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An Optimal Linear Income Tax with a Subsidy on Housing

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1. Introduction

Housing subsidies, currently in force in the income tax legislation of many countries, are the focus of a longstanding debate among academics and policy makers. They have been criticized for several reasons. Some authors, e.g., Laidler (1969), Gahvari (1984, 1985), Hamilton and Whalley (1985), Poterba (1992) and Berkovec and Fullerton (1992) consider that these subsidies induce homeowners to over-consume housing services. According to Skinner (1996), tax subsidies on owner-occupied housing deliver a windfall bonus to existing homeowners at the expense of future generations, which have large efficiency costs. In turn, Gervais (2002) shows that individuals prefer to live in a world without housing subsidies. Other authors, such as Feldstein (1982), Poterba (1984), Zubiri (1990) and López-García (2004), see little efficiency gains from subsidies since they are completely transferred from buyers to sellers through increases in prices. Also, Rosen (1985) argues that subsidies do not induce the acquisition of a first necessity good properly, since it means discriminatory protection against those that have lower income and consequently need to be helped.

On the other hand, several authors favor the use of housing subsidies. Cremer and Gahvari (1998) prove that the differential tax treatment of housing may be justified on grounds of optimal tax policy, creating the conditions under which consumption of housing by the poor must be subsidized. Also Nakagami and Pereira (1995) show that first-time home buyers would suffer great utility loss from the elimination of mortgage-interest deductibility.

The aim of the present paper is to study the optimality of including a housing allowance in the income tax. Two different approaches can be considered. On the one hand, we have the theory of optimal taxation, which has formalized the design of a tax system that maximizes social welfare; see, e.g., Mirrlees (1971), Sheshinski (1972), Atkinson (1970), Atkinson and Stiglitz (1980) and Slemrod (1994). The resulting optimal tax system takes

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account of the trade-off between efficiency and a more equal distribution of wealth. However, these models working with heterogeneous agents represent a highly simplified economy.

On the other hand, there are dynamic macroeconomic models that examine the effect of a certain tax change on the steady state utility of a representative agent, as in (Turnovsky – Okuyama, 1994). The main concern of these models is to analyze the effect of a favorable tax treatment to housing on the stock of housing. In other words, they are concerned about the efficiency of the housing allowance, but not about its equity.

Our objective is to improve previous models on two main grounds. First, instead of the classic static optimal-tax model we use an infinite-horizon model following Turnovsky and Okuyama (1994). Second, we introduce equity objectives that are usually absent in dynamic macroeconomic models.

The rest of the paper is structured as follows. The model is laid out in Section 2. The optimal linear income tax and the optimal subsidy on housing are presented in Section 3. Section 4 offers some concluding remarks.

2. The Model

The production side is as simple as possible. Labor is used in this economy to produce a good (considered the numeraire commodity) that is consumed, and the stock of housing that is used to produce housing services. We assume that all the stock of housing is residential and owner-occupied. The economy consists of a number of infinitely-lived individuals that are equal in all regards except for their ability, represented by w (individual ability). The utility function has the usual concavity properties. Individuals decide the number of hours they want to work, their consumption of both the numeraire good and the housing services, and their accumulation of housing stock and government bonds, in order to maximize their utility subject to the budget constraint. On the other side, the government decides the structure of a progressive linear income tax that maximizes the social welfare function subject to the revenue constraint consisting of three elements: the minimum guaranteed income, the marginal tax rate, and the rate of subsidy on housing.[1]

Hence, each individual maximizes:

$$\sum_{t=0}^{00} \beta^t U(C_t, L_t, H_t) \quad U_C > 0, \quad U_L < 0, \quad U_H > 0 \tag{1.a}$$

where:
U = utility function,
C = per capita consumption of the non-housing good,
L = per capita labour supply,
H = per capita consumption of housing services,
β = consumer rate of time preference,

$U_C = \dfrac{\partial U}{\partial C}$,

[1] Notice that a linear income tax with a minimum guaranteed income is progressive because the average tax rate increases with income.

