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Skewness in Financial Returns: Evidence from the Portuguese Stock Market

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1. Introduction

Utility maximization is the principle behind investment choice. The conventional mean-variance (MV) equilibrium framework (two-parameter model) requires either the normality of the return distributions or quadratic utility functions. Researchers have proposed different statistical distributions for pricing financial assets. However, the pertinence of symmetry analysis exceeds the pure determination of the statistical distributions. The traditional CAPM assumes that investors care only about the mean and variance of returns, implying that upside and downside risks are viewed with equal distaste. Other authors have shown that CAPM-based valuation measures are problematic when market timing strategies and their subsequent non-normal returns are considered. Also, investors typically distinguish between upside and downside risk. Thus, the basic underpinnings of the CAPM are suspect, and its risk measure beta is equally dubious.

Models that allow for some asymmetry of the returns (two or three-parameter models) and require logarithmic or cubic utility functions have been proposed. Rubinstein (1976) attempted to model the asymmetry in a portfolio context by deriving an equilibrium pricing formula "similar" to the traditional CAPM, although under the assumption that the market portfolio returns follow a lognormal distribution. However, the lognormal curve, although allowing for some asymmetry, "is a two-parameter family of distribution. In this sense, the lognormal is just as restrictive as the normal." (Sortino – Forsey, 1996, p. 38) Moreover, over relatively short time periods both approaches will yield identical estimates for systematic risk, which, in turn, will provide equal performance estimates. Markowitz (1959), realizing that investors frequently associate risk with the failure to achieve some minimum target return, offered an alternative risk proxy known as smivariance, which is calculated only in those periods where the returns are

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less than the mean. This downside risk characteristic is recognized by Markowitz (1991) who states that "smivariance seems more plausible than variance as a measure of risk, since it is concerned only with adverse deviations". Also Harlow (1991) concludes that deviations below some fixed value must be earned at a minimum in order to prevent bad outcomes. However, Sortino and Forsey (1996) although believing that downside risk is a valuable measure of risk, also recognize that smivariance does not provide all the information necessary to manage risk.

Alternatively, some financial models allow skewness to affect the required return of financial assets. For example, Kraus and Litzenberger (1976), (1983) constructed a three-moment capital asset pricing model that includes the effect of skewness on valuation. Arguments in favour of three-moment portfolio choice rely on the fact that it is appropriate for investors who have cubic utility functions, that is, their risk aversion decreases as wealth increases and, consequently, have preference for positive skewness.

Therefore, based upon the arguments that other (higher) moments of return distributions are not negligible, at least in major international financial markets (e.g., USA, UK and Japan), it is unreasonable to assume that investors will ignore them as implied by the quadratic utility assumption. Consequently, the objective of this study is to extend the previous work and investigate whether such results hold in small markets. The remainder of the paper is organized as follows: Section 2 presents the relevant literature on skewness. Section 3 describes the data and methodology. Section 4 discusses the tests and the empirical results. Concluding remarks are provided in the last section (Section 5).

2. Literature Review

Since the seminal article of Markowitz (1952) and the extensions of Tobin (1958), most contributions to securities analysis have been based on the MV approach, which provided the basis for the theory of the valuation of risky assets by Sharpe (1964) and Lintner (1965). The requirements for MV to successfully describe the behaviour of such assets have been widely investigated and, in general, one of the conditions that provides for the validity of the theory, is the possibility that the investment returns follow a normal distribution. Thus, MV utilizes the first two distribution moments – the expected return and the variance – of each asset being considered.

The use of the conventional MV approach imposes strong assumptions regarding the investors' utility function and the shape of return distribution, i.e., MV requires that the investors' utility function is quadratic and all return distributions are normal. These requirements have seriously damaged the potential applicability of the MV. Difficulties with the quadratic utility function were pointed out by Hanoch and Levy (1970) and Levy and Sarnat (1972). One major drawback of this function is that it is not defined over the entire range of possible outcomes, and implies increasing absolute risk aversion which conflicts with general acceptance – see (Pratt, 1964), (Arrow, 1964, 1971).

Such objections have introduced a growing debate among researchers over the issue of whether higher moments should be accounted for securities valuation. Despite this, academics and practitioners have used MV measures to manage and access the valuation of individual stocks and portfolios, many researchers ((e.g., (Arditti, 1967, 1971), (Samuelson, 1970) and (Rubinstein, 1973)) have argued that the higher moments cannot be neglected unless there is a reason to believe that the asset returns are normally distributed and the utility function is quadratic, or that the higher moments are irrelevant to the investor's decision.

