5**	***m* = 2**	***m* = 3**	***m* = 4**	***m* = 5**
Returns	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
Ar(1)	**0.000**	**0.000**	**0.000**	**0.000**	**0.002**	**0.000**	**0.000**	**0.000**
GARCH(1,1)_norm	**0.004**	**0.000**	**0.000**	**0.000**	0.588	0.268	0.466	0.588
GARCH(1,1)_Stud	**0.002**	**0.000**	**0.002**	**0.002**	0.606	0.288	0.514	0.662
GARCH(1,3)_norm	0.168	0.160	0.244	0.286	0.700	0.670	0.502	0.504
GARCH(1,3)_Stud	0.404	0.296	0.386	0.402	0.548	0.536	0.370	0.378
AR(1)-GARCH(1,1)_norm	**0.002**	**0.000**	**0.000**	**0.000**	0.248	0.130	0.280	0.418
AR(1)-GARCH(1,1)_Stud	**0.002**	**0.000**	**0.000**	**0.002**	0.442	0.210	0.394	0.532
AR(1)-GARCH(1,3)_norm	0.178	0.156	0.242	0.334	0.890	0.912	0.712	0.700
AR(1)-GARCH(1,3)_Stud	0.336	0.304	0.390	0.464	0.780	0.646	0.456	0.462
AR(1)-GARCH(1,3)_GED	0.244	0.224	0.352	0.432	0.896	0.662	0.464	0.496
AR(1)-GARCH(1,3)_skStud	0.296	0.210	0.300	0.340	0.820	0.730	0.544	0.516
AR(1)-GARCH-M(1,3)_Stud	0.314	0.294	0.404	0.470	0.794	0.656	0.426	0.434
AR(1)-GJR(1,3)_Stud	0.190	0.120	0.190	0.238	0.962	0.946	0.716	0.718
AR(1)-EGARCH(1,3)_Stud	0.708	0.812	0.932	0.782	0.828	0.708	0.508	0.514
AR(1)-IGARCH(1,3)_Stud	0.312	0.230	0.302	0.348	0.750	0.648	0.472	0.468
AR(1)-FIGARCH(1,3)_Stud	0.270	0.188	0.114	**0.094**	0.958	0.852	0.822	0.936
AR(1)-PGARCH(1,3)_Stud	0.746	0.860	0.984	0.928	0.716	0.522	0.398	0.406
AR(1)-CGARCH(1,1)_Stud	0.398	**0.078**	0.140	0.228	0.670	0.970	0.736	0.706
VAR(3)-BEKK(1,1)_Stud	**0.004**	**0.000**	**0.004**	**0.014**	0.374	0.262	0.580	0.818
VAR(3)-BEKK(1,2)_Stud	**0.046**	**0.002**	**0.008**	**0.044**	0.680	0.406	0.646	0.816

Notes: The symbol *m* stands for the embedding dimension in the BDS test. The VAR-BEKK model was constructed for the WIG20 index and the EUR/PLN exchange rate.

