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Information Efficiency of the Capital Market: a Stochastic Calculus Approach

Evidence from the Czech Republic*

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Abstract
This paper deals with an important characteristic of the capital market: information efficiency. With the use of geometric Brownian motion, we run several projections of stock prices based on varying amount of historic information and compare these projections with the real behavior of the stock prices, examining for predictability. This enables us to verify the condition of the weak-efficiency hypothesis in the form of a Markov process. We conduct the empirical part of our analysis in the environment of the Czech capital market, thus providing additional information on the development of transition economies.

1. Introduction

Capital market efficiency is an important concept in modern financial theory and practice. Efficiency ensures that funds are transferred to such uses that yield the highest risk-adjusted returns through the exchange of financial assets between buyers and sellers with the least possible transaction costs. There is a clear analogy between the concept of an efficient market and that of a perfect market known from microeconomic theory.

Information efficiency is defined by the condition that prices fully reflect all relevant information at every instant. The fulfilment of this condition results in prices evolving in a random way. The process of such a price evolution may be described by the concept of a random walk in a discrete-time setting or by the Brownian motion in a continuous-time context. If the condition of prices fully reflecting all relevant information holds, then we refer to the market as being strong efficient. Strong-form efficiency relates to the situation when prices fully reflect past and present information, the latter regardless of the fact of whether it is publicly available. We think of a market as being semi-strong efficient when the prices fully reflect all past and publicly known present information. If the prices have absorbed the past information only, we refer to such a market as being weak-efficient. We will test the weak-efficiency hypothesis in this paper.

Information efficiency or the level of information efficiency has significant microeconomic and macroeconomic consequences. We have already mentioned that information efficiency is a necessary condition for funds to be allocated effectively. This, of course, has serious economic implications on both the micro and macro levels.

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On the micro level it is mainly the accessibility to new funds by firms and the possibility to invest funds. This, in turn, affects the macroeconomic output itself. The efficiency and stability of the capital market is a necessary condition for the efficiency and stability of the economy as a whole.

The concept of information efficiency plays a key role in one kind of microeconomic decision-making processes – portfolio management. The materialization of the strong-form and semi-strong form efficiency prevents investors from earning returns that systematically exceed those generated by the market index portfolio. Thus, portfolio management is reduced to its passive form only. In the latter form it is possible to earn extra yields by using private information, which is usually illegal. Under the circumstances of the weak form of information efficiency, it is possible to make use of the methods of fundamental analysis. In such a market the security prices reflect the past information only. If the market is proved efficient, regardless of the form, technical analysis as a means to systematically earn extra yields is always useless.

There are many papers dealing with the issue of information efficiency. However, most of them take the rather statistical view of the problem, which, of course, requires the use of relatively longer time series. However, this might, in our opinion, cast some shadow on the results of the analyses as we consider the capital market to be under the process of swift evolution. Many authors agree on the fact that the Czech capital market is a weak-efficient one. Diviš and Teplý (2004) use the Cowles-Jones ratio, run tests and a variance-ratio test to show that the Czech capital market is at least weak-efficient. This is supported by Horská (2003) who uses the random walk model with drift and technical analysis approach (Alexander's Filter test, Relative Strength Index). However, she proves by means of a regression model that the market is not semi-strong efficient. The weak efficiency had already been implied by Hanousek and Kočenda (1997) with the use of a variance-ratio test. They draw an analogy between the tests run for the Latin-American countries and the Czech market as there were not enough data for the Czech market at that time. Podpiera (2000) examines the effectiveness of the Czech financial market. He makes use of the continuous decrease in the repo rate conducted by the Czech National Bank in the years 1998–1999. However, the sensitivity of the stock market to these changes was low, so no particular conclusions about the efficiency of the stock market may be drawn. The money market showed signs of weak efficiency. Filáček, Kaplička and Vošvrda (1998) proved the market inefficient.

In this paper we propose a possible way of examination of the concept of information efficiency and conduct the empirical analysis based on this approach in the environment of the Czech capital market. As our approach is rather technical, we feel it is necessary to provide a background to the way we handle the empirical analysis itself. Therefore, our paper is divided into four parts: theoretical framework, empirical analysis methodology, empirical analysis and conclusion.

In the second part – theoretical framework – we lay down the mathematical bases necessary for the approach we take later in the empirical part of the paper. In the following part 3 dedicated to the methodology of the empirical analysis we first turn our attention to the description and characterization of the data needed for the tests of the information efficiency and then on the tests themselves. We clearly state the principles of the empirical approach as a whole and of each test to be taken later and relate them to the concept of information efficiency. In doing so, we formulate the objective to be tested in the following part of our paper. In the fourth part we conduct the empirical analysis and mainly just present the output of the analysis. The output is subject to assessment in the last part 5 – the conclusion, where we draw comparisons between the results of the empirical analysis and our objectives formulated in the third and fourth parts of the paper.

2. Theoretical Framework

In this part we will present the mathematical background of our empirical analysis. The general starting-point of our analysis is the *m*–dimensional Itô process, which we first state in matrix notation:

$$d\mathbf{X} = \boldsymbol{\alpha} dt + \boldsymbol{\beta} d\mathbf{B} \tag{1}$$

where $\mathbf{X}$ is a m-dimensional column vector of Itô stochastic processes: $\begin{bmatrix} dX_1 \\ ... \\ dX_m \end{bmatrix}$

$\boldsymbol{\alpha}$ is a m-dimensional column vector of an adapted stochastic process: $\begin{bmatrix} \alpha_1 \\ ... \\ \alpha_m \end{bmatrix}$

$\boldsymbol{\beta}$ is a $m \times n$-dimensional matrix of an adapted stochastic process: $\begin{bmatrix} \beta_{11} & ... & \beta_{1n} \\ ... & ... & ... \\ \beta_{m1} & ... & \beta_{mn} \end{bmatrix}$

and $\mathbf{B}$ is a n-dimensional column vector of the Brownian motion: $\begin{bmatrix} dB_1 \\ ... \\ dB_n \end{bmatrix}$

We assume that all necessary conditions, especially those about the integrability of the functions α and β hold. This also implies that the multidimensional Itô integral contained in the multidimensional Itô process as defined by (1) exists. For the purpose of clarity, we state it as follows:

$$I_t(\omega) = \int_0^t \begin{bmatrix} \beta_{11} & ... & \beta_{1n} \\ ... & ... & ... \\ \beta_{m1} & ... & \beta_{mn} \end{bmatrix} \begin{bmatrix} dB_{11} \\ ... \\ dB_n \end{bmatrix} \tag{2}$$