In this context, a number of authors have proposed analysing portfolios on the basis of the first three moments of return distributions, rather than the traditional two moments. The positive sign of the third derivative of the utility function ((Arrow, 1964), (Pratt, 1964)) gives rise to the intuition (assumption) that investors' risk aversion decreases as wealth increases and, therefore, has cubic utility functions.

In fact, Arrow (1971) argues that desirable properties for an investor's utility function are (1) positive marginal utility for wealth, (2) decreasing marginal utility for wealth and (3) decreasing absolute risk aversion. The first two conditions are consistent with MV preference. Arditti (1967) has shown that condition (3) implies preference for positive skewness. The author was one of the first to attempt to model the possible preference for skewness, using a moment investment model to test the impact of some variables on the returns of a wide variety of stocks. Regressions with skewness and variance indicate that both variables are significantly correlated. Arditti provided evidence concerning investors' preference for positive skewness. The reason is that, all else being constant, they should prefer portfolios with a larger probability of very large payoffs, i.e., they maintain full upside potential.

Essential references about the type of empirical returns were provided by Fama and Roll (1968) and Fama (1971). Also, a theoretical work on third moments was developed by Jean (1971), by addressing the question of skewness preference in a portfolio context. Also Arditti (1971), Tsiang (1972) and Arditti and Levy (1975), extending the previous research, found the predicted relationship between skewness and return for individual security and portfolio returns. Later, Beedles and Simkowitz (1978) showed that the relatively low returns generated by high risky (beta) stocks might be attributed to positive skewness.

Recently, further evidence has been provided in several studies ((e.g., (Chunhachinda et al., 1997), (Basci – Zaman, 1998), (Peiró, 1999)), which demonstrate that investors' preference for asymmetry is an extremely important factor for risky assets valuation and should not be ignored. Leland (1999) suggests a new methodology, which was tested by Fernandes and Machado-Santos (2002) with very interesting results when compared with "traditional" methodologies.

3. Data and Methodology

The data used in this study consists of daily returns from the Portuguese Stock Index PSI-20 (PSI) and a sample of 20 stocks selected among the most representative of the Portuguese Stock Market (as they belong to the PSI composition), displaying high market values and, as a consequence, changes in its prices have altered drastically the market expectations (and returns), for which they can be considered fairly representative of this market.

Concerning the returns estimation, as pointed out by Strong (1992, p. 353), "there are both theoretical and empirical reasons for preferring logarithmic returns. Theoretically, logarithmic returns are analytically more tractable when linking together sub-period returns to form returns over long intervals. Empirically, logarithmic returns are more likely to be normally distributed and so conform to the assumptions of the standard statistical techniques." Thus, if the returns of the individual stocks (adjusted for dividends)[1] and PSI are computed in logarithmic form, it reinforces our analysis, which we intend to test, precisely, of whether they are normally distributed or, instead, show evidence of skewness (asymmetry). The computation formula is as follows:

$$R_{i,t} = Ln\left(\frac{P_{i,t} + D_{i,t}}{P_{i,t-1}}\right) \tag{1}$$

where $R_{i,t}$ is the return of security i in period t; $P_{i,t}$ is the price of security i at the end of period t; $D_{i,t}$ are the dividends paid by security i during period t and $P_{i,t-1}$ is the price of security i at the end of period t–1[2]. Thus, it provided a final time series of daily returns with a range of 1043 to 1251 observations for each stock, from the period of March 1997 (approximately) to March 2002. The returns on the PSI-20 were obtained similarly to equation (1), except that dividends were not considered, as the Index already includes such adjustments. The series also has daily returns from the period of March 1993 to March 2002, in a total of 2265 observations.

Concerning the skewness estimation, and according to Peiró (1999), "the returns are symmetric about μ if (for any k) $f(\mu + k) = f(\mu - k)$, where f is the density function of the returns. If this relation is true, then μ is the mean of the distribution and coincides with the median." However, to test for symmetry, most authors have used several terminologies for skewness ((e.g., (Aggarwal et al., 1989)). Nevertheless, as can be proved, they result exactly in the same standardized skewness estimates broadly presented in the literature. In fact, with regard to skewness defined as the third central moment, most authors use the term skewness and third central moments to indicate asymmetry in general. To be exact, skewness should be computed as the average cubic deviation from the sample mean, divided by the standard deviation raised to the third power. Thus, we have calculated the sample skewness as follows:

$$Sk = \frac{\frac{1}{N}\sum_{t=1}^{N}\left(R_t - \bar{R}\right)^3}{\sigma^3} \tag{2}$$

[1] The stock dividends were considered exactly in the respective payment date.