$$U_L = \frac{\partial U}{\partial L},$$

$$U_H = \frac{\partial U}{\partial H};$$

subject to:

$$P_t H_t + (b_{t+1} - b_t) + C_t + (1-\delta)(h_{t+1} - h_t) = [wL_t + r_{h_t} h_t + r_t b_t](1-\tau) + G_t$$
$$C_t \geq 0, \qquad 1 \geq L_t \geq 0 \tag{1.b}$$

with the initial conditions:

$$b(0) = b_0 h(0) = h_0 \tag{1.c}$$

where:
P = (imputed) price of housing services, in terms of the numeraire good,
b = per capita stock of government bonds, denominated in terms of the numeraire good,
δ = rate of subsidy on housing,
h = per capita stock of housing,
w = individual ability,
r_h = real rental rate on housing,
r = real rate of return of government bonds,
τ = marginal tax rate,
G = minimum guaranteed income.

Equation (1.b) states that individuals spend their after-tax income from labor, bonds and housing stock, plus the government's minimum guaranteed income, on housing services, consumption, and accumulation of bonds and of housing stock.

Solving the intertemporal optimization problem defined in equations (1.a) to (1.c) we obtain the following optimality conditions:

$$U_C = \lambda \tag{2.a}$$
$$U_L + w(1 - \tau)U_C = 0 \tag{2.b}$$

$$\frac{U_H}{U_C} = \mathrm{P} \tag{2.c}$$

$$r_{h_t} = r_t (1-\delta) \tag{2.d}$$

where λ, the costate variable associated with the accumulation equation (1.b) is the marginal utility of wealth measured in terms of the numeraire commodity. Equation (2.a) equals the marginal utility of consumption of the numeraire good to the marginal utility of wealth. Equation (2.b) defines the labor supply, where there is a critical wage w_0 such that:

$$\begin{array}{l} L_T > 0 \text{ when } w > w_0 \\ L_T > 0 \text{ when } w \leq w_0 \end{array} \tag{2.e}$$

Equation (2.c) can be interpreted as defining the price of housing services as the marginal rate of substitution between housing services and consumption. Finally, equation (2.d) equals the rate of return of the housing stock to the interest rate of bonds taking into account the subsidy on housing.

We assume that housing services (H) are produced using housing stock (h) with a linear technology:

$$H_t = \alpha h_t$$

Equilibrium pricing requires that:

$$\frac{\partial H}{\partial h} P_t - r_{h_t} = 0 \Leftrightarrow$$

$$\alpha P_t = r_{h_t}$$

i.e., the marginal income of housing services equals the marginal cost of housing services.

Without loss of generality we assume $\alpha = 1$ so that:

$$H_t = h_t \tag{2.f}$$
$$P_t = r_{h_t} \tag{2.g}$$
$$P_t H_t = r_{h_t} h_t \tag{2.h}$$

In addition, the following transversality conditions must hold:

$$\lim_{t\to\infty} \beta^t U_c h_{t+1} = \lim_{t\to\infty} \beta^t U_c b_{t+1} = 0 \tag{2.i}$$

On the other hand, on the production side we assume constant prices and the absence of profits. We denote the government revenue per capita as R_0, and the production constraint is:

$$\int_w^\infty C_t \mathrm{d}F + R_0 + \int_w^\infty (h_{t+1} - h_t)\,\mathrm{d}F = \int_{w_0}^\infty wL_t \mathrm{d}F \tag{2.j}$$

where F is the density function.

We assume $\int_w^\infty \mathrm{d}F = 1$. Using constraint (1.b), (2.j) can be rewritten as:

$$R_0 + G + \int_w^\infty [\delta(h_{t+1} - h_t) + r_t b_t]\,\mathrm{d}F = \int_w^\infty [\tau(wL_t + r_{h_t} h_t + r_t b_t) + (b_{t+1} - b_t)]\,\mathrm{d}F \tag{2.k}$$

which is the usual government budget constraint, where the revenue from taxes and government bonds must equal expenditure.
Finally, we assume a continuously balanced budget

$$b_{t+1} = b_t \tag{2.l}$$

so that the government constraint becomes:

$$R_0 + G + \int_w^\infty [\delta(h_{t+1} - h_t) + r_t b_t]\,\mathrm{d}F = \int_w^\infty [\tau(wL_t + r_{h_t} h_t + r_t b_t)\mathrm{d}F \tag{2.m}$$

The macroeconomic equilibrium consists of a set of equations that holds true at all times, and jointly determines C_t, H_t, L_t, h_{t+1}, P_t, r_t, r_{ht} and λ_t; see *Appendix A*. From there, the steady-state equilibrium is attained when $\lambda_t = \lambda_{t+1}$ and $h_t = h_{t+1}$, determining the steady-state values of C, H, L, h, P, r, r_h, and λ.