The multidimensional Itô process as defined by (1) based on the multidimensional Itô integral (2) represent the theoretical basis of our approach. However, in the empirical part of our analysis we will operate only in one dimension. Thus, we can reduce the general forms (1) and (2) to one-dimensional notations. First, we will introduce the one-dimensional Itô process. We can think of this process as a sum of two integrals: the first one is an ordinary integral while the second one is a stochastic integral as defined by Itô. For our purposes we can interpret this in such a way that the first part is of a deterministic nature while the second part is of a stochastic nature.[1] Let's define the one-dimensional Itô stochastic process as follows:

$$Z(t,\omega) = \int_0^t \alpha(s,\omega)\, ds + \int_0^t \beta(s,\omega)\, dB(s,\omega) \tag{3}$$

where: t, s denotes time so that $s < t$,

ω denotes the state of the economy,

α and β are functions adapted to filtration ξ_t, which may be generated by the Brownian motion; α and β both satisfy the integrability condition,

B denotes the Brownian motion.

The second part of expression (3) is the one-dimensional Itô integral:

$$I_t(\omega) = \int_0^t \beta(s,\omega)\, dB(s,\omega) \tag{4}$$

For the purpose of the empirical part of this paper, we will define the stochastic differential equation, which is closely related to the Itô integral and the Itô process. The starting-point is the one-dimensional Itô process (3). If we consider two functions: $\mu(t, x)$ and $\sigma(t, x)$ that satisfy certain conditions[2] and a stochastic process $X(t, \omega)$, we can write a new Itô process which transforms the stochastic process $X(t, \omega)$ into the stochastic process $Z(t, \omega)$ as defined above (3):

$$Z(t,\omega) = \int_0^t \mu\left[s, X(s,\omega)\right] ds + \int_0^t \sigma\left[s, X(s,\omega)\right] dB(s,\omega) \tag{5}$$

We can now consider the following stochastic equation and its solution in the form of an Itô process[3]:

$$X(t,\omega) = \int_0^t \mu\left[s, X(s,\omega)\right] ds + \int_0^t \sigma\left[s, X(s,\omega)\right] dB(s,\omega) \tag{6}$$

$$X(t,\omega) = X_0 + \int_0^t \mu\left[s, X(s,\omega)\right] ds + \int_0^t \sigma\left[s, X(s,\omega)\right] dB(s,\omega) \tag{7}$$

The integral stochastic equation (6) may be rewritten as a stochastic differential equation:

$$dX(t,\omega) = \mu\left[t, X(t,\omega)\right] dt + \sigma\left[t, X(t,\omega)\right] dB(t,\omega) \tag{8}$$

The basic type of stochastic differential equation is a linear stochastic equation whose common example is the arithmetic Brownian motion[4]:

$$dX_t = \mu\, dt + \sigma\, dB_t \tag{9}$$

where μ and σ are constants.

[1] This is not necessarily true. Both parts may be of a stochastic nature, represented by stochastic processes.

[2] The conditions are: the linear growth condition and Lipschitz condition.

[3] The linear growth and Lipschitz conditions must be satisfied.

[4] Formally, (sole) Brownian motion is defined as, based on Shreve (2004, p. 94),: Let (Ω, F, P) be a probability space. For each ω from Ω, suppose there is a continuous function $B(t)$ of $t \geq 0$ that satisfies $B(0) = 0$ and that depends on ω. Then $B(t)$, $t \geq 0$, is a Brownian motion if for all $0 = t0 < t1 < \ldots < tm$ the increments: $B(t1) - B(t0)$, $B(t2) - B(t1)$, $B(tm) - B(tm - 1)$ are independent and each of these increments is normally distributed with $E[B(ti + 1) - B(ti)] = 0$ and $\text{Var}[B(ti + 1) - B(ti)] = ti + 1 - ti$.

The first part of the expression (9) – μ is a drift parameter representing the deterministic characteristic of the evolution of stochastic process X_t,[5] while the second part σdB_t represents the stochastic part of the process X_t.

In the empirical part of the paper we will use the concept of the geometric Brownian motion to model the security prices. The geometric Brownian motion is defined as follows:

$$dX_t = \mu X_t dt + \sigma X_t dB_t \tag{10}$$

The geometric Brownian motion can be easily transformed into a linear stochastic differential equation using the Itô formula:

$$dY_t = \left(\frac{\partial g}{\partial t} + \frac{\partial g}{\partial x}\alpha + \frac{1}{2}\frac{\partial^2 g}{\partial x^2}\beta^2 \right) dt + \frac{\partial g}{\partial x}\beta dB_t \tag{11}$$

where $Y_t = g(t, X_t)$ and α and β are variables.

The Itô formula expresses one Itô process, for example one defined by Y_t, which is a function of another Itô process – X_t. With the help of the Itô formula we can transform the geometric Brownian motion into linear form. If we take $Y = \log X$, according to the Itô formula (11), we obtain:

$$\frac{\partial g}{\partial t} = 0$$

$$\frac{\partial g}{\partial x} = \frac{1}{x}$$

and

$$\frac{\partial^2 g}{\partial x^2} = -\frac{1}{x^2}$$

Let's suppose that the variable α represents a drift parameter and the variable β stands for volatility. Then we can state the geometric Brownian motion as follows:

$$d \log X_t = \left(\mu - \frac{1}{2}\sigma^2 \right) dt + \sigma\, dB_t \tag{12}$$

This lognormal process can be rewritten as:

$$X_t = X_0^{\left\{ \left(\mu - \frac{1}{2}\sigma^2 \right) t + \sigma B_t \right\}} \tag{13}$$

Let's go back to the expression (10) for a moment. The geometric Brownian motion as expressed by equation (10) operates in a continuous time setting, which is useless for our purposes. Using an Euler discretization scheme (Euler approximation) we can restate expression (10) for use within a discrete-time setting. The principal of the Euler approximation is the replacement of the first derivative by the first difference. Thus, we obtain:

$$\Delta X_{t+1} = \mu X_t \Delta t + \sigma X_t \Delta B_t \tag{14}$$

[5] The drift parameter may, of course, be variable. However, we will consider it a constant.

