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Trading Intensity and Intraday Volatility on the Prague Stock Exchange: Evidence from an Autoregressive Conditional Duration Model

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1. Introduction

The availability of both large data sets on ultra-high-frequency data and powerful computational devices that can process them have generated a new wave of interest in the theories of market microstructure. Underlying an extensive number of studies on the subject is a shared focus on the information implicit in market data as well as on the processes that translate this information into market prices. In this study, we focus on the information content implicit to the waiting times between market events, the bid-ask quotes.

As emphasized in the studies by Diamond and Verrecchia (1987), Easley and O'Hara (1992), and Easley et al. (1996), among others, the waiting times (or durations) between events such as trades, quote updates, and price changes play a key role in understanding the processing of private and public information in financial markets. These models – called information-based models of market microstructure – further extend the previous inventory-based models of market microstructure created by Garman (1976) and Stoll (1978), as well as other information-based models such as that of Glosten and Milgrom (1985).

The inventory-based models all apply to the same foundation that the uncertainties in order flow can result in inventory problems for the market maker. Due to the unpredictability of orders the demand and supply are not always balanced and hence, should an order be submitted immediately, it requires that the market makers who stand ready and waiting to trade the incoming orders are compensated in some way for bearing the costs. This compensation is reflected by the price for immediacy: the bid-ask spread. Consequently, in each inventory-based model where the market maker is risk averse (e.g., (Stoll, 1978)) the inventory introduces risks for the market maker who then adjusts his pricing strategy to reduce those risks at least partly.

The information-based models use asymmetric information to explain how even in competitive markets a bid-ask spread will exist; hence, they provide a significantly different point of view of the adjustment process of prices

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compared to the inventory-based models. In these models, three types of transactors are distinguished: the information-motivated transactors with superior information, the liquidity-motivated transactors without the superior information, and the transactors who believe they possess the information but have none. The market maker, who is in the middle of all trades, does not know with which type of transactor he trades with and will consistently lose money to the informed investors. To compensate for this loss, the market maker will need to constantly fix different buy and sell prices so that his buy price (the bid) is then smaller than his sell price (the ask). This way, the market-maker fixes his prices conditionally on the type of trade and, effectively, makes money from the liquidity-motivated transactors who are willing to pay the spread for immediacy. This explains why even in competitive markets a bid-ask spread will exist as long as there are information-motivated traders.

Intuitively one can see that since the information-motivated transactor has superior information, each trade can reveal a special information and thus have a signal value. Glosten and Milgrom (1985) were the first to incorporate the information that the trade itself can reveal in their own microstructure model. They showed that since the type of trade has signal value, following the trade the market maker will revise her beliefs based on what she has just learned from the trade outcome and set new trading prices.[1]

The study of Glosten and Milgrom, while important in its own right, served as an even more important basis for the model of Easley and O'Hara (1992). Unlike their predecessors who assume that the time does not matter to the price process, Easley and O'Hara focus on the role of time in price adjustment process.

They explicitly argue that in the markets characterized by the presence of traders with different levels of information, the duration between two trades conveys information and hence plays a plausible role in leading markets to price discovery. Specifically, their model implies that long durations are likely to be associated with no news, whereas short durations, and hence high trading activity, will most probably reveal the presence of asymmetric information in the market. In case of long durations the probability of dealing with an information-motivated trader is then small and hence the market-maker is to decrease the bid-ask spread. On the contrary, the release of news should lead to an increase in the intensity of trading and hence more frequent revisions of the bid-ask prices posted by the market-makers.

In (Diamond – Verrecchia, 1987), the notion of time as a signal is considered from the short-sale constraints point of view. The authors show that if short sale restrictions do indeed affect information-motivated trader's behavior, two results are possible. First, observing an absence of trade serves as a signal of bad news. Relatively long durations are then likely to be associated with bad news, inducing a negative re-evaluation of the asset value.

[1] Glosten and Milgrom (1985) use standard inferences of Bayesian learning to determine the bid and ask prices. They show that the bid-ask spread is determined by the simple fact that someone wishes to buy, as well as the nature of the underlying information, the number of informed traders, the trader's elasticity, and the trade size.

Consequently, long durations are more likely to appear when informed traders would sell the asset but short-sell constraints prevent them from doing so. Second, since prices adjust more slowly to information in their model, it takes them longer to reflect full information values.[2]

The primary objective of this study is to provide an empirical evidence that the intensity of transaction arrivals as measured and forecasted by the time between quotes carries information about the state of the market. Merging the trade and quote datasets and thus linking the price duration process to the characteristics of the trade process obtained over the price durations (i.e. the intensity of trading, the average volume per trade, and the average spread), we also provide evidence of the relevance of information-based models for the local stock market, the Prague Stock Exchange (PSE).

Building upon the data from the PSE, we extend the existing empirical literature on the high-frequency data which has been until now largely concentrated only on the Western stock markets. Still, this does not mean that the intraday behavior of Czech stock returns has never been studied before. Hanousek and Němeček (2002), for example, investigated the relationship between liquidity and information based trading and the possible impact of market microstructure changes on this relationship. Similarly, Hanousek and Podpiera (2003a) explored the impact of informed trading on the composition of the bid-ask spread from an information based point of view. However, as of today no study has appeared that would analyze specifically the transaction durations on the Czech stock market. Extending the existing empirical work on the local dealership market should be all the more relevant as the PSE has only recently become the largest stock exchange in Central and Eastern Europe by turnover and total market value.

In the analysis, we make use of the logarithmic autoregressive conditional duration (log-ACD) model as first introduced by Bauwens and Giot (2000) to model the durations between successive trades (quotes). The log-ACD model is more flexible than the original version of the ACD model developed by Engle and Russell (1998). The logarithmic version of the ACD model avoids the non-negativity constraints on the coefficients implied by the original specification and thus greatly facilitates the testing of market microstructure hypotheses put forward in our study.

The rest of the study is organized as follows. In Section 2 we provide a detailed description of the PSE, including an introduction into its structure and trading rules. In Section 3, we describe basic methodology and develop the estimation techniques employed in the analysis. In the section that follows, Section 4, we provide a general description of the dataset as well as the basic summary statistics for the price durations. The empirical results of the study are then included in Section 5. Section 6 summarizes the findings and concludes the study.

[2] As mentioned by O'Hara (1995), this hypothesis has been investigated before in several empirical studies by examining how markets with and without options adjust to information – e.g., (Jennings – Starks, 1985). The notion here is that options replace the inability to short sell and thereby should increase efficiency.