3. Government Taxing Decisions

In this section the government decides the structure of a linear progressive income tax, i.e. a minimum guaranteed income, a marginal tax rate, and the subsidy on housing. Therefore, we are generalizing the current standard model of the optimal linear income tax to include the decision of fixing a certain subsidy on housing. In addition, we have extended the standard model to include the effect that the marginal tax rate may have on the real rate of return of government bonds and the real rate of return of the per capita stock of housing.

The government is assumed to maximize the social welfare function (Ψ)

$$\int_w^\infty \tilde{\Psi}(V)\,\mathrm{d}F \tag{3.a}$$

where V is the indirect utility function, subject to:

$$R_0 + G + \tilde{r}\int_w^\infty b\mathrm{d}F = \tau\int_w^\infty [w\tilde{L} + \tilde{r}\,(1-\delta)\tilde{H} + \tilde{r}b]\mathrm{d}F \tag{3.b}$$

We denote the steady state values by the symbol ~. The Lagrangean of this problem is:

$$L = \int_w^\infty \big[\tilde{\Psi}(V) + \mu\big(\tau[w\tilde{L} + \tilde{r}\,(1-\delta)\tilde{H} + \tilde{r}b] - R_0 - G - \tilde{r}b\big)\big]\mathrm{d}F \tag{3.c}$$

from which we can derive the first-order conditions with respect to G, τ and δ:

$$\int_w^\infty \Big[\tilde{\Psi}'\frac{\partial V}{\partial G} + \mu\big(\tau[w\frac{\partial\tilde{L}}{\partial G} + \tilde{r}\,(1-\delta)\,\frac{\partial\tilde{H}}{\partial G}] - 1\big)\Big]\mathrm{d}F = 0 \tag{4.a}$$

$$\begin{aligned}\int_w^\infty \Big[\tilde{\Psi}'\frac{\partial V}{\partial \tau} + \mu\big([w\tilde{L} + \tilde{r}\,(1-\delta)\tilde{H} + \tilde{r}b] + \tau[w\frac{\partial\tilde{L}}{\partial\tau} + \tilde{r}\,(1-\delta)\,\frac{\partial\tilde{H}}{\partial\tau}] + \\ + \tau\,\frac{\partial\tilde{r}}{\partial\tau}\,[b + (1-\delta)\tilde{H}] - \mathrm{b}\,\frac{\partial\tilde{r}}{\partial\tau}\,\big)\Big]\mathrm{d}F = 0\end{aligned} \tag{4.b}$$

$$\int_w^\infty \Big[\tilde{\Psi}'\frac{\partial V}{\partial \delta} + \mu\big(\tau[w\frac{\partial\tilde{L}}{\partial\delta} + \tilde{r}\,(1-\delta)\,\frac{\partial\tilde{H}}{\partial\delta} - \tilde{r}\tilde{H}]\big)\Big]\mathrm{d}F = 0 \tag{4.c}$$

It is possible to simplify these three expressions as follows (see *Appendix B*):

$$\int_w^\infty \Big[\tilde{\Psi}'\frac{\gamma}{\mu} + \tau[w\frac{\partial\tilde{L}}{\partial M} + \tilde{r}\,(1-\delta)\,\frac{\partial\tilde{H}}{\partial M}] - 1)\Big]\mathrm{d}F = 0 \tag{5.a}$$

$$\int_{w}^{\infty} \Big[(-\tilde{\Psi}'\frac{\gamma}{\mu} + 1)\, Z + \tau[w(-\, wS_{LL} - Z\,\frac{\partial\tilde{L}}{\partial M}) + \tilde{r}\,(1-\delta)\,[\tilde{r}\,(1-\delta)S_{HH} - \mathrm{Z}\,\frac{\partial\tilde{H}}{\partial M}] +$$