[2] The price of security i at the end of period $t-1$ should be corrected for any capital adjustments in order to make it comparable to $P_{i,t}$ (Strong, 1992). In our sample, three securities (BCP, BPI and TLE) have been subjected to such capital adjustments due to a stock split.

TABLE 1 Sample Return Statistics

Sample	*N*	Mean (in %)	*t*-stat	Std. Dev. (in %)	Skew-ness	Sk S.E.	Kurtosis	St-R	J-B
BCP	1 195	0.107	1.562	1.779	0.152	0.092	4.087	9.703	70
BES	1 205	0.088	1.312	1.843	0.009	0.091	4.233	10.568	85
BPA	1 121	0.074	0.952	2.713	4.742	0.097	55.138	16.443	134 799
BPI	1 181	0.068	0.968	2.128	0.139	0.093	3.521	9.876	20
BRI	1 066	0.074	1.453	1.536	0.429	0.102	1.971	8.171	73
BSM	1 209	0.187	2.365	2.217	0.621	0.091	3.156	8.641	70
CPR	1 043	0.003	0.106	1.572	0.169	0.104	5.345	11.229	260
EDP	1 179	0.038	0.136	1.713	0.600	0.093	2.848	8.766	62
JMT	1 180	0.001	0.218	2.292	0.737	0.093	6.875	10.097	873
MCF	1 148	0.207	1.926	2.682	1.962	0.095	14.562	12.964	7 288
MEL	1 047	0.056	0.698	2.205	1.593	0.103	14.850	14.645	6 750
MOC	1 115	0.098	1.023	2.487	2.517	0.098	34.824	16.699	49 674
PTC	1 251	0.140	1.865	2.146	0.212	0.088	2.509	8.122	19
PTI	1 195	0.043	0.623	2.019	0.828	0.092	4.052	8.723	181
PTM	1 047	0.012	0.083	1.578	0.159	0.103	5.185	10.245	213
SEM	1 039	0.055	0.762	1.813	0.610	0.104	6.616	11.190	655
SOA	1 059	0.032	0.418	2.071	–0.017	0.102	5.729	11.682	350
SON	1 120	0.095	1.207	2.119	0.238	0.097	4.107	9.592	73
SOP	1 117	0.007	0.089	2.232	0.172	0.097	5.259	10.144	260
TLE	1 209	0.173	1.791	2.804	0.119	0.091	3.148	9.191	4
PSI	2 265	0.086	3.452	1.051	–0.880	0.057	10.521	15.356	5 825

Notes: The skewness was calculated as shown in equation (2) and the Kurtosis as m_4/S^4. For the significant tests on the individual mean return estimates we used the standard *t*-statistics, which were obtained by dividing the mean over all (daily) periods by the standard error of the mean. The remaining tests were computed according to Peiró (1999), where the Studentized Range (St-R) is (max $\{Rt\}$ – min$\{Rt\}$)/s and Jarque-Bera (J-B) is $N(Skewness^2/6 + (Kurtosis\text{-}3)^2/24)$ where N is the number of observations. The Standard Errors (Sk S.E.) of the skewness estimates under the null hypothesis of normality were computed as $(6/N)^{1/2}$. (The asymptotic distribution of the sample skewness, under normality, is given by $Sk \rightarrow Normal\ (0; 6/N)$.

where, N is the number of observations, R_t is the return of period t, $\bar{R}$ is the arithmetic mean of returns and σ is the standard deviation of returns.

4. Empirical Evidence

Some preliminary descriptive statistics and previously performed tests computed for the 20 stocks and the PSI index, using the sample data earlier defined, are summarized in *Table 1*. First, we can observe that the mean sample returns for all the individual stocks is positive, although not significant at the 5% level, i.e., we cannot reject the hypothesis of zero mean returns for 19 of them. On the contrary, the mean sample returns of the PSI index are significantly positive. More relevant to the analysis, and although the tests presented (St-R and J-B) reject the normality hypothesis of the distribution for most of the sample returns, a simple inspection of the skewness and kurtosis estimates, with a few exceptions (BPA, MCF, MEL and

MOC), appear not to be far from the values of the standardized normal distribution parameters.

In such a context, for a more consistent analysis, we tested whether the returns reject the normality assumption under other conditions. To that purpose, a χ-Square test was run, with the stocks and PSI returns tested against alternative distributions. The χ-Square test for goodness-of-fit is a measure of how well the sample data fits a hypothesized probability density function (e.g., the normal distribution), allowing the comparison between observed and expected frequencies (Gujarati, 1995). It was observed[3] that at a 5% confidence level the null hypothesis of normality was clearly rejected in relation to the whole sample. The ranking test positioned most of the stocks and the PSI in the alternative Logistic and Loglogistic distributions (which allow for some asymmetry) as the ones that best fit their returns.