Table 7 Bootstrap *p*-Values for the BDS Test for the SWIG80 Index

Model	BDS							
	Standardized residuals				Modules of standardized residuals			
	***m* = 2**	***m* = 3**	***m* = 4**	***m* = 5**	***m* = 2**	***m* = 3**	***m* = 4**	***m* = 5**
Returns	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
ARMA(4,1)	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
GARCH(1,1)_Stud	**0.002**	**0.012**	**0.010**	**0.012**	0.866	0.674	0.484	0.256
ARMA(4,1)-GARCH(1,1)_norm	0.286	0.264	0.260	0.236	0.808	0.672	0.704	0.794
ARMA(4,1)-GARCH(1,1)_Stud	0.866	0.860	0.898	0.844	0.472	0.380	0.438	0.622
ARMA(4,1)-GARCH(1,1)_GED	0.574	0.618	0.670	0.612	0.614	0.506	0.592	0.726
ARMA(4,1)-GARCH(1,1)_skStud	0.812	0.812	0.800	0.762	0.340	0.220	0.248	0.366
ARMA(4,1)-GARCH-M(1,1)_Stud	0.482	0.446	0.416	0.334	0.626	0.566	0.522	0.574
ARMA(4,1)-GJR(1,1)_Stud	0.790	0.800	0.828	0.944	0.254	0.158	0.204	0.384
ARMA(4,1)-EGARCH(1,1)_Stud	0.650	0.644	0.706	0.656	0.888	0.780	0.908	0.824
ARMA(4,1)-IGARCH(1,1)_Stud	0.106	**0.018**	**0.010**	**0.008**	**0.094**	**0.026**	**0.014**	**0.010**
ARMA(4,1)-IGARCH(1,2)_Stud	0.442	0.414	0.420	0.466	0.154	**0.080**	0.112	0.132
ARMA(4,1)-FIGARCH(1,1)_Stud	0.604	0.706	0.826	0.992	0.240	0.176	0.240	0.476
ARMA(4,1)-PGARCH(1,1)_Stud	0.762	0.770	0.782	0.742	0.536	0.452	0.502	0.644
ARMA(4,1)-CGARCH(1,1)_Stud	0.818	0.868	0.934	0.798	0.578	0.384	0.400	0.542
VAR(4)-BEKK(1,1)_Stud	0.260	0.124	**0.072**	**0.044**	0.698	0.614	0.392	0.304
VAR(4)-BEKK(1,2)_Stud	**0.088**	**0.048**	**0.040**	**0.028**	0.306	0.300	0.188	0.166

Notes: The symbol *m* stands for the embedding dimension in the BDS test. The VAR-BEKK model was constructed for the SWIG80 index and the EUR/PLN exchange rate.

Table 2 Bootstrap *p*-Values for the MI, LB and LM tests for EUR/PLN

Model	MI		LB(3) LBM(3)	LB(12) LBM(12)	LM(3)	LM(12)
	Stan. resid.	Modules				
Returns	**0.000**	**0.000**	**0.091**	0.318	**0.000**	**0.000**
AR(1)	**0.000**	**0.000**	0.288	0.330	**0.000**	**0.000**
GARCH(1,1)_Stud	0.333	0.363	**0.072**	0.158	**0.003**	**0.030**
GARCH(1,2)_Stud	0.207	0.476	**0.089**	0.132	**0.009**	0.130
AR(1)-GARCH(1,1)_norm	0.246	0.389	0.252	0.350	**0.013**	0.111
AR(1)-GARCH(1,1)_Stud	0.363	0.362	0.177	0.307	**0.005**	**0.043**
AR(1)-GARCH(1,2)_norm	0.195	0.354	0.211	0.132	**0.026**	0.371
AR(1)-GARCH(1,2)_Stud	0.536	0.682	0.220	0.277	**0.012**	0.153
AR(1)-GARCH(1,2)_GED	0.396	0.505	0.206	0.269	**0.023**	0.293
AR(1)-GARCH(1,2)_skStud	0.506	0.469	0.157	0.227	**0.012**	0.132
AR(1)-GARCH-M(1,2)_Stud	0.484	0.556	0.201	0.300	**0.011**	0.151
AR(1)-GJR(1,2)_Stud	0.721	0.731	0.245	0.302	**0.027**	0.329
AR(1)-EGARCH(1,2)_Stud	0.241	0.423	0.114	0.188	**0.002**	**0.011**
AR(1)-IGARCH(1,2)_Stud	0.559	0.568	0.224	0.292	**0.019**	0.265
AR(1)-FIGARCH(1,2)_Stud	0.408	0.654	0.152	0.267	**0.049**	0.505
AR(1)-PGARCH(1,2)_Stud	0.533	0.522	0.279	0.328	**0.047**	0.553
AR(1)-CGARCH(1,1)_Stud	0.444	0.520	0.205	0.265	**0.035**	0.430
VAR(3)-BEKK(1,1)_Stud	0.399	0.373	0.355	0.559	**0.003**	**0.014**
VAR(3)-BEKK(1,2)_Stud	0.241	0.354	0.571	0.580	**0.016**	**0.018**

Notes: The columns indicated by the symbols MI, LB, LBM, and LM contain the *p*-values obtained from, respectively, the test based on the mutual information measure, the Ljung-Box test, the modified Ljung-Box test (adequate in the case of conditional heteroskedasticity), and the Engle test for the presence of the ARCH effect. The LB, LBM, and LM tests are applied up to orders 3 and 12. The VAR-BEKK model was constructed for the EUR/PLN exchange rate and the WIG20 index.