$$+ \frac{\partial\tilde{r}}{\partial\tau}\, b + (1-\delta)\tilde{H}] - \mathrm{b}\,\frac{\partial\tilde{r}}{\partial\tau}\,)\Big]\mathrm{d}F = 0 \qquad (5.b)$$

$$\int_{w}^{\infty} \Big[\,\tilde{\Psi}'\frac{\gamma}{\mu}\;\tau\,\tilde{r}\tilde{H} + \tau[w\,\frac{\partial\tilde{L}}{\partial\delta} + \tilde{r}\,(1-\delta)\,\frac{\partial\tilde{H}}{\partial\delta} - \tilde{r}\tilde{H}\,]\Big]\mathrm{d}F = 0 \qquad (5.c)$$

where $M = rb + G$, $\dfrac{\partial V}{\partial M} = \gamma$ and $Z = w\tilde{L} + \tilde{r}b + \tilde{r}(1-\delta)\tilde{H}$.

We now define B to be the net social marginal valuation of income, measured in terms of government revenue, and modified to include the effect of the increase in income on the housing stock. That is, B measures the benefit from transferring one monetary unit to the household, allowing for the marginal tax paid on this extra monetary unit.

$$B = \tilde{\Psi}'\,\frac{\gamma}{\mu} + \tau\,\Big[w\,\frac{\partial\tilde{L}}{\partial M} + \tilde{r}\,(1-\delta)\,\frac{\partial\tilde{H}}{\partial M}\Big]$$

Using this condition as well as equation (5.a) the optimal tax policy can be characterized as:

$$\int_{W}^{\infty}[B-1]\,\mathrm{d}F = 0 \qquad (6.a)$$
$$\Leftrightarrow \bar{B} = 1$$

where $\bar{B}$ is the mean value of B.

Using the definition of B in (5.b) we obtain:

$$\int_{W}^{\infty}[(B-1)Z + \tau[\frac{w\tilde{L}}{1-\tau}\varepsilon_{LL} - \frac{\tilde{r}(1-\delta)}{1-\tau}\,\tilde{H}\varepsilon_{HH} - \frac{\tilde{r}}{1-\tau}\,\varepsilon_{r\tau}\,(b + (1-\delta)\tilde{H})] + b\,\frac{\partial\tilde{r}}{\partial\tau}]\mathrm{d}F = 0$$

where $\varepsilon_{LL} = S_{LL}\,\dfrac{(1-\tau)w}{\tilde{L}}$ is the compensated wage elasticity of labor, $\varepsilon_{HH} = S_{HH}\,\dfrac{r(1-\delta)(1-\tau)}{\tilde{H}}$ is the compensated price elasticity of housing, and $\varepsilon_{r\tau} = \dfrac{\partial\tilde{r}}{\partial\tau}\,\dfrac{(1-\tau)}{\tilde{r}}$ is the tax-rate elasticity of the interest rate, all of these expressed in wage-equivalent units and S_{LL}, S_{HH} are the substitution terms. S_{LL} is the compensated response of labor to the net wage and S_{HH} is the compensated response of housing demand to its net price.

Finally (5.b) and (5.c) become:

$$\frac{\tau}{1-\tau} = \frac{\text{-cov}(Z, B) - \int_{W}^{\infty}\Big[\mathrm{b}\dfrac{\partial\tilde{r}}{\partial\tau}\Big]\mathrm{d}F}{\int_{W}^{\infty}[w\tilde{L}\varepsilon_{LL} - \tilde{r}(1-\delta)\tilde{H}\varepsilon_{HH} - \tilde{r}(b + (1-\delta)\tilde{H})\varepsilon_{r\tau}]\mathrm{d}F} \qquad (6.b)$$

$$\int_w^\infty \left[\tilde{\Psi}' \frac{\gamma}{\mu} \tau\tilde{r}\tilde{H} + \tau\tilde{r}(1-\delta)\frac{\partial\tilde{H}}{\partial\delta}\right] \mathrm{d}F = \tau\int_w^\infty \left(\tilde{r}\tilde{H} - w\frac{\partial\tilde{L}}{\partial\delta}\right)\mathrm{d}F \qquad (6.c)$$

Condition (6.a) can be interpreted as in the model of Atkinson and Stiglitz (1980). It states that the minimum guaranteed income should be adjusted in such a way that the net social valuation of income B should, on average, be equal to its cost (one monetary unit).