TABLE 1 Characteristics of PSE

	2000	2001	2002	2003	2004
Total Trade Value (CZK billions)	264,145	128,799	197,398	257,442	479,662
Main + Secondary Markets	259,564	124,053	181,281	238,114	450,332
Daily Average (bil. CZK)	1,060.8	515.2	789.6	1,025.7	1,903.4
Main + Secondary Markets	1,042.4	496.2	725.1	948.7	1,787.0
Trade Volume (mil. pieces)	822,911	546,544	804,105	830,771	1,179,107

Notes: Total trade value, average daily trade value, and total number of shares traded on Prague Stock Exchange during the years 2000–2004.
The years correspond to the period assessed later in the study.
Source: PSE

2. Prague Stock Exchange

Founded in 1992, the Prague Stock Exchange (PSE) (see *Table 1*) is now the leading securities market organizer in Central and Eastern Europe (CEE), covering more than 99 % of the total trade value in the Czech Republic and at times up to 50 % of the total trade value in the CEE countries. Securities registered on the PSE are traded on three markets: main, secondary, and free markets, where the main market is the most prestigious market on the Exchange.[3]

Only members of the PSE are allowed to trade directly on the stock exchange, either on their own account or on the account of their clients. Other persons can only trade indirectly through a member of the stock exchange.

2.1 Trading System

The Prague Stock Exchange is a fully electronic exchange with the trading based on automated processing of its member's orders and instructions for the purchase and sale of securities. Trading on the PSE is segmented into two distinct sub-systems with distinct prices:[4]

– (a) quote driven system (referred to simply as SPAD), and
– (b) order driven system (described by automatic trades).

In general, PSE members send electronic buy or sell instructions to the PSE and if conditions for matching opposite instructions within the above subsystems are met, a trade gets immediately recorded. In addition, PSE members must also report their over-the-counter (OTC) trades concluded without direct usage of either of the above price-determining mechanisms (usually over the phone). OTC trades must be registered with the PSE in order to preserve transparency of prices on the market.[5]

Dissemination of trading information is provided in real time to all participants of the market; namely, the information is immediately sent to

[3] The main market has the most stringent conditions regarding the admission of securities to trading in the market.

[4] This does not mean that the same instruments cannot be traded in either of the two sub-systems.

TABLE 2 SPAD Issues

Issue	ISIN	Std Qt	Max Spread	Al Qt
Cesky Telecom	CZ0009093209	5,000	6	95,000
CEZ	CZ0005112300	10,000	2	100,000
Erste Bank	AT0000652011	2,000	8	34,000
Komercni banka	CZ0008019106	1,000	20	11,000
Orco Property Group	LU0122624777	500	10	31,000
Philip Morris CR	CS0000841869	100	200	2,200
Unipetrol	CZ0009091500	10,000	3	300,000
Zentiva	NL0000405173	3,000	8	51,000

Notes: Parameters of issues in SPAD as of April 5, 2005.
Values for *Std Qt* (Standard Quantity) and *AL Qt* (Above-Limited Quantity) are in pieces.

Source: PSE

the members of the PSE, the Czech Securities Commission (CSC), and the data vendors mentioned. It is also available for free on the PSE's internet pages (www.pse.cz). Settlement of all trades in the PSE database of trades occurs through UNIVYC, a 100% subsidiary of the PSE used for clearing. Trading at the PSE generally conforms to the T+3 standard settlement cycle. Trades in the SPAD system and automatic trades are settled in T+3 by UNIVYC. Block trades can be settled in a period from T+0 to T+15, and guarantees by the PSE's Guarantee Fund do not apply.

Each of the above-mentioned trading subsystems operates on its own timetable. Trading through the PSE occurs only during open phase hours. However, even outside open trading session hours, PSE members are still required to report their OTC trades. The time period after 17:00 technically belongs to the following business day; therefore, all OTC trades concluded from 17:00 to 20:00 and registered within the PSE database of trades are designated as belonging to the following trading day rather than to the day just ended at 16:00.

2.2 Stock and Bond Market Support System (SPAD)

The Stock and Bond Market Support System (SPAD) is a price-driven trading system based on the activity of market makers. As already mentioned, the whole system is screen-based. This way, all the market makers as well as other members of PSE can see all the quotes and trades. Today, the system accommodates eight of the most liquid Czech securities (called blue-chips) supported by ten market makers.[6]

[5] All trades (both those concluded on the PSE within the two subsystems and OTC trades registered with the PSE) enter a central PSE database of trades that were concluded on a given day and are immediately published on anonymous basis (that is, no name of the involved members is visible) to PSE members and to external data vendors such as Bloomberg or Reuters. Daily summaries of trades are published, too.

[6] Hanousek and Podpiera (2003b) studied the functioning of SPAD since its launch in 1998. They provide evidence that the new system has succeeded in increasing the transparency of the market and that it has improved the price discovery function of the exchange by attracting a large portion of order-flow to the main market.

The SPAD system operates in two phases: an open phase and a closed phase. During an open phase (from 9:30 to 16:00 CET), all market makers are obliged to publish their quotations (buying and selling prices) for issues for which they act as market makers. As the actual trading occurs during this period only, we assume just this phase as directly relevant to our study. We will describe the closed phase later.

A member of the PSE who supports trading in assigned securities traded in SPAD and thus increases liquidity of the securities within SPAD is called a market maker. For each security in SPAD, there has to be a minimum of three market makers[7] quoting prices. Each market maker is required to quote a buy and a sell price at all times during the open phase for a standardized number of shares (a round lot or its multiple) to be delivered on a T+3 basis. The Trading Committee also sets a maximum bid-offer spread for quotes of the same market maker to bring prices of purchases and sales closer.

All quotes of all market makers take the form of a buy or sell instruction sent to the PSE and are immediately displayed via electronic means to all PSE members. Any member can immediately take the best quote displayed within SPAD, resulting in instructions being matched and a trade being recorded and published. Quotes worse than the current best quotes within SPAD are informative only. Once a given market maker's quote becomes the best bid or offer available within SPAD (so called best quote), such quote becomes binding for that market maker. This means that the market maker is obliged to conclude a trade at such bid or offer price quoted should any other PSE member choose to accept it. On the other hand, the market maker is free to adjust his quotes up or down based on its assessment of demand and supply in the stock at any time. Once the best quote is accepted and a trade is thus recorded, the respective market maker is allowed up to a 3-minute recovery period to recalculate its positions and reset quotes. The second best indicative quote of another market maker immediately becomes the best quote and is binding from that moment on, until that quote is accepted too, or until a better quote appears within SPAD.

During the open phase, PSE members are allowed to conclude OTC trades with respect to SPAD securities with each other (OTC SPAD trade) only within a narrow price range often referred to as a permitted range. The permitted range is constantly changing as market makers publish their quotes and it is defined as an up-to-the-minute price range limited by the price 0.5 % below the best SPAD bid at the bottom and by the price 0.5 % above the best SPAD offer at the top. However, OTC SPAD trades with large blocks of shares with total value larger than CZK 40 million can be registered with any price even outside the current permitted range. In addition, OTC SPAD trades must be reported to the PSE within a 5-minute deadline after such trade is concluded in order to preserve market transparency. The reason for mandatory reporting of OTC SPAD trades is to direct PSE members to SPAD and curb OTC trading among them, as OTC is considered less transparent for the market.

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[7] Non-market-making members are subject to limitations as to their daily maximum trading volume with the market makers, as a protection measure for market makers against excessive settlement risks.