Table 4 Bootstrap *p*-Values for the MI, LB and LM Tests for USD/PLN

Model	MI		LB(3) LBM(3)	LB(12) LBM(12)	LM(3)	LM(12)
	Stan. resid.	Modules				
Returns	**0.000**	**0.000**	0.372	0.381	**0.000**	**0.000**
GARCH(1,1)_norm	0.502	**0.097**	0.521	0.982	**0.096**	0.593
GARCH(1,1)_Stud	0.746	**0.087**	0.520	0.984	**0.040**	0.420
GARCH(2,1)_norm	0.294	0.175	0.581	0.989	0.171	0.668
GARCH(2,1)_Stud	0.191	0.197	0.582	0.987	0.100	0.528
GARCH(2,1)_GED	0.163	0.390	0.592	0.987	0.115	0.586
GARCH(2,1)_skStud	0.458	0.179	0.541	0.986	**0.045**	0.418
GARCH-M(2,1)_Stud	0.484	0.316	0.624	0.985	0.101	0.530
GJR(2,1)_Stud	0.392	0.567	0.644	0.994	0.292	0.750
EGARCH(2,1)_Stud	0.301	**0.054**	0.620	0.988	**0.016**	0.207
IGARCH(2,1)_Stud	0.548	0.167	0.550	0.988	0.126	0.634
FIGARCH(1,1)_Stud	0.440	0.612	0.580	0.987	0.133	0.674
PGARCH(2,1)_Stud	0.197	0.216	0.580	0.987	0.105	0.530
CGARCH(1,1)_Stud	0.700	0.189	0.500	0.984	0.167	0.692
VAR(1)-BEKK(1,1)_Stud	0.192	0.207	0.109	0.527	**0.006**	**0.082**
VAR(1)-BEKK(1,2)_Stud	0.308	**0.064**	0.103	0.769	**0.014**	0.193

Notes: The columns indicated by the symbols MI, LB, LBM, and LM contain the *p*-values obtained from, respectively, the test based on the mutual information measure, the Ljung-Box test, the modified Ljung-Box test (adequate in the case of conditional heteroskedasticity), and the Engle test for the presence of the ARCH effect. The LB, LBM, and LM tests are applied up to orders 3 and 12. The VAR-BEKK model was constructed for the USD/PLN exchange rate and the WIG20 index.

Table 6 Bootstrap *p*-Values for the MI, LB and LM Tests for the WIG20 Index

Model	MI		LB(3) LBM(3)	LB(12) LBM(12)	LM(3)	LM(12)
	Stan. resid.	Modules				
Returns	**0.000**	**0.049**	0.296	0.409	**0.000**	**0.000**
AR(1)	**0.000**	**0.067**	0.375	0.444	**0.000**	**0.000**
GARCH(1,1)_norm	0.105	**0.099**	0.114	0.306	**0.005**	**0.015**
GARCH(1,1)_Stud	0.397	**0.053**	0.112	0.369	**0.006**	**0.009**
GARCH(1,3)_norm	**0.020**	0.167	**0.059**	0.347	0.991	0.970
GARCH(1,3)_Stud	**0.047**	**0.029**	**0.045**	0.314	0.852	0.949
AR(1)-GARCH(1,1)_norm	0.756	**0.041**	0.740	0.911	**0.003**	**0.008**
AR(1)-GARCH(1,1)_Stud	0.656	**0.087**	0.603	0.888	**0.006**	**0.007**
AR(1)-GARCH(1,3)_norm	0.106	0.160	0.731	0.877	0.996	0.960
AR(1)-GARCH(1,3)_Stud	0.176	0.326	0.524	0.832	0.909	0.946
AR(1)-GARCH(1,3)_GED	**0.069**	0.230	0.385	0.738	0.967	0.952
AR(1)-GARCH(1,3)_skStud	0.296	0.493	0.567	0.841	0.919	0.956
AR(1)-GARCH-M(1,3)_Stud	0.218	0.526	0.529	0.826	0.905	0.949
AR(1)-GJR(1,3)_Stud	**0.061**	**0.087**	0.564	0.857	0.903	0.954
AR(1)-EGARCH(1,3)_Stud	0.223	0.201	0.602	0.884	0.635	0.926
AR(1)-IGARCH(1,3)_Stud	0.253	**0.068**	0.570	0.828	0.899	0.949
AR(1)-FIGARCH(1,3)_Stud	**0.060**	0.157	0.501	0.817	0.744	0.930
AR(1)-PGARCH(1,3)_Stud	**0.011**	0.136	0.566	0.831	0.581	0.899
AR(1)-CGARCH(1,1)_Stud	**0.033**	**0.084**	0.567	0.856	**0.035**	0.212
VAR(3)-BEKK(1,1)_Stud	**0.033**	0.198	0.730	0.929	**0.006**	**0.000**
VAR(3)-BEKK(1,2)_Stud	**0.011**	**0.038**	0.765	0.943	**0.010**	**0.007**