In turn, condition (6b) determines the optimal tax rate and can be compared to the corresponding expression from the Atkinson-Stiglitz model:

$$\frac{\tau}{1-\tau} = \frac{-\mathrm{cov}(w\tilde{L}, B)}{\int_w^\infty \left[w\tilde{L}\varepsilon_{LL}\right]\mathrm{d}F}$$

In our model $\mathrm{cov}(w\tilde{L} + \tilde{r}b + \tilde{r}(1-\delta)\tilde{H}, B)$ replaces cov $(w\tilde{L}, B)$ because $w\tilde{L} + \tilde{r}b + \tilde{r}(1-\delta)\tilde{H}$ and not $w\tilde{L}$ is pre-tax income, given that we have introduced government bonds and housing in the original model. We can interpret the covariance as a marginal measure of inequality. The greater the inequality aversion is, the higher the marginal tax rate will be. When there is no aversion to inequality, $\mathrm{cov}(w\tilde{L} + \tilde{r}b + \tilde{r}(1-\delta)\tilde{H}, B)$ will be zero and the marginal tax rate will be zero as well.

The second term that appears in the numerator of equation (6.b) is new. It can be interpreted as the disincentive the government has to increase marginal tax rates when the interest rate depends on the tax rate as in our model. When the government raises the tax rate, the interest rate and government debt increase.

The denominator is also modified. In the Atkinson-Stiglitz version the denominator is the compensated labor-supply elasticity weighted by labor income. In our version we also include the compensated price elasticity of housing weighted by housing expenditure, and the tax-rate elasticity of the interest rate weighted by the expenditure on government bonds and housing expenditure. The compensated price elasticity of housing tends to reduce the marginal tax rate. An increase in the tax rate raises the price of housing, reducing the investment in housing stock and consequently the government's income. On the other hand, the tax-rate elasticity of the interest rate will increase the marginal tax rate. This is due to the fact that an increase in the tax rate will increase the interest rate as well as government revenue coming from income taxation. This equation shows that the decision about the optimal progressivity of the income tax must take into account efficiency effects other than the labor-supply effect. An increase in the tax rate will probably have effects on the interest rate that we cannot ignore.

Finally, condition (6.c) tells us that the optimal value for the deduction δ is such that the social marginal benefit from increasing the subsidy on housing must be equal to its marginal cost. The marginal benefit appears on the left-hand side of the equation and consists of two elements. The first one is the increase in the consumer's utility due to a reduction in the price of per capita housing services. The second one is extra revenue collected because individuals raise their per capita stock of housing and pay more income taxes on it. The marginal cost also consists of two elements: the decrease in government revenue because the real rate of return of the per capita stock of housing has decreased, and the decrease in government revenue due to the fact that per capita labor supply will be lower.

4. Concluding Remarks

This paper has analyzed an optimal linear tax with a subsidy on housing. Some motivation for this analysis has been provided by the major controversy that has emerged about whether a subsidy on housing should be included in the income tax and its effects. The novelty of this analysis is the inclusion of dynamic effects that were absent in previous optimal linear income-tax models. In particular, with regard to macroeconomic models such as Turnovsky and Okuyama (1994) that examine the effect of a certain tax change on the steady state utility of a representative agent, we have introduced here heterogeneous agents as well as equity objectives, which are absent in that literature.