The last question related to the security price modeling is concerned with the Brownian motion. Among the properties of the standard Brownian motion there is the fact that the stochastic increments are independent and normally distributed. In a discrete-time setting we can make use of this property in the way that we can model the stochastic increments using a simple function of a random number with the normal distribution. With regard to this fact and other characteristics of the standard Brownian motion, we will restate the expression (14) in the following way:

$$X_{t+1} = X_t + \mu X_t \Delta t + \sigma X_t \varepsilon \sqrt{\Delta t} \tag{15}$$

where: $X_t + 1$ is the price of a security at time t+1,

X_t is the price of a security at time t,

μ is annualized return,

Δt is a step in time $[(t + 1) – (t)]$,

σ is annualized standard variation of returns,

ε is a random number with the normal distribution.

The restatement (15) is fully operational in the sense that it may be used to model the evolution of a security price without any need of sophisticated mathematical software. The use of expression (15) for the simulation of prices is based on the principles of the Monte Carlo method as described in Glasserman (2004).

We have introduced the background to the way we will model the stock prices for the needs of our analysis. We will now proceed with the introduction of some other concepts necessary for the empirical part. In particular, we are concerned with the Markov and martingale processes.

The key theoretical process when describing the weak efficiency of the capital market is a Markov process. Shreve (2004, p. 74) defines it as:

Let (Ω, F, P) be a probability space, let T be a fixed positive number, and let F(t), 0 ≤ t ≤ T be a filtration of sub-σ-algebras of F. Consider an adapted stochastic process X(t), 0 ≤ t ≤ T. Assume that for all 0 ≤ s ≤ t ≤ T and for every non-negative, Borel-measurable function f, there is another Borel-measurable function g such that E[f(X(t))/F(s)] = g(X(s)). Then we say that X is a Markov process.

This definition may be rewritten as follows:

$$E\,[f\,(t, X\,(t))/F(s)] = f\,(s, X\,(s)),\ 0 \leq s \leq t \leq T \tag{16}$$

In (16) we just stress the fact that the functions f and g depend on time, s and t. Then, it is evident that we consider two different functions of X at different times and, therefore, it is not necessary to use the function g. We interpret (16) as the condition by which the probability distribution of the process X at time t given the filtration $F(s)$, which here denotes the evolution of the process up to time s, depends only on the value of the process X at time s. If the stock prices follow the Markov process, the market is said to have no memory. There is no use trying to make better estimates of the probability distribution of future prices of, say, a stock, using the series of its past prices, regardless of its length.

Another concept linked with the Markov process is a martingale. Shreve (2004, p. 74) defines it as follows:

Let (Ω, F, P) be a probability space, let T be a fixed positive number, and let F(t), 0 ≤ t ≤ T be a filtration of sub-σ-algebras of F. Consider an adapted stochastic process M(t), 0 ≤ t ≤ T. If E[M(t)/F(s)] = M(s) for all 0 ≤ s ≤ t ≤ T, we say this process is a martingale. It has no tendency to rise or fall. If E[M(t)/F(s)] ≥ M(s) for all 0 ≤ s ≤ t ≤ T, we say this process is a submartingale. It has no tendency to fall; it may have a tendency to rise. If E[M(t)/F(s)] ≤ M(s) for all 0 ≤ s ≤ t ≤ T, we say this process is a supermartingale. It has no tendency to rise; it may have a tendency to fall.

If a process is a martingale it is a fair game. Every martingale is a Markov process, though not every Markov process is a martingale. Brownian motion is a martingale, which may be proved as follows, based on Shreve (2004, p. 98):

Let 0 ≤ s ≤ t be given.

Then:

$$E[B(t)/F(s)]=E[(B(t)-B(s)+B(s))/F(s)]$$
$$=E[(B(t)-B(s))/F(s)+E[(B(s)/F(s)]$$
$$=E[B(t)-B(s)]+B(s)$$
$$=B(s)$$ [6]

Therefore, a Brownian motion is also a Markov process.[7] Now, let's return to expression (13) for a moment. The process:

$$H(t) = \exp\left\{\sigma B(t) - \frac{1}{2}\sigma^2 t\right\}$$

is a (exponential) martingale. We could prove this along the same lines as in the case of a simple Brownian motion.[8] Thus, the process described by expression (13) has this property together with the fact it has a mean rate of return μ. This upward drift (generally, it is positive) indicates that the process is a submartingale. From what has already been discussed it follows that it has a Markov property.

In the next part of this paper we use the theoretical concepts introduced here to describe the methodology of the testing.

3. Methodology

As already implied in the previous part, we build our analysis on the theory of stochastic processes. We introduced the Itô process, which represents the basis for the actual price modeling used in this paper. The model to be used for the analysis of the behaviour of stock prices was derived using the process of simplification and discretization of the general multi-dimensional Itô process. Thus, we introduced the technique of stock price modeling and together with the definition of the Markov process and martingale we set the stage for the empirical part of the analysis. Here we will focus on the definition of the object of the empirical analysis.

[6] We used the fact of linearity of conditional expectations and independence of conditional expectations. For more, see (Shreve, 2004, pp. 69–70).

[7] Formal proof is in (Shreve, 2004, pp. 107–108).

[8] Formal proof is in (Shreve, 2004, p. 109).

Reflecting back on the ideas brought up in the introduction, we set the objective of this paper as the verification of the weak-efficiency hypothesis both in theoretical and empirical terms, the latter carried out under the conditions of the Czech capital market. The concept of the Itô process specified by the special case of the geometric Brownian motion represents a useful tool for stock price modelling for two reasons. First, it takes account of the fact that stock prices tend to rise from the long-term point of view and, second, it contains the key idea behind the behaviour of stock prices which is the stochastic nature of the price process.

We have already stated that a market is weak-efficient if it is impossible, using information on the past evolution of prices, to produce better estimates of future development of prices and thus to systematically earn extra profits. The key idea of our empirical approach rests on exploitation of the discretized geometric Brownian motion (15) to model future prices while changing the drift and volatility parameters according to the amount of past information (prices) considered. The random number with normal distribution ε in (15) is generated by a generator of pseudonumbers with normal distribution.[9] The comparison of the projections and real development of prices under the various drift and volatility parameters shows how much the predictability of future prices is sensitive to the amount of past information.

First we will introduce the data which were necessary to run the tests. As we have already mentioned, we tested the hypothesis of the information efficiency under the conditions of the Czech capital market. We limited the environment to stocks which are subject to trading on the Prague Stock Exchange (PSE). There are 36 firms whose stocks are traded on the PSE. However, we only considered 33 of them because three firms did not meet our requirements for the data necessary to run the tests.