Notes: The columns indicated by the symbols MI, LB, LBM, and LM contain the *p*-values obtained from, respectively, the test based on the mutual information measure, the Ljung-Box test, the modified Ljung-Box test (adequate in the case of conditional heteroskedasticity), and the Engle test for the presence of the ARCH effect. The LB, LBM, and LM tests are applied up to orders 3 and 12. The VAR-BEKK model was constructed for the WIG20 index and the EUR/PLN exchange rate.

Table 8 Bootstrap *p*-Values for the MI, LB and LM Tests for the SWIG80 Index

Model	MI		LB(3) LBM(3)	LB(12) LBM(12)	LM(3)	LM(12)
	Stan. resid.	Modules				
Returns	**0.000**	**0.000**	**0.000**	0.125	**0.000**	**0.000**
ARMA(4,1)	**0.000**	**0.000**	0.440	0.398	**0.000**	**0.000**
GARCH(1,1)_Stud	**0.000**	0.539	**0.000**	**0.000**	0.417	0.606
ARMA(4,1)-GARCH(1,1)_norm	0.643	0.352	0.645	0.300	0.776	0.738
ARMA(4,1)-GARCH(1,1)_Stud	0.634	0.797	0.313	0.303	0.805	0.865
ARMA(4,1)-GARCH(1,1)_GED	0.359	0.502	0.189	0.218	0.828	0.854
ARMA(4,1)-GARCH(1,1)_skStud	0.595	0.669	0.105	0.152	0.769	0.864
ARMA(4,1)-GARCH-M(1,1)_Stud	0.599	0.978	0.195	0.225	0.654	0.646
ARMA(4,1)-GJR(1,1)_Stud	0.652	0.927	0.468	0.432	0.675	0.892
ARMA(4,1)-EGARCH(1,1)_Stud	0.498	0.779	0.390	0.362	0.883	0.910

continued

Table 8 *(continued)*

Model	MI		LB(3) LBM(3)	LB(12) LBM(12)	LM(3)	LM(12)
	Stan. resid.	Modules				
ARMA(4,1)-IGARCH(1,1)_Stud	0.488	0.895	0.182	0.190	0.114	**0.020**
ARMA(4,1)-IGARCH(1,2)_Stud	0.234	0.631	0.100	0.160	0.482	**0.068**
ARMA(4,1)-FIGARCH(1,1)_Stud	0.831	0.772	0.265	0.271	0.605	0.778
ARMA(4,1)-PGARCH(1,1)_Stud	0.652	0.747	0.296	0.286	0.820	0.865
ARMA(4,1)-CGARCH(1,1)_Stud	0.786	0.824	0.347	0.352	0.714	0.827
VAR(4)-BEKK(1,1)_Stud	0,189	0,886	0.553	**0.038**	0.574	0.542
VAR(4)-BEKK(1,2)_Stud	0,145	0,527	0.613	**0.047**	0.366	0.496

Notes: The columns indicated by the symbols MI, LB, LBM, and LM contain the *p*-values obtained from, respectively, the test based on the mutual information measure, the Ljung-Box test, the modified Ljung-Box test (adequate in the case of conditional heteroskedasticity), and the Engle test for the presence of the ARCH effect. The LB, LBM, and LM tests are applied up to orders 3 and 12. The VAR-BEKK model was constructed for the SWIG80 index and the EUR/PLN exchange rate.

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