The opening price of an instrument traded in SPAD is equal to the midpoint of the permitted range as of the start of the open phase; in other words, it is calculated based on the quotes of market makers at the start of the session. The closing price of an instrument traded in SPAD is equal to the midpoint of the permitted range at the close of the open phase (16:00 CET). If during the open phase the arithmetic midpoint of the up-to-the-minute permitted range deviates by more than 20 % from the arithmetic midpoint of the permitted range as of the start of the open phase, all quotes by the market makers become informative only, including the best quotes. Naturally, trades already concluded are not affected by this rule. The reason for this rule is to protect market makers against excessive losses once dramatic price moves occur.

Closed phase stands for the period between two open phases (or, more exactly, from 16:00 to 17:00 and from 7:30 to 9:30 CET). During the closed phase, market makers are not obliged to quote any prices. Technically, the closed phase allows the market makers to clear the trades that they did not manage to conduct during the preceding (open) phase. However, over-the-counter SPAD trades must still be reported (and get published), although the reporting limitations are softer than during an open phase.

The permitted range is a 5% wrap around the best quotes at the close of the last open phase, compared to a 0.5% wrap during an open phase. All OTC SPAD Trades during a Closed trading session must be reported before a new open phase begins, compared to a 5-minute reporting time limit during an open phase. Unmatched OTC instructions sent to the PSE for registration are automatically cancelled before an open phase begins.

2.3 Automatic Trades

Automatic trades occur in an order-driven system in which trades are concluded on the basis of matching orders for the purchase and sale of securities in a PSE order book, as entered via electronic means into relevant PSE subsystem by member firms. Matching of orders takes place under two regimes: the auction regime (with price priority queuing), followed immediately by the continuous regime (queue priority: price first, then order entry time).

Due to different price defining strategies, securities within the SPAD subsystem can have different prices from those within the automatic trades subsystem, i.e., there is no direct technical interaction between the two subsystems as to prices. Trading hours for automatic trades are from 7:30 through 15:45. We will not describe the automatic trade system any further, as the transactions that take place in it are not directly relevant to our analysis.

2.4 Block Trades

Members of the PSE must report not only OTC trades between them but also trades they make with non-PSE members. The latter are generally referred to as block trades and must be registered within one hour after such trades are concluded (the PSE system is open for registration between 7:30 until 20:00 CET). No price limitations apply in this case (except for the OTC SPAD trade limitations described above). For all OTC trades, members are free to agree on a settlement different from the T+3 standard (including OTC SPAD trades) within the range of T+0 to T+15 (T+1 to T+15 in case of OTC SPAD trades).

OTC trades registered with the PSE are regarded as a tool to protect market transparency rather than as a special trading subsystem of its own. Thus, through the OTC trades, PSE members effectively engage in "off-exchange" transactions with securities listed on the PSE, meaning that these transactions are concluded outside the PSE price-setting systems. For transparency reasons, these transactions are reported to the PSE and are registered in the relevant PSE databases.

3. Methodology

3.1 The ACD Model

In this study we focus on the autoregressive conditional duration model (ACD) originally introduced in (Engle – Russel, 1998). This time-series model is suitable for analysis of high-frequency financial data because it allows for explicit modeling of irregularly spaced events. The model treats the time between events as random and the arrival of events is governed by a point process with dependent arrival rates. It can therefore capture clustering of events, which is observed empirically, e.g. that short duration between consecutive trades tends to be followed by short duration and similarly for large durations. The ACD model can also accommodate exogenous explanatory variables thereby providing a framework for testing various market microstructure hypotheses. Finally, as shown in (Giot, 1999), when events are properly defined, the ACD model can be a powerful alternative to the well-know GARCH model.

Underlying the ACD model is a conditional intensity process defined by:

$$\lambda\left(t \mid N(t), t_1, ..., t_{N(t)}\right) = \lim_{\Delta t \to 0} \frac{P\left(N(t + \Delta t) > N(t) \mid N(t), t_1, ..., t_{N(t)}\right)}{\Delta t} \qquad (1)$$

where $\{t_0, t_1, t_2, ..., t_n, ...\}$ with $t_0 < t_1 < ... < t_n < ...$ is a sequence of times and $N(t)$ is the associated counting function which is the number of events that have occurred by time t. The conditional intensity is, loosely speaking, the probability that an event occurs in the short interval $t + \Delta t$ given that it has not occurred before t. Let $x_i = (t_i - t_{i-1})$ denote the interval between the $(I - 1)$th and i events, which is called duration. Then the ACD model can be specified in terms of the distribution of x_i. In particular, the expectation of the ith duration, denoted by ψ_i, is given by:

$$\psi_i = \psi_i\left(x_{i-1}, ..., x_1; \theta\right) = E\left[x_i \mid x_{i-1}, ..., x_1\right] \qquad (2)$$

The ith duration itself is then given multiplicatively as $x_i = \psi_i\ \varepsilon_i$, where ε_i is $i.i.d.$ with density $p(\varepsilon,\varphi)$, and θ and φ are variation free parameters. To derive the conditional intensity of the ACD model, define the baseline hazard as $\lambda_0 = p_0(t) / S_0(t)$, where $p_0(t)$ is the density of ε and $S_0(t) = 1 - (\int_0^t p_0(s)ds)$ is the associated survival function. Then the conditional intensity of the ACD model is expressed generally as:

$$\lambda\left(t \mid N(t), t_1 \ldots, t_{N(t)}\right) = \lambda_0\left(\frac{t - t_{N(t)}}{\psi_{N(t)+1}}\right)\frac{1}{\psi_{N(t)+1}} \tag{3}$$

Equations (2) and (3), and the fact that $x_i = \psi_i \varepsilon_i$, describe the ACD class of models in general. The most common specification found in the literature assume that the expected conditional duration is given by:

$$\psi_i = \omega + \sum_{j=1}^{m} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j} + \gamma^T \mathbf{z}_i \tag{4}$$

which is called the ACD(m, q) model. In this model, expected duration depends on past m durations, past q expected durations and a vector of non-negative exogenous variables $\mathbf{z}_i$. Since durations are by definition non-negative we require that $\omega > 0$, $\alpha_j \geq 0$, $\beta_j \geq 0$ for all j and $\gamma_j \geq 0$ for all i. Stationarity requires that $\Sigma(\alpha_j + \beta_j) < 1$ and the unconditional mean of durations is:

$$E[x_i] = \frac{\omega + \gamma^T E[\mathbf{z}_i]}{1 - \Sigma(\alpha_j + \beta_j)} \tag{5}$$

The specification in (4) is limited by the non-negativity restrictions and the use of positive exogenous variables. Bauwens and Giot (1997) introduce a logarithmic ACD model where:

$$\log(\psi_i) = \omega + \sum_{j=1}^{m} \alpha_j \log(x_{i-j}) + \sum_{j=1}^{q} \beta_j \log(\psi_{i-j}) + \gamma^T \mathbf{z}_i \tag{6}$$

In this equation for ψ_i non-negativity of duration is ensured regardless of parameter values. In the empirical part of the study, we focus on the log-ACD model (6) because it is more suitable for testing market microstructure hypotheses. One such hypothesis is that bid-ask spread is negatively related to expected duration due to the presence of informed traders.[8] Since the bid-ask spread is non-negative we expect the partial effect γ to be negative. But if equation (4) were used, large bid-ask spreads might, *ceteris paribus*, predict negative durations which is inconsistent.