We used prices of those stocks for the years 2003 and 2004 to carry out five simulations, each having fifty paths.[10] Each simulation projects the evolution of a particular stock price for the year 2005. The simulations differ in the amount of past information which was used to project the evolution of the stock price in the year 2005. Simulation 1 is based on the years 2003 and 2004, Simulation 2 on the year 2004, Simulation 3 on the second half of the year 2004, Simulation 4 on only the last change of price (i.e. between the last trading day of the year 2004 and first trading day of the year 2005) and, finally, Simulation 5 with no drift. For the purpose of clarity we again describe the differences between the simulations in the following section.

Regarding the model derived in the previous section – expression (15), the data from the particular periods were used to compute the drift and volatility. In the case of Simulation 5, volatility based on the 2004 data was used (computation of volatility based on one trading day is impossible). This might seem to be imprecise, but there are only slight differences in the volatility computations based on various time intervals. We can conclude that the volatility whose computation is based on past data is quite a stable measure.

[9] All sorts of pseudonumber generators are used in Monte Carlo simulations (for more see (Glasserman, 2004)). We used the pseudogenerator supplemented with an Excel spreadsheet.

[10] Increasing the number of simulated paths enhances the quality of the simulation as a whole. However, increasing the number of paths also makes the simulation and files hardly manageable (with respect to the particular software used). The number we chose is a compromise between these two factors.

The varying amount of information which serves for the computation of the drift and volatility, which are inserted into the model derived in the previous part, enables us to examine the effect of this varying amount of information on the possibility to predict future prices and thus systematically earn extra returns.

Now we focus on the methods we use to measure our capability to predict future prices. We use four tests.

The first test – Probability Distribution Test A – uses the simulated daily prices to compute average monthly prices. Each simulation consists of 50 paths. Let $\left\{p_i^j\right\}$ be a series which is a j-th projection, $j \in \langle 1,50 \rangle$, of prices for i-th stock, $i \in \langle 1,33 \rangle$, given a particular month. Such a series has k elements according to the particular month. Averaging these elements, we obtain an average projected price for j-th projection and i-th stock. Taking account of all 50 projections, we get a set of 50 average projected prices for i-th stock and a particular month. Using Sturges's rule[11] we divide the set into seven intervals and assess whether or not the real average price for the particular stock and month fits the interval with the highest frequency. Based on these findings, we compute the probability with which the real average monthly prices fit, generally speaking, the prediction.

The second test – Probability Distribution Test B – has a similar background. Let $\left\{p_i^j\right\}$ be a series which is a j-th projection, $j \in \langle 1,50 \rangle$, of prices for i-th stock, $i \in \langle 1,33 \rangle$, given a particular month. Such a series has k elements according to the particular month. Averaging these elements, we obtain an average projected price for j-th projection and i-th stock. Taking account of all 50 projections and computing their average, we get the average projected price for a particular month and stock. We use this average projected price to create an interval whose borders are computed in such a way that the lower limit equals the average value less one percent of the value and the upper limit equals the average value plus one percent of the value: $\langle p_i - 0,01p_i, p_i + 0,01p_i \rangle$, where p_i is the average projected monthly price for i-th stock. We assess whether or not the real monthly averages fit the interval. Thus, we obtain the probability of our capability to predict the monthly average prices.

The third test, in the form of two subtests – Correlation Tests A and B, moves from monthly to daily prices. Let $\left\{p_i^j\right\}$ be a series which is a j-th projection, $j \in \langle 1,50 \rangle$, of prices for i-th stock, $i \in \langle 1,33 \rangle$ for the whole period, the year 2005. We compute the daily average projected prices and correlate them with the actual daily prices.[12] Afterwards, using the average daily data, we compute growth indices (day-over-day indices) and correlate them with real growth indices. Both of the variants measure our capability to predict daily prices.

[11] $k = 1 + 3,3\log_{10}(n)$, where k is the number of intervals and n is the number of observations.

[12] $\rho_{XY} = \dfrac{\mathrm{COV}(X,Y)}{\sigma_X \sigma_Y}$, where COV stands for covariation and σ denotes standard deviation.

The fourth test – Relative Discrepancies Test – is focused on the predictability of the very first projected price. Let $\left\{p_i^j\right\}$ be a series which is a *j*-th projection, $j \in \langle 1,50 \rangle$, of prices for *i*-th stock, $i \in \langle 1,33 \rangle$ for the whole period, the year 2005. We take the 50 projections for the first trading day of 2005 and compute the relative difference between the projected and real prices:

$$rd_i = \frac{\left|p_i^p - p_i^r\right|}{p_i^r}$$

where rd_i stands for a relative difference for the *i*-th stock, while p^p and p^r are the projected and real prices, respectively. Then we average these relative differences for each stock to get an average relative difference for the particular simulation. Comparing across simulations, it shows how the capability to predict the first of all the simulated prices varies with the amount of past information used. We use this test because it directly follows the definition of the Markov process as stated by expression (16).

In a nutshell, we assess the predictability of the evolution of future prices and its dependence on the information set.

In the following section we present the results of the tests. However, we do not focus on the interpretation of the results in the following part. The output of the empirical analysis is to be assessed in the final part of the paper.

Appendix 1 contains tables with the detailed results of the tests which were run.

4. Empirical Analysis

In this section we will present the results of the analysis of the weak-form efficiency hypothesis we performed in the environment of the Czech capital market.

The results will be presented in the following way. As previously mentioned, we ran five simulations. Simulation 1, whose estimates of drift and volatility are based on 2003 and 2004 data; Simulation 2, which draws from the year 2004; Simulation 3, whose estimates are based on data from the second half of 2004; Simulation 4, whose estimate of drift is based on the last relative change in price and estimate of volatility based on 2004 data; and Simulation 5, which is driftless while the volatility estimation is based on 2004 data. The output of the analysis will be given for each simulation in turn. In each case we will present the results for all the four tests in turn.