However, one of the weaknesses of the χ-Square test is that there are no clear guidelines for selecting intervals, and different conclusions can be reached from the same data, depending on how intervals have been specified. Therefore, and because of the extreme importance that symmetry assumes in our study, we have also run other goodness-of-fit tests, such as the Kolmogorov-Smirnov, which tends to be more powerful than χ-Square tests against many alternative distributions, and is defined as the maximum difference between the cumulative distribution from the actual sample and the cumulative distribution of the hypothesized (assumed) population (Kanji, 1995). This test works by comparing an empirical distribution function with the distribution of the hypothesized function and does not require us to group the data in any way, being valid for any sample size n when all parameters are known. The results reveal that under this distribution and at a 5% confidence level the null hypothesis of normality was rejected in relation to all the stocks and PSI, despite some statistics being closer to the critical value. Once again, the rank positioned most of the sample in the Logistic, Loglogistic and Lognormal distributions.

Further, the Anderson-Darling test was run as a complement to the previous two, because it is designed to detect discrepancies in the tails of distributions. The statistic is defined by:

$$A_n^2 = n \int_{-\infty}^{\infty} \left[F_n(x) - \hat{F}(x)\right] \psi(x)\, \hat{f}(x)\, dx \tag{3}$$

where the weight function is:

$$\psi(x) = 1/\left\{\hat{F}(x) \left\lfloor 1 - \hat{F}(x) \right\rfloor\right\} \tag{4}$$

and $F_n(x)$ is the cumulative distribution from the actual sample and $\hat{F}(x)$ is the cumulative distribution of the assumed population. Therefore, A_n^2 is just the weighted average of the squared differences, where the weights are larger close to either tail (Anderson – Darling, 1954). The results seem to reinforce even more the normality rejection of the sample returns. In fact,

[3] Full results concerning all the applied tests can be provided upon request.

TABLE 2 *F*-Test and Kruskal-Wallis Test for Differences between Positive and Negative Returns

Sample	*F*-stat	*p*-value	*H*-stat	*p*-value	Sample	*F*-stat	*p*-value	*H*-stat	*p*-value
BCP	11.109	0.00089	19.374	0.00001	MOC	8.057	0.00462	14.098	0.00017
BES	10.617	0.00115	18.543	0.00002	PTC	12.647	0.00039	22.099	0.00000
BPA	6.063	0.01395	11.858	0.00057	PTI	8.472	0.00367	14.834	0.00012
BPI	11.804	0.00061	20.563	0.00001	PTM	11.109	0.00089	18.987	0.00001
BRI	9.211	0.00246	16.069	0.00006	SEM	14.493	0.00015	25.603	0.00000
BSM	8.372	0.00388	14.667	0.00013	SOA	11.039	0.00092	19.178	0.00001
CPR	13.187	0.00030	22.975	0.00000	SON	9.890	0.00171	17.258	0.00003
EDP	8.480	0.00366	14.844	0.00012	SOP	11.623	0.00067	20.205	0.00001
JMT	14.023	0.00019	24.124	0.00000	TLE	10.763	0.00107	18.790	0.00001
MCF	6.976	0.00837	12.233	0.00047	PSI	14.838	0.00012	25.603	0.00000
MEL	7.570	0.00604	13.235	0.00027					

at a 5% confidence level the null hypothesis of normality was rejected, again, for the entire sample.

In such a context, and where the sample returns do not clearly conform to the normal distribution, it seems important to verify whether there is asymmetry around the mean, in order to get deeper insights into the skewness analysis. According to the literature, the rejection of normality does not imply the rejection of symmetry. Following the procedure proposed by Peiró (1999), we have created two sub-samples for each series. One is formed by negative excess returns in absolute values, and the other formed by positive excess returns:

$$|R^{-}| = \{\bar{R} - R_t | R_t < \bar{R}\}; \quad R^{+} = \{R_t - \bar{R} | R_t > \bar{R}\} \tag{5}$$

Thus, to detect symmetry, the same distribution has to be observed for the excess returns of the two sub-samples. To test the hypothesis of significant differences between the two pairs (sub-samples), we carried out the parametric F-test and the non-parametric Kruskal-Wallis rank test. Although each one checks for different mean values among various populations, the primary difference is the assumption of the nature of the distributions for the test variable. Therefore, because of their complementarily, the use of the two statistics seems appropriate.