There is a number of possible specifications for the density of ε, the most common are exponential, Weibull, generalized gamma and Burr.[9] The choice of $p_0(t)$, of course, affects the shape of the conditional intensity. For exponential distribution, the hazard function is constant, and thus for given expected duration the probability that an event occurs in an interval $(t + \Delta t)$ given that it has not occurred by t is constant in t. The Weibull distribution

[8] We discuss this effect in greater detail in the later part of the study.

[9] See e.g., (Engle – Russel, 1998) for the exponential and Weibull models, (Lunde, 1999) for the generalized gamma model and (Grammig – Maurer, 2000) for the Burr model.

exhibits monotonic hazard function and hence $\lambda\ (t \mid N(t), t_1, ..., t_{N(t)})$ is either increasing or decreasing in t depending upon the value of the parameter of the Weibull distribution. The most flexible choice is the generalized gamma of Burr distribution which both have hazard functions that can be either constant, monotonic or U-shaped, depending upon the parameters.[10] Using these distributions we can therefore model situations where the conditional probability of an event occurring first increases in t, but eventually reaches a maximum and starts to decline. In this study, we focus on the exponential model.

As shown in many empirical studies[11], transactions durations exhibit significant diurnal patterns. The deterministic component of expected durations can be either estimated jointly with the ACD model or it can be first removed from raw durations and then the ACD model can be fitted to the seasonally adjusted durations. In this study, we take the second approach. We write the raw duration as $x_i = \varphi_i(t)\ \psi_i\ \varepsilon_i$, and assume that the seasonal component $\varphi_i(t)$ can be approximated by cubic splines

$$\varphi_i(t_i) = \sum_{j=1}^{K} I_j[c_j + d_{1j}\,(t_i - k_{j-1}) + d_{2j}\ (t_i - k_{j-1})^2 + d_{3j}\ (t_i - k_{j-1})^3] \qquad (7)$$

where I_j is an indicator for the j-th segment of the spline, i.e. $I_j = 1$ if $t_i \in (k_{j-1}, k_j)$ and zero otherwise, and the parameters c_j, d_{1j}, d_{2j}, d_{3j}, where $j = 1, ..., K$, are restricted by the usual differentiability conditions. We set the nodes $(k_0, ..., k_K)$ at 9:30, 10:30, 11:30, 12:30, 13:30, 14:30, 15:30 and 16:00. The reason for having two nodes over the last trading hour is to allow for more flexibility during the period of overlapping trading with the U.S. market. The cubic splines are estimated by ordinary least squares and the raw durations are standardized according to $\widetilde{x}_i = x_i\ /\ \hat{\varphi}\ (t_i)$.

Using the Berndt-Hall-Hall-Hausman (1974) algorithm, we will estimate the ACD(m, q) model for diurnally adjusted durations $(\widetilde{x}_i)$ by the method of maximum likelihood. As shown in Gourieroux, Monfort and Trognon (1984), if the conditional model is properly specified the ML parameter estimators are consistent if and only if the distribution used to obtain them belongs to the linear exponential family, regardless of the true density. Since the exponential distribution belongs to this family, we can interpret the ML estimates of the exponential model as quasi maximum likelihood estimates (QMLE). This is, however, not the case with generalized gamma distribution. We thus face the common trade-off between consistency and efficiency when estimating the generalized gamma model.[12]

3.2 Price Durations and Volatility

Besides transactions durations we will also investigate the behavior of price durations which are defined as duration between events for which

[10] Both generalized gamma and Burr distribution nest the exponential and Weibull distributions as special cases.

[11] See e.g., (Engle – Russel, 1998), (Bauwens – Giot, 1997), and the references therein.

[12] Gramming and Maurer (2000) show in a Monte Carlo analysis that a misspecification in the error distribution can severely deteriorate the accuracy of duration forecast.

price has changed. With each transactions time is associated a price and such process is called market point process with price being the mark. By selecting only those points for which price has changed we perform thinning of the point process for transaction arrival times and obtain a point process for price arrival times. To avoid small price changes caused by recording or quoting errors and the impact of large trades that temporarily move the price by a small amount, we consider as a price change a movement in the mid-price by at least c, a constant. Then the probability that the price changes by at least c in an interval Δt is by equation (1) $\lambda\,(t \mid t_i, \ldots, t_0)\,\Delta t + o(\Delta t)$ and otherwise there is no change.

As Engle and Russell (1998) demonstrate, by applying the ACD model to the price durations, we obtain a model for the inverse of volatility. To see this, define the instantaneous volatility as:

$$\sigma^2\,(t) = \lim_{\Delta t \to 0} E\left\{\frac{1}{\Delta t}\left[\frac{P(t+\Delta t) - P(t)}{P(t)}\right]^2\right\} \tag{8}$$

Substituting $c(\Delta t)$ for $P(t{+}\Delta t) - P(t)$ in (8) and taking limits we obtain a model for the expected instantaneous conditional volatility per second given by:

$$\sigma^2\,(t \mid t_{N(t)}, \ldots, t_1) = \left(\frac{c}{P(t)}\right)^2 \lambda\,(t \mid t_{N(t)}, \ldots, t_1) \tag{9}$$

The relation between instantaneous volatility and price duration is simple: in a given time period, high price intensity $\lambda\,(t \mid t_{N(t)}, \ldots, t_1)$ implies high number of transactions with price change of at least c, and by equation (9) high volatility. High number of transactions is associated with short price durations and thus there is an inverse relationship between price durations and volatility. Since the ACD model can capture the clustering of durations, it can therefore also capture volatility in the clustering, which has been observed ever since the advent of the ARCH model.

4. Data Description

We base the analysis on the data from the Prague Stock Exchange (PSE). Similarly to other data of this kind, the data come in two datasets: one that carries the intra-day trade data and one that consists of the intra-day quotes data. The former set contains a detailed record of every single trade in a specific stock, including the time and the price at which the trade took place, the amount of the stock traded, as well as other information relevant to the trade such as type of trade. The latter set includes the time of the quote posted, the corresponding bid and ask prices, and the depth – that is, how many shares the market maker is willing to buy/sell at the given bid/ask quote. Both datasets contain the information for all stocks traded on PSE's main market (SPAD) from January 2000 to November 2004.

Of the four years of data available, we select the most recent year for the analysis. The period assessed then begins with the first trading day of 2004 (January 5) and ends with the last day the data were available for

TABLE 3 General Statistics

	CEZ	KB	Telecom
– Average price	323.1	2.884,7	206.8
# of *trade* observations	14,297	20,133	15,615
– Open phase*	14,092	19,896	15,390
# of *quote* observations	42,004	49,534	26,215
– Open phase*	41,763	49,273	25,980
– W/out multiple record	39,609	46,424	24,846

Notes: The data summary data contains the information on the number of trade and quote observations for CEZ, Komercni banka (KB), and Cesky Telecom (Telecom) from January 5 to November 11, 2004.
Open phase shows number of transactions during open phase (from 9:30 to 16:00).
In case of quote observations, the line *w/out multiple records* relates to number of transactions ignoring overnight duration and without simultaneous trades.

that year (November 11). The 218 day trading sample is long enough to allow reasonably precise estimations (see (Easley – Kiefer – O'Hara, 1993)). We work with three securities: two non-financial (CEZ, Cesky Telecom) and one financial (Komercni banka). These stocks were the most actively traded of the total of eight titles present on SPAD in 2004.