Let's turn our attention to the results of the tests for Simulation 1. From Probability Distribution Test A we can see that the probability of the real average monthly prices fitting the interval with the highest frequency is 20.5 %. In five cases out of 33 there is no match. The variability of the probabilities measured by standard deviation is 15.5. With regard to the results of Probability Distribution Test B, we see that the probability with which the real average monthly prices fit the target interval is 12.9 %. The variability of the probabilities measured by standard deviation is 18.2 and in 14 cases (out of 33) there is no match. Let's now focus on the Correlation Test. There is a correlation of 0.2251 between the absolute projected and real prices; on the other hand, the correlation coefficient between the growth indices of real and projected daily prices is only 0.0160. The last test was the Relative Discrepancies Test, which measures the average percentage discrepancy between the projected price and the real price for the first trading day of 2005. The average relative discrepancy for Simulation 1 is 0.46.

We will proceed with the results of the tests for Simulation 2. The probability with which the real average monthly prices fit the interval with the highest frequency is 25.2 %. This information is derived from Probability Distribution Test A. The standard deviation of the probabilities is 26.1 and in ten cases there is no match. The probability of the real average monthly prices fitting the target interval is 14.6 % – the result of Probability Distribution Test B. The standard deviation of the probabilities is 24.1 and there are 14 cases with no match. With regard to the Correlation Test, we can see that the correlation coefficient between the absolute projected and real prices is 0.2399 while the one between the growth indices is –0.0112. The average relative discrepancy for Simulation 2 is 0.43.

Let's now focus on Simulation 3. From the results of Probability Distribution Test A we can read that the probability of the real average monthly prices fitting the interval with the highest frequency is 24.7 %. The variability of the probabilities measured by standard deviation is 29.8. There are nine cases with no match. From the output of Probability Distribution Test B we can see that the probability with which the real average monthly prices fit the target interval is 19.9 %. The standard deviation of the probabilities is 30.5 and in 15 cases there is no match. The correlation coefficient between the absolute real and projected daily prices is 0.1869 while the one between the growth indices is 0.0030. The average relative discrepancy for Simulation 3 is 0.47.

The main results for Simulation 4 are as follows: the output of Probability Distribution Test A states that the probability with which the real average monthly prices fit the interval with the highest frequency is 25.0 %. The standard deviation of the individual probabilities is 28.4 and there are ten cases with no match. From the output of Probability Distribution Test B we can see that the probability of the real average monthly prices fitting the target interval is 18.7 %. The variability of the probabilities measured by standard deviation is 28.3. There are 15 cases with no match. The correlation coefficient between the absolute real and projected prices is –0.0407 and the one between growth indices is –0.0018. The average relative discrepancy for Simulation 4 is 0.53.

Finally, we will present the results of the test for Simulation 5. Taking into account the output of Probability Distribution Test A, we can conclude that the probability with which the real average monthly prices fit the interval with the highest frequency is 26.3 %. The standard deviation of the individual probabilities is 27.4. There is no match in ten cases. From the results of Probability Distribution Test B it can be concluded that the probability with which the real average monthly prices fit the target interval is 22.2 %. The variability of the individual probabilities measured by standard deviation is 30.3 and there are 13 cases with no match. With regard to the output of the Correlation Test, we can see that the correlation coefficient between the absolute real and projected prices is –0.0369 and the correlation coefficient between the growth indices is –0.0034. The average relative discrepancy for Simulation 5 is 0.39.

FIGURE 1 Probability Distribution Test A

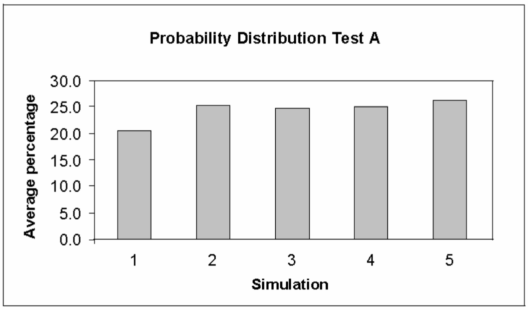
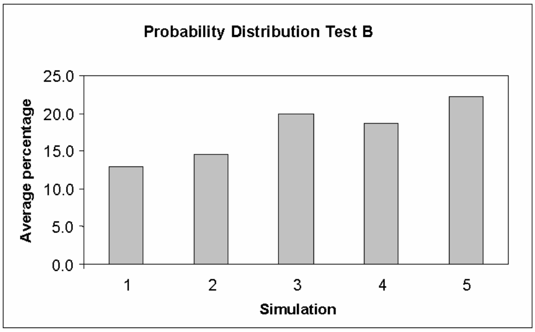


FIGURE 2 Probability Distribution Test B



We remind the reader of the fact that the detailed presentation of the output of the tests can be found in *Appendix 1*. In the last section we focus on the interpretation of the results presented in this part.

5. Conclusion

To make the interpretation as clear as possible we will summarize the most important output of the empirical analysis presented in the previous section in figures.

Figure 1 depicts the average probability of the real average monthly prices fitting the interval with the highest frequency according to Probability Distribution Test A.

In *Figure 1* we can see that the average probability with which the real average monthly values fit the interval with the highest frequency is between 20.5 % and 26.3 %. First, this means that our capability of predicting future prices is very low. We must take account of the fact that it is not the probability of predicting values which is depicted in the *Figure 1* but the probability of fitting the highest-frequency interval with the real value. The interval spread varies from firm to firm and in time. A high percentage could be interpreted as being able to predict the behaviour of future prices, but even this is not the case; we do not consider a probability of 20 % to be high.

The second point to be made is that the average probability of being able to predict the evolution of future prices is not affected by the information set on which

FIGURE 3 Correlation Test A

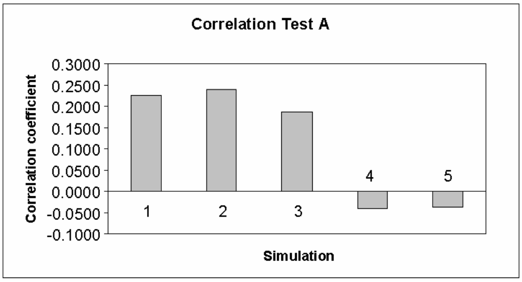
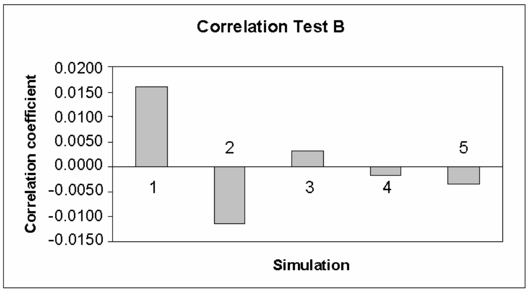


FIGURE 4 Correlation Test B



the estimates of drift and volatility are made. Of course, we do not mean that there is no variability in the average probabilities, but the variability is low. Furthermore, we cannot interpret the results by claiming that with a broader information set the ability to predict the evolution of future prices is better. The highest probability comes from Simulation 5, which is driftless.