The discussion suggests that when a more complex economy is considered new efficiency effects from increasing the marginal tax rate or the subsidy on housing appear that should be taken into account:

- First of all, we have the disincentive of the government to increase marginal tax rates when the interest rate depends on the tax rate in our model. When the government raises the tax rate, the interest rate and government debt increase.
- Secondly, the compensated price elasticity of housing tends to reduce the marginal tax rate because an increase in the tax rate raises the price of housing, reducing investment in housing stock and consequently the government's income.
- On the other hand, the tax-rate elasticity of the interest rate will increase the marginal tax rate. This is due to the fact that an increase in the tax rate will increase the interest rate as well as government revenue coming from income taxation.
- The marginal benefit from the housing subsidy consists of two elements: the increase in the consumer's utility due to a reduction in the price of per capita housing services, and the extra revenue collected because individuals raise their per capita stock of housing and pay more income taxes on it.
- Finally, the marginal cost also consists of two elements: the decrease in government revenue because the real rate of return of the per capita stock of housing is smaller and the decrease in government revenue due to the fact that the per capita labor supply will be lower.

As should be obvious, the importance of all these efficiency effects depends on their compensated elasticity, but they should not be discarded a priori.

To conclude, it goes without saying that the analysis is still based on many restrictive assumptions. We have considered that there is a numeraire commodity that can be used for consumption or alternatively as a stock of housing to produce housing services. A possible way to improve the model could be by introducing two different goods in the economy: housing and a composite good, which we could call non-housing, as well as two productive sectors: the housing and the non-housing sectors. This would probably allow us to see the effect of the subsidy on housing on the price of housing services, which is one of the most discussed effects of the housing subsidy.

Appendix A

The macroeconomic equilibrium consists of the following set of equations that holds at all times and jointly determines C_t, H_t, L_t, h_{t+1}, P_t, r_t, r_{h_t} and λ_t:

$$U_C = \lambda_t \tag{A1.a}$$

$$U_L + w(1-\tau)U_C = 0 \tag{A1.b}$$

$$\frac{U_H}{U_C} = P_t \tag{A1.c}$$

$$r_{h_t} = r_t\,(1-\delta) \tag{A1.d}$$

$$-\lambda_t + \beta\lambda_{t+1}\,[1 + r_{t+1}(1-\tau)] = 0 \tag{A1.e}$$

$$P_t H_t + C_t + (1-\delta)\,(h_{t+1} - h_t) = [wL_t + r_{h_t}\,h_t + r_t b]\,(1-\tau) + G \tag{A1.f}$$

$$P_t = r_{h_t} \tag{A1.g}$$

$$H_t = h_t \tag{A1.h}$$

The steady-state equilibrium is attained when: $\lambda_t = \lambda_{t+1}$ and $h_t = h_{t+1}$, which implies the following set of equations:

$$\tilde{r} = \frac{(1-\beta)}{\beta}\;\frac{1}{(1-\tau)} \tag{A2.a}$$

$$\tilde{r}_h = \tilde{r}(1-\delta) \tag{A2.b}$$

$$U_L + w(1-\tau)U_C = 0 \tag{A2.c}$$

$$\frac{U_H}{U_C} = \tilde{P} \tag{A2.d}$$

$$\tilde{P}\tilde{H} + \tilde{C} = [w\tilde{L} + \tilde{r}_h\,\tilde{h} + \tilde{r}b]\,(1-\tau) + G \tag{A2.e}$$

$$\tilde{P} = \tilde{r}_h \tag{A2.f}$$

$$\tilde{H} = \tilde{h} \tag{A2.g}$$

$$\tilde{U}_C = \tilde{\lambda} \tag{A2.h}$$

These long-run equilibrium equations jointly determine the steady-state values of C, H, L, h, P, r, r_h, and λ, denoted by the symbol ~. We can solve the system in a very simple recursive way. First, equation (A2.a) determines the value of in terms of β and τ. An increase in the marginal tax rate will increase the rate of return of government bonds. Equation (A2.b) yields $\tilde{r}_h$ in terms of β, τ, and δ. Once $\tilde{r}_h$ is obtained, equation (A2.f) will determine $\tilde{P}$. As before, a higher marginal tax rate raises the rate of return of the per capita stock of housing. Equations (A2.c), (A2.d) and (A2.e) can be solved jointly to determine the value of $\tilde{C}$, $\tilde{H}, \tilde{L}$. Replacing $\tilde{C}$ in (A2.h) the value of $\tilde{\lambda}$ is obtained. Once we know $\tilde{H}$, then $\tilde{h}$ immediately follows from (A2.g).