Summary statistics for the quote data for CEZ, Komercni banka (KB), and Cesky Telecom (Telecom) from January 5 to November 11, 2004. The number (#) of price durations shows the number of durations after thinning the original set of quote observations.

Two days were deleted from the 218 day trading sample. A halt occurred on Tuesday, May 16, when the trading in SPAD was interrupted for nearly three hours due to technical error. Another interruption took place in case of Komercni banka on Friday, June 10, when the trading was postponed for four hours owing to several wrong order submissions. After deleting these two days from each of the two datasets, we adjust the data as follows. First, by focusing solely on open trading hours (from 9:30 to 16:00), we remove the overnight transactions. This way, we effectively ignore the overnight duration and treat the data consecutively from day to day.[13] Furthermore, in case of quote data, we only consider unique quotation times and hence regard the simultaneously recorded quotes as a single quotation. Despite these adjustments the sample remains very large. For CEZ, we are left with 14,092 open-phase observations on trades and 39,609 unique open-phase observations on quotes. Komercni banka (KB) and Cesky Telecom (Telecom) show much higher number of trades versus quotes, as it appears from *Table 3*.

4.1 Price Durations

In order to estimate the model for the price process and ultimately test the market microstructure hypotheses discussed in introduction, we also

[13] To be specific, if the last transaction on day j takes place at 15:55:40, and the first transaction of the next day at 9:30:35, the duration between these two events is not used, so that the first duration for day $(j + 1)$ will effectively be the one between 9:30:35 and the next transaction.

TABLE 4 Summary Statistics

	CEZ	KB	Telecom
# of adjusted quotes	39,609	46,424	24,846
# of price durations	7,728	4,901	7,001
Thinning (in %)	19.51	10.56	28.17
c (CZK)	0.25	5.00	0.50
Mean	500.2 (0.99)	671.9 (0.99)	520.6 (0.99)
Std. Deviation	1,240.5 (1.98)	1,429.2 (1.64)	1,252.8 (2.07)
Overdispersion	2.48 (2.01)	2.14 (1.66)	2.41 (2.09)
Minimum	1 (0.001)	1 (0.001)	1 (0.001)
Maximum	18,778 (32.22)	16,648 (21.67)	21,439 (37.99)
Q(10)	594.3 (1,047.6)	171.4 (299.1)	428.1 (586.9)

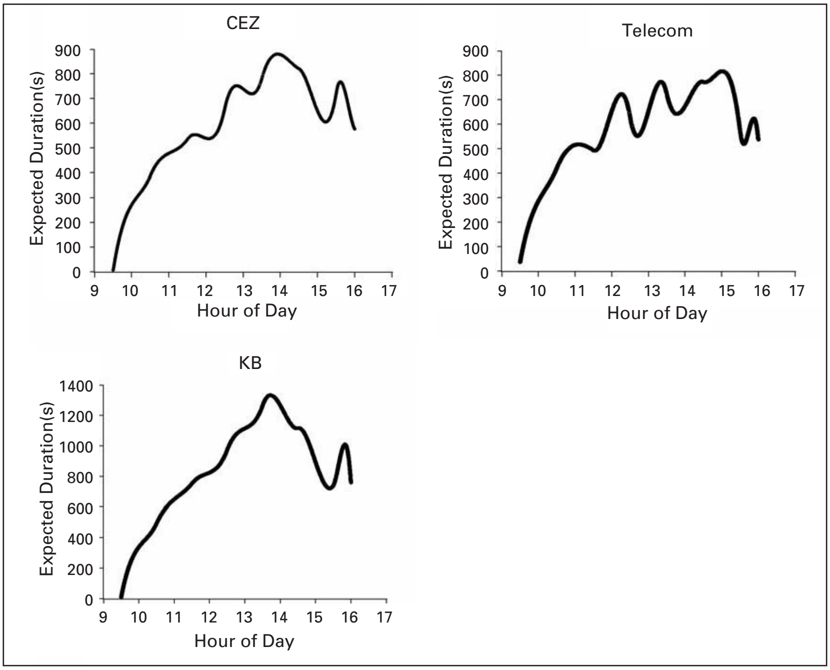
Notes: Summary statistics for the quote data for CEZ, Komercni banka (KB), and Cesky Telecom (Telecom) from January 5 to November 11, 2004.
The number (#) of price durations shows the number of durations after thinning the original set of quote observations.

compute the price durations for each of the three stocks under analysis. As described in the previous section, we define price durations by filtering the bid-ask quote durations and retaining only those leading to a significant cumulated change in the midpoint (or midprice, defined as $p_i = (\text{bid}_i + \text{ask}_i)/2$) of the bid-ask quotes. Specifically, in our study we calculate a significant cumulated change in the midprice as a change leading to at least a CZK 0.25 cumulated change in the midprice for CEZ, CZK 5.00 for KB, and CZK 0.50 for Cesky Telecom. Illustrating the calculation on example of CEZ, a cumulated change of CZK 0.25 may result from two successive positive changes of CZK 0.125, just as it can follow from four successive changes of CZK 0.125 with the first two going in opposite directions and the next two in the same direction.

Defining the price durations on the midpoint of the bid-ask quotes and thinning the process with respect to a given cumulative change allows us to eliminate the problem of "bid-ask" bounce, an inherent feature of quote transactions data that is nevertheless annoying to work with as it gives relatively little information. Thinning the process by effectively filtering the numerous small changes as in the example with CEZ (CZK 0.125 changes) can be justified on the assumption of the transitory nature of such changes. In fact, Biais, Hillion and Spatt (1995) show that the bid-ask quote process is often characterized by information events that lead to similar (successive) changes in the quotes and thus move significantly the midpoint. In addition, thinning the bid-ask quote process also allows us to avoid small price changes caused by recording or quoting errors and the impact of large trades that temporarily move the price by a small amount, effectively extracting a process where only meaningful price changes are retained in the end.

In *Table 4*, we provide characteristics of price durations computed using different thresholds. In case of CEZ, the sample size is reduced to 7,728 after thinning with threshold of CZK 0.25. This corresponds to about 19.5 % of the size of the original sample. The mean duration is 500 seconds (or

FIGURE 1 Diurnal Components for the Price Durations of CEZ, Price Thinning Threshold of c = CZK 0.25, Cesky Telecom (Telecom), Threshold of c = CZK 0.25, and Komercni banka (KB), Threshold of c = CZK 5.00



Note: The durations are based on the sample period January 5, 2004 to November 11, 2004.