The results of the parametric F-test are shown in *Table 2*. Analyzing the statistics presented, the null hypothesis of equality between positive excess returns and negative excess returns is rejected for all the individual stocks as well for the PSI index at the 5% significant level. Moreover, the null hypothesis of equal means cannot be rejected for only one stock (BPA) at the 1% significant level. In this context, by displaying a different dispersion in the two sub-samples, such results provide strong evidence of asymmetry in the return distributions, where statistically significant differences of returns above and below the mean are detected.

The results of the Kruskal-Wallis test, which is a distribution-free test and assumes only similar distributions among the population groups, are also shown in Table 2. The statistics (*H*-stat) seem to show, once again, that the null hypothesis of equal positive and negative excess return means is strongly rejected for all the individual stocks and PSI index at the 5% and 1% significant levels. In fact, the *p*-values display very low values for the entire sample, exhibiting the biggest value for the stock BPA (again the stock that shows less asymmetry), but of only about 0.057 %. In such a context, this non-parametric test seems to provide further and stronger evidence of different dispersion in the return distributions, showing that returns above and below the mean are asymmetric.

Therefore, from the statistics presented for all the parametric and distribution-free tests, both the hypothesis of normality (first) and the hypothesis of symmetry around the mean (second) are clearly rejected at very high levels of significance. These results suggest that the observed sample skewness is a consistent finding in the major stocks and stock index returns for the Portuguese market. Given that the sample of our study came from a small market, it is interesting to note that such results confirm the findings of Peiró (1999) where, from a sample of eight international stock market indexes, only Milan and Madrid – low capitalization and trading volume markets – exhibit similar significant asymmetric return distributions. Bekaert et al. (1998) also provide useful findings on the distributional characteristics of returns in emerging markets.

5. Summary and Conclusions

This study addresses the issue of symmetry in financial returns. Traditional methodologies have priced financial assets based simply on the first two moments of the distributions (mean and variance), assuming, implicitly, the normality of their returns. However, a vast literature suggests that the inclusion of the skewness effects on valuation (three-moment models), argues that higher moments of return distributions are not negligible and, therefore, one cannot assume that investors will ignore them as implied by the quadratic utility assumption.

In such a context, this paper investigates the presence of skewness in the distributions of 20 major (daily) stock returns traded in the Portuguese Market as well as the PSI-20 Index. We have verified that the computed sample skewness is positive for 19 individual stocks and negative for the index. As in other studies, we started by computing the parametric Studentized Range and Jarque-Bera tests, which rejected the normality hypothesis of the distributions for most of the sample returns. The symmetry of the returns was also tested against alternative distributions, using goodness-of-fit statistics, for which the results show that the null hypothesis of normality is now rejected for all the stocks and PSI index return distributions.

According to the literature, the rejection of normality does not imply, necessarily, the rejection of symmetry. Thus, we have created two sub-samples for each series, formed by negative and positive excess returns. To test the hypothesis of significant differences between returns below and above the mean, the *F*-test and the Kruskal-Wallis rank test were run. The results provided confirm that the null hypothesis of equal positive and negative excess returns is highly rejected for all the individual stocks and PSI index at the 5% and 1% significant levels.

Summarizing, from the analysis conducted, we may conclude that there is strong evidence of skewed return distributions in the Portuguese Stock Market. Moreover, such findings apply to different time periods[4] and, as mentioned previously, are similar to those of low capitalization and trading volume markets.

Finally, it should be mentioned that more advanced asset pricing approaches, such as those introduced by Glosten – Jagannathan (1994) and Leland (1999), might apply better in this context of asymmetric returns, such as the Portuguese. Also, it would be interesting, as a piece of further research, to extend this study of skewness to differently sized portfolios obtained from the main stock sample.

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[4] Two sub-sample periods were created from the main sample. Applying the same procedures, we verify that the skewness estimates tend to persist and do not change significantly. Full results can be provided upon request.

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SUMMARY

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Skewness in Financial Returns:
Evidence from the Portuguese Stock Market

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This paper addresses the issue of symmetry in financial returns. The return distributions of the major stocks traded on the Portuguese market and included in the PSI-20 Index are examined for periods from four to nine years. The results show that the symmetry of the returns is rejected against several alternative distributions. Statistically significant differences between returns below and above the mean are detected, which provides additional evidence of skewness in the return distributions. In addition, as observed in other studies, it is interesting to note that such results are similar to other low-capitalization and low-volume markets, which also exhibit asymmetric return distributions.