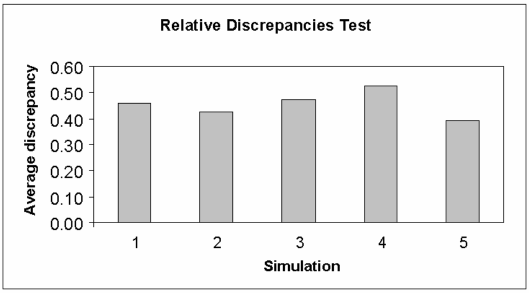
Let's now turn our attention to Probability Distribution Test B. *Figure 2* depicts its main results.

The results show that the spread of the probabilities is broader. However, there seems to be a trend in the average probability values with respect to the amount of information which was taken into consideration in the process of modelling the evolution of future prices. It seems that the less past information is used, the higher the probability. We do not think this is conclusive. We cannot claim that the predictability grows with the amount of past information considered going to zero because the average probability for Simulation 3 is higher than that for Simulation 4, which clearly contradicts the seeming negative relationship between the average probability and the amount of information. We conclude that the results of the first and second test are much the same.

We will proceed with the results of the Correlation Test, which are summarized in *Figures 3* and *4*.

First we will discuss the results of the correlation test between absolute values. The correlation coefficients between the real and average projected daily prices as such are low. However, we can see that the correlation coefficient is much higher for Simulations 1, 2 and 3 than for Simulations 4 and 5. This might imply that

FIGURE 5 Relative Discrepancies Test



the ability to predict future prices is low taking account only the very latest information (or no information as far as drift is concerned – Simulation 5). Nevertheless, it is important to point out the fact that higher correlation in absolute prices does not mean higher predictability. The test measures how similar the evolutions of the absolute real and projected prices are, not how close the absolute prices are. To be able to make conclusions about the predictability of daily prices, one must consider the correlation of absolute values together with relative values – the growth indices. This is depicted in *Figure 4*.

We can see that the correlation of the growth indices is very low regardless of the information set which the simulation is based on. Taking these two correlation analyses together we can say that the ability to predict daily prices is extremely low and, with respect to *Figure 3* and *4*, is significantly affected by the amount of information considered. However, only Correlation Test A indicates a possible positive relationship between the amount of information and the quality of prediction.

Now we turn our attention to the last, fourth, test (*Figure 5*). The relative discrepancies range from 0.39 to 0.53. There is no clear relationship between the amount of past information and the relative discrepancies – quality of prediction.

Let's now use these partial considerations to draw conclusions on the objective of the empirical analysis.

The key question was whether the varying amount of past information used to project future prices influences the quality of prediction. Only Correlation Test A examining the correlation between real and predicted prices in absolute terms indicates that using the past information improves the quality of prediction. However, it is important to stress the fact that by no means does it necessarily mean that it improves the opportunities to systematically earn extra yields. No other test supports this.

We conclude our paper with this finding: the behaviour of stock prices on the Czech capital market is in line with the concept of the weak-form efficient market as we consider the very condition in the form of the Markov process fulfilled. Thus, we consider the possibility to systematically earn extra returns using the techniques of technical analysis in the conditions of the Czech capital market improbable.

REFEENCES

Diviš K, Teplý P (2004): Informační efektivnost burzovních trhů ve střední Evropě. UK FSV-IES Working Paper, no. 52.

Filáček J, Kapička M, Vošvrda M (1998): Testování hypotézy efektivního trhu na BCPP. *Finance a úvěr*, 48(9):554–566.

Focardi SM, Fabozzi FJ (2004): *The Mathematics of Financial Modeling and Investment Management*. J. Wiley. ISBN 0-471-46599-2.

Glasserman P (2004): *Monte Carlo Methods in Financial Engineering*. Springer. ISBN 0-387-00541-3.

Hanousek J, Kočenda E (1997): Efektivnost nových kapitálových trhů: příklad pro střední Evropu. *Finance a úvěr*, 47(9):538–548.

Horská H (2003): Český kapitálový trh – jeho efektivnost a makroekonomické souvislosti. *VŠE v Praze Working Paper*, no. 7.

Podpiera R(2000): Efektivnost českého kapitálového trhu ve světle snižování úrokových sazeb. *Finance a úvěr*, 50(5):270–282.

Shreve SE (2004): *Stochastic Calculus for Finance II: Continuous-Time Models*. Springer. ISBN 0-387-40101-6

APPENDIX

Probability Distribution Test A					
Firm/Simulation	**Simulation 1**	**Simulation 2**	**Simulation 3**	**Simulation 4**	**Simulation 5**
Česká námořní plavba	41.7	58.3	66.7	66.7	58.3
Česká zbrojovka	25.0	0.0	0.0	33.3	16.7
ČEZ	33.3	66.7	8.3	58.3	0.0
Energoaqua	33.3	33.3	16.7	8.3	16.7
Erste Bank	0.0	0.0	0.0	0.0	25.0
Jihočeské papírny Větřní	16.7	33.3	16.7	16.7	25.0
Jihočeská plynárenská	25.0	16.7	91.7	66.7	66.7
Jihomoravská plynárenská	16.7	75.0	75.0	75.0	75.0
Komerční banka	0.0	33.3	25.0	50.0	50.0
Kotva	0.0	8.3	8.3	33.3	8.3
Lázně Teplice v Čechách	16.7	41.7	33.3	8.3	0.0
Léčebné lázně Jáchymov	8.3	33.3	100.0	58.3	75.0
Paramo	25.0	0.0	0.0	0.0	33.3
Philip Morris ČR	33.3	41.7	8.3	8.3	41.7
Pražská energetika	58.3	8.3	0.0	0.0	8.3
Pražská plynárenská	33.3	0.0	0.0	25.0	33.3
Pražské služby	16.7	100.0	100.0	100.0	100.0
RM-S Holding	50.0	16.7	41.7	25.0	25.0
Severočeská plynárenská	16.7	50.0	33.3	16.7	16.7
Setuza	25.0	33.3	16.7	0.0	0.0
Slezan Frýdek Místek	25.0	8.3	16.7	8.3	50.0
Severomoravská plynárenská	25.0	25.0	41.7	66.7	58.3
Severomoravské vodovody a kanalizace Ostrava	8.3	0.0	8.3	8.3	8.3
Spolana	0.0	0.0	0.0	0.0	0.0
Spolek pro chemickou a hutní výrobu	41.7	0.0	16.7	0.0	0.0
Stavby silnic a železnic	0.0	25.0	25.0	8.3	0.0
Středočeská energetická	8.3	66.7	16.7	0.0	25.0
Středočeská plynárenská	8.3	33.3	8.3	8.3	8.3
Telefonica	16.7	8.3	33.3	0.0	0.0
Toma	8.3	0.0	0.0	58.3	41.7
Unipetrol	41.7	0.0	0.0	16.7	0.0
Východočeská plynárenská	8.3	0.0	0.0	0.0	0.0
Západočeská plynárenská	8.3	16.7	8.3	0.0	0.0
Average percentage	**20.5**	**25.2**	**24.7**	**25.0**	**26.3**