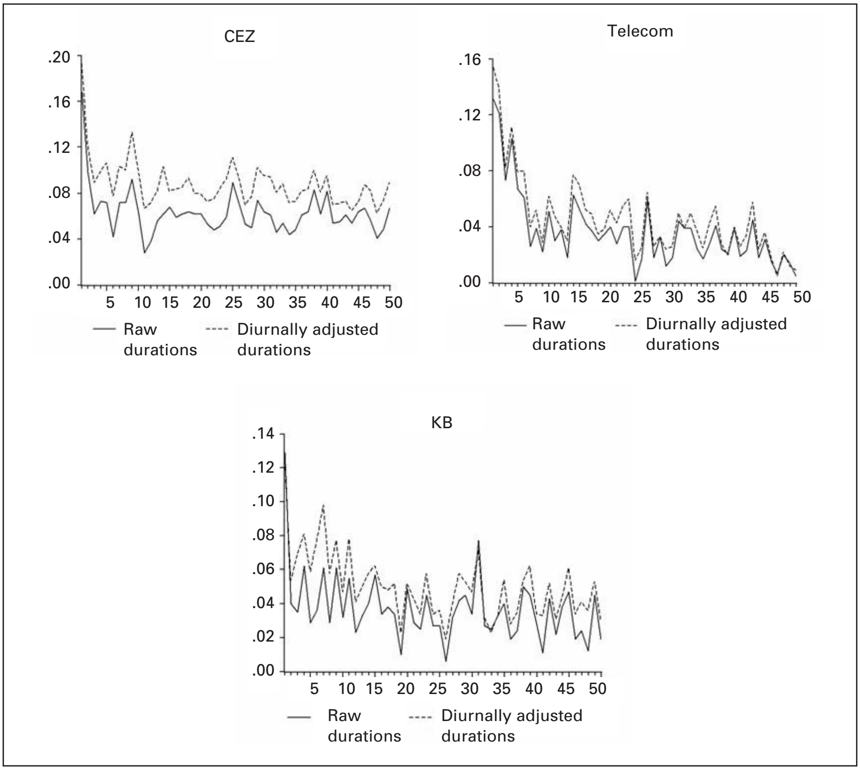
5.3 minutes) with a standard deviation of 1,240.5. The minimum price duration is 1 second while the maximum is 18,778 (5 h 13 min). A relatively high value for overdispersion (2.48) as well as strong autocorrelation associated with CEZ's price durations are both characteristic of duration data filtered at small thresholds. Increasing the relative threshold[14], the values for overdispersion as well as the Ljung-Box statistics for the first ten autocorrelations on price durations become smaller.

The summary statistics are very similar for other two titles, Komercni banka (KB) and Cesky Telecom (Telecom). As in the case of CEZ, for each of these two stocks we choose the price thinning thresholds based on the properties of the underlying stock (i.e., price, median spread, mean, tick size) as well as with regard to consistency of results in the sample (i.e., similar percentage of thinning and overdispersion).

As explained in the previous section, transactions durations exhibit significant diurnal patterns. Price durations – which effectively define an intraday volatility process – are not different in this regard. Indeed, as discussed by McInish and Wood (1992), volatility tends to be systematically

[14] We have examined five different thresholds for CEZ and Cesky Telecom (with c = CZK 0.25, 0.50, 1.00, 1.50, and 2.00) as well as for Komercni banka (with c = CZK 2.5, 5.0, 10.0, 15.0, and 20.0). The results are available on request.

FIGURE 2 Autocorrelations Functions for the Raw and Diurnally Adjusted Price Durations of CEZ, Price Thinning Threshold of c = CZK 0.25, Cesky Telecom (Telecom), Threshold of c = CZK 0.25, and Komercni banka (KB), Threshold of c = CZK 5.00



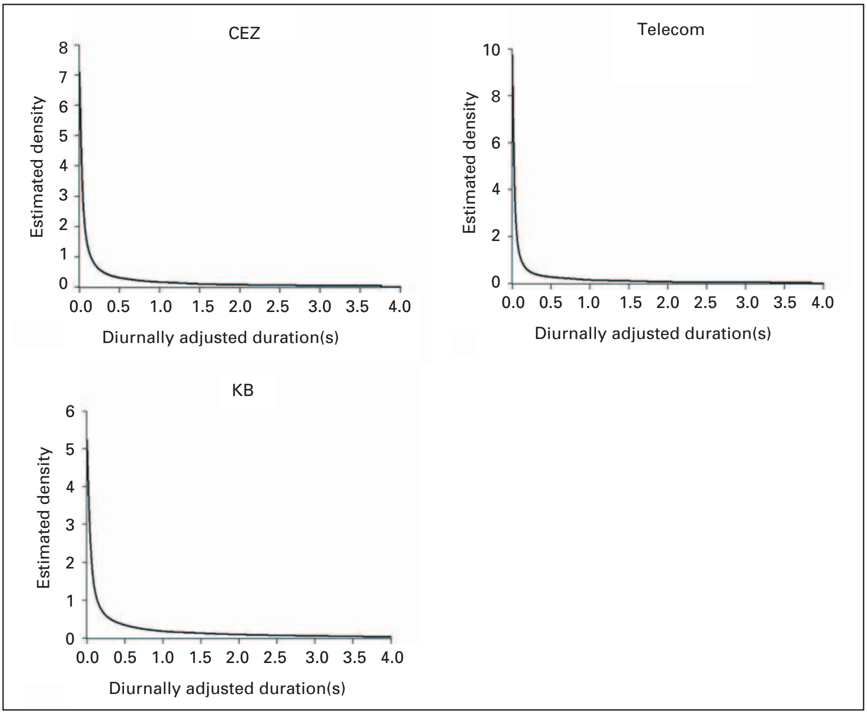
Note: The price durations are based on the sample period January 5, 2004 to November 11, 2004.

higher near the market's open and generally just prior to the market's close. With this in mind, we transform the raw price durations prior to estimation by removing the (deterministic) intraday seasonality.

Figure 1 plots the estimated diurnal components for the price durations as functions of time of the day, where the vertical axis is measured in seconds. The well known inverted *U*-shape is clearly apparent. The opening of the market (9:30 h) is very active with price quotations occurring, on average, every 10 to 20 seconds.[15] The middle of the day tends to be less volatile with peaks in the expected durations being most pronounced just before 14:00 h for CEZ and KB and 12:00 h and 13:00 h for Telecom. At these periods, the quotations tend to occur least frequently, with the durations between quotations of just over 15 minutes (900 s) for CEZ and 23 minutes (1,400 s) for KB. We may notice an important jump in the activity just before the close (16:00 h) corresponding to opening of U.S. markets at 9:00 h in

[15] The plot of expected price durations in Figure 1 seems to start at a zero expected duration. Given the size of the picture, however, this is only an optical illusion.

FIGURE 3 Density Functions of Diurnally Adjusted Price Durations for CEZ, Price Thinning Threshold of c = CZK 0.25, Cesky Telecom (Telecom), Threshold of c = CZK 0.25, and Komercni banka (KB), Threshold of c = CZK 5.00



Note: The durations are based on the sample period January 5, 2004 to November 11, 2004.

the morning. In short, we observe the pattern is very similar for all three stocks. It should be noted, however, that the plots in Figure 1 are merely for illustrating a general pattern, and that the diurnal patterns may differ quite strongly from day to day as well as from week to week.