Appendix B

We simplify (4.a), (4.b) and (4.c) taking into account that:

$$\frac{\partial V}{\partial G} = \frac{\partial V}{\partial M}\frac{\partial M}{\partial G} = \gamma \tag{B1.a}$$

where $M = rb + G$ and $\frac{\partial V}{\partial M} = \gamma$

$$\frac{\partial \tilde{L}}{\partial G} = \frac{\partial \tilde{L}}{\partial M} \tag{B1.b}$$

$$\frac{\partial \tilde{H}}{\partial G} = \frac{\partial \tilde{H}}{\partial M} \tag{B1.c}$$

and use Roy's Identity

$$-\frac{\dfrac{\partial V}{\partial \tau}}{\dfrac{\partial V}{\partial M}} = w\tilde{L} + \tilde{r}b + \tilde{r}(1-\delta)\tilde{H}$$

Now, we define Z as: $Z = w\tilde{L} + \tilde{r}b + \tilde{r}(1-\delta)\tilde{H}$ so that:

$$-\frac{\partial V}{\partial \tau} = \gamma Z \tag{B1.d}$$

$$-\frac{\dfrac{\partial V}{\partial \tau(1-\delta)\tilde{r}}}{\dfrac{\partial V}{\partial M}} = \tilde{H}$$

$$\Leftrightarrow \frac{\partial V}{\partial \delta} = \frac{\partial V}{\partial \tau(1-\delta)\tilde{r}}\,\frac{\partial \tau(1-\delta)\tilde{r}}{\partial \delta} = \tau\tilde{r}\gamma\tilde{H} \tag{B1.e}$$

Finally we have the Slutsky equations:

$$\frac{\partial \tilde{L}}{\partial \tau} = -\,wS_{LL} - \mathrm{Z}\,\frac{\partial \tilde{L}}{\partial M} \tag{B1.f}$$

$$\frac{\partial \tilde{H}}{\partial \tau} = \tilde{r}(1-\delta)S_{HH} - \mathrm{Z}\,\frac{\partial \tilde{H}}{\partial M} \tag{B1.g}$$

where S_{LL} and S_{HH} are the substitution terms: S_{LL} is the compensated response of labor to the net wage and S_{HH} is the compensated response of housing demand to its net price.

We can rearrange (4.a), (4.b) and (4.c) taking account of these two equations, (B1.f) and (B1.g) and obtain:

$$\int_w^\infty \Big[\tilde{\Psi}' \frac{\partial V}{\partial G} + \mu(\tau[w\frac{\partial \tilde{L}}{\partial G} + \tilde{r}\,(1-\delta)\,\frac{\partial \tilde{H}}{\partial G}] - 1)\Big]\mathrm{d}F = 0$$

$$\int_w^\infty \Big[\tilde{\Psi}' \frac{\partial V}{\partial \tau} + \mu\big([w\tilde{L} + \tilde{r}\,(1-\delta)\tilde{H} + \tilde{r}b] + \tau[w\frac{\partial \tilde{L}}{\partial \tau} + \tilde{r}\,(1-\delta)\,\frac{\partial \tilde{H}}{\partial \tau}] + \\ + \tau\,\frac{\partial \tilde{r}}{\partial \tau}\,[b + (1-\delta)\tilde{H}] - \mathrm{b}\,\frac{\partial \tilde{r}}{\partial \tau}\big)\Big]\mathrm{d}F = 0$$

$$\int_w^\infty \Big[\tilde{\Psi}' \frac{\partial V}{\partial \delta} + \mu\big(\tau[w\frac{\partial \tilde{L}}{\partial \delta} + \tilde{r}\,(1-\delta)\,\frac{\partial \tilde{H}}{\partial \delta} - \tilde{r}\tilde{H}]\big)\Big]\mathrm{d}F = 0$$

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SUMMARY

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An Optimal Linear Income Tax with a Subsidy on Housing

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This paper generalizes the standard model of the optimal linear income tax to include a subsidy on housing. Unlike previous literature, we start from a dynamic equilibrium model and examine the steady state equilibrium. We then analyse first order conditions for our linear tax structure. Given the higher complexity of our model, some new efficiency effects appear, coming from both the marginal tax rate and the subsidy on housing.