Probability Distribution Test B					
Firm/Simulation	**Simulation 1**	**Simulation 2**	**Simulation 3**	**Simulation 4**	**Simulation 5**
Česká námořní plavba	50.0	66.7	58.3	66.7	58.3
Česká zbrojovka	0.0	8.3	0.0	41.7	41.7
ČEZ	0.0	0.0	0.0	16.7	0.0
Energoaqua	33.3	25.0	0.0	8.3	8.3
Erste Bank	8.3	8.3	0.0	0.0	8.3
Jihočeské papírny Větřní	8.3	0.0.	0.0	0.0	0.0
Jihočeská plynárenská	50.0	50.0	91.7	91.7	91.7
Jihomoravská plynárenská	75.0	75.0	75.0	75.0	75.0
Komerční banka	8.3	8.3	0.0	8.3	8.3
Kotva	41.7	16.7	41.7	41.7	41.7
Lázně Teplice v Čechách	0.0	0.0	8.3	0.0	0.0
Léčebné lázně Jáchymov	8.3	8.3	100.0	33.3	75.0
Paramo	0.0	0.0	0.0	0.0	8.3
Philip Morris ČR	8.3	8.3	0.0	0.0	8.3
Pražská energetika	0.0	0.0	0.0	0.0	0.0
Pražská plynárenská	16.7	8.3	8.3	50.0	50.0
Pražské služby	0.0	100.0	100.0	100.0	100.0
RM-S Holding	0.0	0.0	25.0	0.0	0.0
Severočeská plynárenská	8.3	25.0	25.0	16.7	16.7
Setuza	8.3	0.0	0.0	8.3	8.3
Slezan Frýdek Místek	16.7	8.3	16.7	25.0	33.3
Severomoravská plynárenská	16.7	8.3	0.0	8.3	66.7
Severomoravské vodovody a kanalizace Ostrava	25.0	8.3	8.3	8.3	8.3
Spolana	0.0	0.0	0.0	0.0	0.0
Spolek pro chemickou a hutní výrobu	0.0	0.0	8.3	0.0	0.0
Stavby silnic a železnic	16.7	8.3	16.7	0.0	0.0
Středočeská energetická	8.3	16.7	8.3	8.3	16.7
Středočeská plynárenská	16.7	25.0	16.7	0.0	0.0
Telefonica	0.0	0.0	25.0	0.0	0.0
Toma	0.0	0.0	0.0	0.0	8.3
Unipetrol	0.0	0.0	0.0	8.3	0.0
Východočeská plynárenská	0.0	0.0	0.0	0.0	0.0
Západočeská plynárenská	0.0	0.0	25.0	0.0	0.0
Average percentage	**12.9**	**14.6**	**19.9**	**18.7**	**22.2**

Correlation Coefficients					
Firm/Simulation	**Simulation 1**	**Simulation 2**	**Simulation 3**	**Simulation 4**	**Simulation 5**
Česká námořní plavba	0,3491	0,0000	0,0000	0,0000	0,0000
Česká zbrojovka	-0,9209	-0,9213	-0,9170	-0,4431	0,3969
ČEZ	0,9458	0,9446	0,9480	0,9506	0,3684
Energoaqua	0,5490	0,5199	0,5551	0,3924	0,1111
Erste Bank	-0,2787	-0,7546	-0,6794	-0,7319	-0,6050
Jihočeské papírny Větřní	-0,3933	-0,7328	-0,6978	-0,2647	0,1326
Jihočeská plynárenská	-0,3623	0,3695	-	-0,2507	0,0800
Jihomoravská plynárenská	-0,8113	-	-	-	-
Komerční banka	-0,0378	0,0782	0,0574	0,5291	-0,0828
Kotva	0,8449	0,8392	-0,8374	-0,8476	-0,6813
Lázně Teplice v Čechách	-0,6628	-0,6413	-0,6556	-0,7270	0,3662
Léčebné lázně Jáchymov	0,0000	0,0000	1,0000	0,0000	0,0000
Paramo	-0,8352	-0,8274	-0,8307	-0,5853	-0,1974
Philip Morris ČR	-0,1167	0,1782	-0,3309	-0,1344	-0,3257
Pražská energetika	0,9629	0,9619	0,9220	-0,4992	-0,9352
Pražská plynárenská	0,8567	0,8683	0,8610	-0,7961	-0,0208
Pražské služby	-	-	-	-	-
RM-S Holding	0,7709	-0,7663	0,9220	0,8691	-0,6441
Severočeská plynárenská	0,6672	0,8209	0,7397	0,0326	0,6075
Setuza	0,6258	0,6365	0,6135	-0,4610	-0,4123
Slezan Frýdek Místek	-0,0858	0,1071	-0,0719	-0,1291	0,0145
Severomoravská plynárenská	-0,5653	-0,6774	-0,6900	0,1518	0,6275
Severomoravské vodovody a kanalizace Ostrava	0,1648	-0,0004	-0,0093	0,0599	0,1153
Spolana	0,1744	0,4927	0,0562	0,0227	0,0422
Spolek pro chemickou a hutní výrobu	0,5271	0,5597	0,4986	0,1554	-0,4827
Stavby silnic a železnic	0,8891	0,8654	0,8588	0,1642	-0,6104
Středočeská energetická	0,1564	0,0812	0,1135	0,0558	0,2062
Středočeská plynárenská	0,8338	0,8675	0,8546	-0,8894	0,7248
Telefonica	0,9205	0,9197	0,9474	0,8750	-0,4728
Toma	0,1534	0,1408	0,1639	0,1588	0,1453
Unipetrol	0,8516	0,8762	0,8649	0,8739	0,4665
Východočeská plynárenská	0,2761	0,9068		0,6414	-0,4846
Západočeská plynárenská	0,7536	0,7246	0,7702	-0,4353	0,4063
Average correlation coefficient	**0,2251**	**0,2399**	**0,1869**	**-0,0407**	**-0,0369**