Figure 2 plots the autocorrelation functions for each of the three stocks. The ACFs are presented for both raw and diurnally adjusted durations. The series exhibit long sets of positive autocorrelation spanning many quotes even after the deterministic component has been removed. These autocorrelations indicate clustering of durations.

Figure 3 plots the estimated density functions of diurnally adjusted price durations. The estimates are obtained using gamma kernel with the bandwidth set to the rule-of-thumb value proposed by Chen (2000). The empirical densities exhibit substantial overdispersion, which is a finding similar to that of Engle and Russel (1998) and Bauwens and Giot (2000). This does not imply, however, that standardized durations need not be $exp(1)$ as we assume for the purposes of the maximum likelihood estimation. In any case, as mentioned above, the MLE estimator based on the $exp(1)$ hypothesis is consistent regardless of the true density so we retain this assumption in the empirical applications.

4.2 Additional Explanatory Variables

In order to test the hypotheses of market microstructure, we need to link the price durations just discussed to characteristics of the trade process. As explained by Bauwens and Giot (2000), with price durations this becomes possible as they are relatively "long" with respect to trade durations and thus allow for the definition of market characteristics for the trading process over the price durations. We focus on three variables related to characteristics of the trade process as suggested by information-based models discussed in the introduction: a) measure of the intensity of trading, b) average volume per trade, and c) existing spread when the past trades were made. All of these variables can be computed relatively easily based on the information contained in the merged trade and quote datasets. We shortly describe each of them separately in the following paragraphs.

Over each price duration, a measure of *trading intensity* is constructed using the number of trade transactions (or simply trades) per second recorded over each price duration divided by the length of the duration. This way, a small number of trades over a long duration leads to a low trading intensity and *vice versa*. According to the information-based model due to Easley and O'Hara (1992), discussed in more detail in the introduction, an increase in the intensity of trading should lead to more frequent quote revisions. Put differently, according to the model the number of transactions would influence the price process through information based clustering of transactions. Nevertheless, a different model by Admati and Pfleiderer (1988) predicts that the number of transactions would have no impact on the intensity of trading. While casual evidence may be ambiguous, using the variable just discussed we are able to test the hypothesis that the number of transactions would influence the price process through information based clustering of transactions statistically.

Further evidence about the validity of predictions of numerous information-based microstructure models can be found using *average volume per trade* variable. Defined as the average, over previous price duration, of the volume of the trades made during that duration, the average volume per trade is indicative of possible informed trading as shown in the model of Easley and O'Hara (1992). The important role of volume has been highlighted in several other studies as well (e.g. (Easley – Kiefer – O'Hara, 1997)). Still, in all cases the volume is found to exhibit an informational content that is not contained in the price process.

The last variable that we include in the regressions is the *spread*. Defined as the average spread, over previous price duration, corresponding to the trades made during that duration, a high spread is once again indicative of possible informed trading and should be linked to short durations. In addition, the asymmetric information models (e.g., (Glosten – Milgrom, 1985)) generally predict that spreads tend to become wider as the probability increases of informed agent trading. As a particular kind of such situations, we can imagine a specialist reacting to transaction clustering that results from information-based trading. In such cases, the specialist can set narrow spreads when the fraction of informed traders is small and *vice versa* when the fraction of traders is large.

TABLE 5 Estimated Price Models for CEZ

	Model 1	Model 2	Model 3	Model 4	Model 5
ω	0.024 (22.03)	0.024 (20.03)	0.024 (21.88)	0.031 (18.17)	0.034 (17.74)
α_1	0.202 (65.21)	0.202 (65.17)	0.203 (64.95)	0.203 (65.05)	0.203 (64.66)
α_2	–0.185 (–61.57)	–0.185 (–61.47)	–0.185 (–61.39)	–0.184 (–60.81)	–0.183 (–59.66)
β_1	1.187 (84.02)	1.187 (83.55)	1.192 (84.53)	1.172 (80.74)	1.166 (78.21)
β_2	–0.206 (–15.15)	–0.206 (–15.08)	–0.211 (–15.56)	–0.194 (–13.90)	–0.190 (–13.43)
y_1		0.001 (1.11)			0.003 (3.82)
y_2			0.001 (3.71)		0.002 (5.86)
y_3				–0.005 (–5.11)	–0.009 (–7.59)
$Q(10)$	12.61	12.55	12.03	12.39	10.85

Notes: Coefficient estimates and t-statistics (in parentheses) for estimated transaction duration model (model 1) and price duration models (model 2 to 5) for CEZ.
Coefficient y_1 corresponds to first explanatory variable (intensity of trading), y_2 to average volume per trade, and y_3 to spread.
Robust t-statistics are given in parentheses.
$Q(10)$ is the Ljung-Box Q-statistics for serial correlation in standardized residuals for up to ten lags.

5. Empirical Results

The log-ACD model is used as the base model in our analysis. We use *log*--ACD (2,2) model for CEZ and Cesky Telecom (Telecom) and *log*-ACD (3,2) model for Komercni banka (KB). In each case, we use the same model to estimate the price process without explanatory variables (Model 1) as well as with additional explanatory variables related to microstructure hypotheses (Models 2 to 4). In the next paragraph, we first describe the results for Model 1. Subsequently, we turn our attention to the price models with additional variables.

For CEZ, the estimated coefficients and t-statistics for the transaction duration model (Model 1) are presented in *Table 5*. The parameters for the stochastic factor are all highly significant. The sum of coefficients on lagged durations (α_i) and lagged conditional durations (β_i) is 0.998, indicating the duration process has strong persistence as measured in transaction time. Similar results can be found for KB (*Table 6*) and Telecom (*Table 7*). The generally positive values of β_1's – ranging from close to 1.2 for CEZ to 1.7 in case of KB – point out a positive impact of lag-1 conditional duration on the present conditional duration. This impact then generally reverses in the second lag. Finally, the Ljung-Box statistics associated with the standardized residuals lie between 12.61 for CEZ and 15.55 for KB. These statistics suggest that the log-ACD model performs relatively well when accounting for the intertemporal dependence in transaction arrival rates.

TABLE 6 Estimated Price Models for Komercni banka

	Model 1	Model 2	Model 3	Model 4	Model 5
ω	0.003 (5.77)	0.032 (7.89)	0.002 (5.60)	0.003 (3.82)	0.005 (4.25)
α_1	0.138 (29.84)	0.138 (29.96)	0.135 (29.18)	0.137 (29.65)	0.133 (28.58)
α_2	–0.197 (–16.97)	–0.004 (–0.39)	–0.193 (–16.76)	–0.196 (–16.91)	–0.187 (–16.07)
α_3	0.062 (7.44)	–0.102 (–8.71)	0.060 (7.39)	0.061 (7.41)	0.057 (6.90)
β_1	1.737 (51.79)	0.090 (1.21)	1.740 (53.81)	1.737 (51.84)	1.729 (49.42)
β^2	–0.739 (–22.30)	0.872 (11.89)	–0.741 (–23.24)	–0.740 (–22.34)	–0.732 (–21.26)
y_1		0.008 (3.01)			0.000 (–0.25)
y_2			0.001 (4.88)		0.001 (4.68)
y_3				–0.000 (–0.29)	–0.002 (–2.67)
$Q(10)$	15.55	28.36	11.87	15.35	11.78

Notes: Coefficient estimates and *t*-statistics (in parentheses) for estimated transaction duration model (model 1) and price duration models (model 2 to 5) for Komercni banka.