Correlation Coefficients (indices)					
Firm/Simulation	**Simulation 1**	**Simulation 2**	**Simulation 3**	**Simulation 4**	**Simulation 5**
Česká námořní plavba	0,0086	-	-	-	-
Česká zbrojovka	0,0713	-0,0465	0,0041	-0,0559	0,0390
ČEZ	-0,0486	-0,0322	0,0071	-0,0031	-0,1150
Energoaqua	-0,1232	-0,0530	-0,0220	0,0452	-0,0114
Erste Bank	0,0588	0,0023	-0,0430	-0,0308	-0,0062
Jihočeské papírny Větřní	0,0089	0,1144	0,0150	0,0413	-0,0147
Jihočeská plynárenská	0,0092	-0,0675	-	0,0142	0,0546
Jihomoravská plynárenská	-0,0354	-	-	-	-
Komerční banka	0,0073	-0,0085	0,0941	-0,0163	0,0549
Kotva	-0,0361	0,0587	0,0280	-0,0986	0,0843
Lázně Teplice v Čechách	0,0209	0,0048	-0,0014	0,0262	0,1724
Léčebné lázně Jáchymov	-	-	-	-	-
Paramo	0,0736	-0,0868	-0,1093	0,0627	0,0375
Philip Morris ČR	0,0743	0,0102	-0,0661	0,1221	-0,0397
Pražská energetika	-0,0272	0,0046	-0,0056	-0,1670	-0,0755
Pražská plynárenská	-0,0974	-0,0213	-0,0523	-0,0723	-0,0289
Pražské služby	-	-	-	-	-
RM-S Holding	0,0950	0,0069	-0,0622	-0,0396	0,0209
Severočeská plynárenská	0,0088	-0,1313	-0,0066	-0,0471	0,0373
Setuza	0,0242	-0,1013	-0,0620	0,0108	0,0646
Slezan Frýdek Místek	0,0309	-0,0251	0,0370	0,0116	0,0157
Severomoravská plynárenská	-0,0171	0,0228	0,0394	0,0313	-0,0103
Severomoravské vodovody a kanalizace Ostrava	0,1286	0,0037	-0,0262	-0,0283	-0,1214
Spolana	0,0481	0,0283	-0,0294	-0,0122	-0,0519
Spolek pro chemickou a hutní výrobu	-0,0158	-0,0085	-0,0441	-0,0290	-0,0752
Stavby silnic a železnic	0,0212	-0,0334	0,0714	0,1165	-0,0519
Středočeská energetická	0,0982	-0,0608	0,1443	0,0302	0,0369
Středočeská plynárenská	-0,0098	-0,0502	0,0711	-0,0913	-0,0389
Telefonica	0,0296	-0,0504	0,1629	0,0455	0,0365
Toma	0,0564	-0,0016	0,0155	0,0922	-0,0328
Unipetrol	0,0575	-0,0060	-0,0939	0,0620	-0,0847
Východočeská plynárenská	-0,1027	0,1066	-	-0,0097	0,0006
Západočeská plynárenská	0,0777	0,0949	0,0157	-0,0621	0,0059
Average correlation coefficient	**0,0160**	**-0,0112**	**0,0030**	**-0,0018**	**-0,0034**

Relative Discrepancies					
Firm/Simulation	**Simulation 1**	**Simulation 2**	**Simulation 3**	**Simulation 4**	**Simulation 5**
Česká námořní plavba	0,02	0,00	0,00	0,00	0,00
Česká zbrojovka	0,33	0,07	0,25	0,01	0,02
ČEZ	1,95	1,61	1,86	1,39	2,17
Energoaqua	0,30	0,16	0,22	0,04	0,05
Erste Bank	2,68	2,71	2,40	2,20	2,05
Jihočeské papírny Větřní	0,19	0,58	1,40	0,15	0,39
Jihočeská plynárenská	0,01	0,01	0,00	0,00	0,01
Jihomoravská plynárenská	0,00	0,00	0,00	0,00	0,00
Komerční banka	2,12	1,72	2,03	2,71	2,17
Kotva	0,03	0,01	0,00	0,01	0,10
Lázně Teplice v Čechách	0,09	0,30	0,16	0,18	0,13
Léčebné lázně Jáchymov	0,02	0,00	0,00	0,01	0,04
Paramo	0,13	0,27	0,49	4,92	0,28
Philip Morris ČR	1,41	1,78	1,26	0,84	1,83
Pražská energetika	0,02	0,06	0,14	0,04	0,08
Pražská plynárenská	0,03	0,08	0,13	0,07	0,06
Pražské služby	0,03	0,00	0,00	0,00	0,00
RM-S Holding	0,14	0,16	0,08	0,02	0,21
Severočeská plynárenská	0,06	0,10	0,05	0,04	0,02
Setuza	0,30	0,13	0,24	0,19	0,21
Slezan Frýdek Místek	0,04	0,12	0,04	0,03	0,07
Severomoravská plynárenská	0,18	0,03	0,06	0,20	0,15
Severomoravské vodovody a kanalizace Ostrava	0,04	0,07	0,07	0,06	0,02
Spolana	0,18	0,11	0,41	0,10	0,09
Spolek pro chemickou a hutní výrobu	0,68	0,17	0,33	0,20	0,07
Stavby silnic a železnic	0,16	0,03	0,42	1,88	0,06
Středočeská energetická	0,27	0,13	0,56	0,02	0,05
Středočeská plynárenská	0,08	0,37	0,06	0,19	0,21
Telefonica	2,04	1,80	1,83	0,90	1,73
Toma	0,54	0,55	0,32	0,20	0,25
Unipetrol	0,96	0,79	0,75	0,71	0,31
Východočeská plynárenská	0,10	0,10	0,00	0,00	0,08
Západočeská plynárenská	0,01	0,01	0,07	0,05	0,03
Average discrepancy	0,46	0,43	0,47	0,53	0,39