Coefficient y_1 corresponds to first explanatory variable (intensity of trading), y_2 to average volume per trade, and y_3 to spread.

Robust *t*-statistics are given in parentheses.

$Q(10)$ is the Ljung-Box Q-statistics for serial correlation in standardized residuals for up to ten lags.

Turning our attention to price duration models with explanatory variables, we can immediately observe that a great majority of coefficients is again statistically significant. The coefficients on lagged durations (α_i) tend to exhibit the same pattern with lag-1 duration having positive and lag-2 duration negative impact on the expected price duration.

Model 2 includes the lagged intensity of trading as an additional explanatory variable to test for the hypothesis that the number of transactions would influence the price process through information-based clustering of transactions. This coefficient is statistically insignificant in case of CEZ. For the other two stocks, the coefficients are positive for KB and negative for Telecom. The negative sign means that the expected price durations are shorter, and equivalently the volatility is higher, following the periods of higher transaction rates. Thus, only Telecom is in accordance with the hypotheses put forward by Easley and O'Hara (1992).

The same results holds true when the second explanatory variable – lagged average volume per trade – is added to the log-ACD model (Model 3). Although this time all coefficients are statistically significant, they are positive in two out of three cases (CEZ and KB). Consequently, only Telecom's result is in agreement with the hypothesis that higher average volume shortens expected duration.

Finally, the coefficients on third explanatory variable – lagged average

TABLE 7 Estimated Price Models for Cesky Telecom

	Model 1	Model 2	Model 3	Model 4	Model 5
ω	0.024 (18.45)	0.027 (17.45)	0.033 (20.71)	0.047 (18.91)	0.051 (19.32)
α_1	0.173 (68.81)	0.174 (67.50)	0.174 (68.97)	0.173 (66.93)	0.175 (66.70)
α_2	–0.158 (–62.42)	–0.160 (–63.48)	–0.158 (–61.60)	–0.156 (–60.15)	–0.158 (–60.03)
β_1	1.314 (105.5)	1.330 (112.7)	1.267 (100.91)	1.268 (95.66)	1.260 (99.59)
β_2	–0.340 (–29.91)	–0.354 (–32.53)	–0.300 (–26.34)	–0.302 (–25.12)	–0.294 (–25.50)
y_1		–0.004 (–8.73)			–0.003 (–6.13)
y_2			–0.007 (–17.91)		–0.005 (–13.20)
y_3				–0.018 (–15.77)	–0.015 (–13.17)
Q(10)	14.25	12.55	16.87	12.81	13.78

Notes: Coefficient estimates and *t*-statistics (in parentheses) for estimated transaction duration model (model 1) and price duration models (model 2 to 5) for Cesky Telecom.
Coefficient y_1 corresponds to first explanatory variable (intensity of trading), y_2 to average volume per trade, and y_3 to spread.
Robust *t*-statistics are given in parentheses.
Q(10) is the Ljung-Box Q-statistics for serial correlation in standardized residuals for up to ten lags.

spread – are statistically significantly negative only for CEZ and Telecom (Model 4). Thus, the hypothesis that higher average spread shortens expected duration is validated with these two stocks. For KB, the coefficient is insignificant and the results remain inconclusive.

When all three additional variables are included into the ACD model at the same time (Model 5), the results are again mixed. As the coefficients keep their signs, it is only in case of Cesky Telecom that they are all negative. This time, however, all of them are highly significant. Obviously, in CEZ and KB only the coefficients on the average spread variable are negative as in the previous Model 4.

From these results it would seem that only spread seems to have a consistent negative impact on expected duration. In absolute value, the influence of this variable is also the most pronounced of all three additional explanatory variables included in the model. We also note that the likelihood ratio test statistics for joint significance of explanatory variables are 17.44 (p-value of 0.0006) for CEZ and 2.20 (p-value of 0.5319) for KB. Interestingly, it is the highest for Telecom (67.59 with p-value of 0.0000).

The log-ACD model shows an evident success in removing the autocorrelation in the price duration data. For CEZ, the Ljung-Box Q-statistic of order 10 has been reduced from 594.3 (see Table 4) to around 12 although in this case, the residuals are somewhat influenced by the additional explanatory variables. The results are similar to the other two stocks in this regard. The duration process also continues to exhibit a strong persistence throughout the stocks although such conclusion should be considered carefully as price durations are much longer on average than transaction durations.

6. Conclusions

In this study, we apply the logarithmic version of the original ACD model developed by Engle and Russell (1998) to price duration process of three of the most liquid securities traded on the Prague Stock Exchange in order to examine whether the intensity of bid-ask quote arrivals carries any information about the state of the market. The preliminary empirical analysis provides evidence of clustering effect in price durations: that is, short (long) durations tend to be followed by short (long) durations, respectively. In fact, we show that large autocorrelations in price durations tend to persist even after the time-of-day effects have been removed from the process.

We take the duration analysis a step further when we empirically test the predictions of market microstructure as demonstrated by Bauwens and Giot (2000) and Engle and Russell (1998). Given the price durations are "longer" than transaction durations, we can define three market characteristics as suggested by the information-based models of market microstructure in order to test for the hypotheses. The variables concerned are the intensity of trading, the average volume per trade, and the average spread. We obtain the following empirical results: of the three variables, only the average spread seems to have a consistent negative impact on the next expected duration among all stocks. With the other two variables, the results are not clear. Both the coefficients on the intensity of trading and the average volume per trade remain positive when tested individually/jointly in two of the three stocks analyzed. In abstract, our results tend to favor the conclusions of information models, however any straightforward judgments remain at best ambiguous.

In the estimated empirical model we used only three of a large number of variables possible to test for the hypotheses of market microstructure. Other possibilities include the depth of the bid and ask, or the changes in price or in spread. The model could also be extended in form of the explanatory variables. For example, we could include their non-linear transformations. This and similar topics are left for further empirical research.

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SUMMARY

JEL: G14, G18
Keywords: autoregressive conditional duration; instantaneous volatility; market microstructure

Trading Intensity and Intraday Volatility on the Prague Stock Exchange: Evidence from an Autoregressive Conditional Duration Model

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Using trade and quote data from the Prague Stock Exchange, this study investigates the empirical behavior of price durations defined as the time needed for a quote midpoint to move by a given amount. Focusing on the three most liquid securities traded on the exchange – Cesky Telecom, CEZ, and Komercni banka – the authors estimate autoregressive conditional duration (ACD) models for price-duration series and test several market-microstructure hypotheses suggested by the information-based models of market microstructure. Similar to earlier studies, the authors find that price durations exhibit diurnal patterns, overdispersion, and substantial persistence, which can be adequately captured by the ACD model. The market-microstructure hypotheses, however, find little empirical support in the authors' results.